FE Civil Review

with 800 Solved Problems



Based on Version 9.5 of the Reference Handbook

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Preface

I am delighted to introduce this Fundamentals of Engineering (FE) Civil Engineering Review Manual to civil engineering students who want to pass the FE exam smartly with no hardship. It is well known that students are not required to receive 100% or near that. Although the exact passing score is not reported by the National Council of Examiners for Engineering and Surveying (NCEES) or varies batch to batch, the author believes that the passing score is between 50% and 70%. Therefore, it is smarter to prepare such that 70-90% questions can be answered correctly. This book is constructed in a way that such a score can be achieved if students properly master this book. In addition, instead of compiling all the theoretical materials, this book includes more than 800 practice problems. This is done so that students can practice sufficient problems and learn the effective way of using the NCEES Ref. Handbook, v. 9.5 to answer the examination questions.

Few issues found in the first and second editions have been solved. The author is requesting the readers to report any concern about the book at books.mrislam@gmail.com. Any suggestion to improve this book, or any issue reported, will be fixed in the next edition with proper acknowledgement.

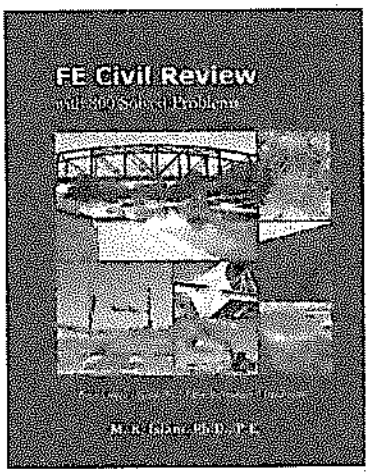
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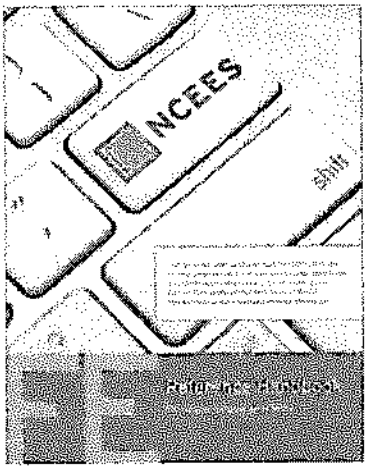
How to Use this Book

To obtain the most benefit from this book, please follow the following steps:

Step 1. Manage the following two books. It is better to have a digital copy of the NCEES Ref. Handbook, v. 9.5.



Current Book



NCEES FE Ref. Handbook

Step 2. Read this book chapter by chapter and practice the solved problems. While doing so, always keep the NCEES FE Ref. Handbook, version 9.5 at your computer for any reference.

Step 3. Try to get accustomed to navigate to any material you need. It is not a good idea to use printed copy the NCEES Ref. Handbook, as you will be using an electronic copy during the actual exam.

Step 4. While practicing problems, use any of the NCEES recommended calculators.

Step 5. Try some samples exams before the NCEES test. One is recommended at the back of this book.

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1 UNITS

This chapter is very important in the sense that a unit is involved in most of the engineering problems. The NCEES FE Ref. Handbook starts with this topic. The NCEES FE exam and the NCEES FE Ref. Handbook use both the metric system and the U.S. Customary System (USCS) units. Therefore, knowledge on both units and conversion factors between them is essential for the NCEES FE exam. Read this chapter from the NCEES FE Ref. Handbook. You do not need to memorize the units or unit conversion factors. However, make sure you know which information is available and at which page it is available. Some important materials are presented in Tables 1.1 to 1.4.

Table 1.1. Some Quantities, Prefix, Symbols, and Symbol

METRIC PREFIXES		
Multiple	Prefix	Symbol
10^{-18}	atto	a
10^{-15}	femto	f
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^{-2}	centi	c
10^{-1}	deci	d
10^{1}	deka	da
10^{2}	hecto	h
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T
10^{15}	peta	P
10^{18}	exa	E

COMMONLY USED EQUIVALENTS	
1 gallon of water weighs	8.34 lb
1 cubic foot of water weighs	62.4 lb
1 cubic inch of mercury weighs	0.491 lb
The mass of 1 cubic meter of water is	1,000 kg
1 mg/L is	8.34 lbf/Mgal

TEMPERATURE CONVERSIONS
°F = 1.8 (°C) + 32
°C = (°F − 32)/1.8
°R = °F + 459.69
K = °C + 273.15
Celsius and Fahrenheit units are particularly important

Problem 1.1

10^6 Pa = ?

A. 1 GPa B. 1 kPa C. 1 μPa D. 1 MPa

Solution: D

10^6 Pa = 1 MPa

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Problem 1.2

20 °C = ?

A. 86 °F B. 58 °F C. 78 °F D. 68 °F

Solution: D

$°F = 1.8\ (°C) + 32 = 1.8\ (20) + 32 = 68\ °F$

Table 1.2. Relationship between US Units and Metric Units

US Units	Metric Units	US Units	Metric Units
1 lb =	454 gm	1 hectare =	10^4 m^2
1 lb =	4.448 N	1 hectare =	2.47 acres
1 ft =	0.3048 m	1 psi =	6.89 kPa
1 mile =	5280 ft	g= 32.2 ft/s^2	g = 9.81 m/s^2
1 inch =	25.4 mm	γ_w = 62.4 pcf	γ_w = 9.81 kN/m^3

Table 1.3. Some Other Popular Quantities

1 ft^3 =	7.48 gallon	1 liter =	61.02 in^3
I gallon =	3.785 liter	1 liter =	10^{-3} m^3
1 ton =	1000 kg	1 bar =	10^5 Pa
1 ton =	2000 lb	1 bar =	0.987 atm
1 liter =	61.02 in^3	1 hp =	746 watt

Table 1.4. Some Common Derived Units

Parameters	Unit	Derivation	Derived Unit
Force	N	F = ma	kg.m/s^2
Work	J	W=Fs	kg.m^2/s^2
Power	watt	P=W/t	kg.m^2/s^3

Problem 1.3

The unit of force is –

A. kg.m^2/s^3
B. kg.m/s^2
C. J
D. watt

Solution: B

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Problem 1.4

45 miles per hour = ?

A. 66 fps
B. 60 fps
C. 45 fps
D. None of the above

Solution: A

$$45\ \frac{\text{mi}}{\text{hr}} = \frac{45\,\text{mi}(5,280\ \frac{\text{ft}}{\text{mi}})}{(1\text{hr})\left(3,600\ \frac{\text{sec}}{\text{hr}}\right)} = 66\ \frac{\text{ft}}{\text{s}} = 66\ \text{fps}$$

Problem 1.5

45 gallons per hour = ?

A. $7.4\text{x}10^{-5}$ m^3/s
B. $7.4\text{x}10^{-5}$ L/s
C. $4.7\text{x}10^{-5}$ m^3/s
D. None of the above

Solution: C

$$45\ \frac{\text{gal}}{\text{hr}} = \frac{45\,\text{gal}(3.785\ \frac{\text{liter}}{\text{gal}})\left(10^{-3}\ \frac{\text{m}^3}{\text{liter}}\right)}{(1\text{hr})\left(3,600\ \frac{\text{sec}}{\text{hr}}\right)} = 0.047\text{x}10^{-3}\ \frac{\text{m}^3}{\text{s}} = 4.7\text{x}10^{-5}\ \frac{\text{m}^3}{\text{s}}$$

Problem 1.6

The unit of work is –

A. $kg.m^2/s^3$
B. $kg.m/s^2$
C. J
D. watt

Solution: C

Problem 1.7

1 meter = ?

A. 36.36 in B. 1.094 ft C. 3.281 ft D. None of the above

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Solution: C

Problem 1.8

1 liter = ?

A. 61 in^3 B. 0.264 gal C. 10^{-3} m^3 D. All of the above

Solution: D

Problem 1.9

1 psi = ?

A. 6.895 Pa B. 2.307 ft of water C. 2.036 ft of Hg D. None of the above

Solution: B

Problem 1.10

1 acre = ?

A. 43,560 sft B. 5,280 ft C. 46,530 sft D. None of the above

Solution: A

Problem 1.11

A pump can extract 10 liters of water per second. The correct expression of it is most nearly:

A. 21.19 ft^3/min
B. 0.6 m^3/min
C. 36,614 in^3/min
D. All of the above

Solution: D

1 L/s = 2.119 ft^3/min; 1 L = 10^{-3} m^3; 1 L = 61.02 in^3; 1 min = 60 sec

The exam appointment time is 6 hours long and includes the following:

a) A nondisclosure agreement (2 minutes)
b) Tutorial (8 minutes)
c) The exam (5 hours and 20 minutes)
d) A scheduled break (25 minutes)

Visit http://ncees.org/engineering/fe/ for more information.

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2 ETHICS

Fundamentals of Engineering (FE) CIVIL Exam Specification mentions, 4–6 questions may appear in the exam based on Ethics and Professional Practice as follows:

A. Codes of ethics (professional and technical societies)
B. Professional liability
C. Licensure
D. Sustainability and sustainable design
E. Professional skills (e.g., public policy, management, and business)
F. Contracts and contract law

NCEES FE Ref. Handbook, Ethics Section, mentions the importance of ethical practice in engineering, the three major sections of the Model rules, and so on. The three major sections of the Model Rules addressed are as follows:

(1) Licensee's Obligation to the Public,
(2) Licensee's Obligation to Employers and Clients, and
(3) Licensee's Obligation to Other Licensees.

Please read the NCEES FE Ref. Handbook, Ethics Section, carefully and practice the following problems.

Problem 2.1

Which code does give immediate and mechanical answers to all ethical and professional problems that an engineer may face?

A. NCEES Ethics Code
B. NSPE Ethics Code
C. Federal Code
D. None of the Above

Solution: D

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No code gives immediate and mechanical answers to all ethical and professional problems that an engineer may face.

Problem 2.2

Given a licensing board requires 4 years of professional experience. A candidate has 3 years and 6 months of professional experience. Can the candidate mention that he has 4 years of experience as it may take 6 months to process the application?

A. Yes, as it is almost 4 years
B. Yes, if the candidate thinks it is safe
C. No, it is against Licensee's Obligation to the Public
D. No, it is not ethical

Solution: C

It is against Licensee's Obligation to the Public. The ninth point of Licensee's Obligation to the Public says, "Licensees shall not knowingly provide false or incomplete information regarding an applicant in obtaining licensure."

Problem 2.3

A professional engineer is in charge of a construction project. The local political leader advises him to hire his recommended person. That recommended person has the minimum amount of credentials. In that case, what should the engineer do?

A. Hire that person as he has the minimum qualifications
B. Should not hire that person as he is a political person
C. Hire that person if the engineer's employer agrees
D. Nothing, it is not ethical

Solution: C

The responsible engineer can hire that person if the engineer's employer agrees. There is no obligation that a political person cannot be hired ethically. As that person has the minimum qualification, the company may hire him.

Problem 2.4

Two professional engineers are working with two different projects which are not connected. The first engineer found anyway that the second engineer's work is believed to contain a material discrepancy, error, or omission that may impact the health, safety, or welfare of the public. In that situation the first engineer should –

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A. do nothing, as he is not responsible for that project
B. do nothing, as he is not the supervisor
C. let the second engineer's supervisor know what is happening
D. let the second engineer aware first what is happening

Solution: D

The 4th point of 'Licensee's Obligation to Other Licensees' says "Licensees shall make a reasonable effort to inform another licensee whose work is believed to contain a material discrepancy, error, or omission that may impact the health, safety, or welfare of the public, unless such reporting is legally prohibited." Therefore, the 4th option should be the answer. The first two options cannot be the answer. The third option can be adopted if the second engineer does not respect the warning of the 1st engineer.

Problem 2.5

Which of the following statements should not be a conflict of interest?

A. A professional engineer in civil engineering signs a major design document in mechanical engineering
B. One of the senior employees of a firm, which is competing for a bid, is hired as a board member of the awarding board
C. The owner of a highway company is nominated the head of the board which awards highway projects
D. Working in an engineering firm without professional engineering license

Solution: D

Working in an engineering firm without professional engineering license should not be unethical. However, he/she should not sign the design document.

Problem 2.6

For which of the following conditions can a professional engineer with expertise in geotechnical engineering sign designs in water resource engineering?

A. If the professional engineer took a course in water resource engineering during his college education
B. If the professional engineer attends all the meetings during the design process
C. If the professional engineer is told that the water resource part of the design is not risky to public anyway
D. If the professional engineer is knowledgeable on that particular water resource part by anyway
E. I don't know it

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Solution: D

If the professional engineer is knowledgeable and competent in that particular water resource, he can sign that part.

Problem 2.7

A design document has two parts: geotechnical and structural. A licensed civil engineer with expertise in geotechnical engineering is in charge of the design and a structural engineer is supposed to look after the structural engineering portion. For this design, which of the following conditions, is ethical?

A. The geotechnical engineer can sign the design document if both of them worked together during the whole design process
B. The geotechnical engineer can sign the design document as both geotechnical and structural engineering are part of civil engineering
C. Geotechnical engineer can sign the geotechnical part and structural engineer can sign the structural engineering part
D. A third engineer is required to sign the document

Solution: C

If the geotechnical engineer is competent in structural part as well, then he can sign both parts. Otherwise, each of them should sign their own part.

Problem 2.8

Which of the following statements is not a reason of setting up ethical standards?

A. Ethical practice is safe for public
B. Ethical practice gives public a confidence in engineering works
C. Guidelines help engineers work smoothly without hesitation
D. Ethical practice helps reduce police actions

Solution: D

The first three statements are obviously true for setting up ethical standards. Although ethical practice reduces police action, it is not the reason for setting up ethical standards.

Problem 2.9

Which of the following is not a profession?

A. Financial consultant B. Law practicing C. Politics D. Engineering

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Solution: C

Medical practicing, law practicing, engineering, teaching, financial consultant etc. are obviously professions.

Problem 2.10

Ethics are –

A. rules set by the law enforcing agency
B. standard rules or moral codes set by a professional society
C. rules and regulation set by the government which must be followed by all
D. codes to maintain balance and peace in society and protection to its citizens

Solution: B

Ethics are moral codes or guidelines every person is expected to follow in a professional society. They come from people's moral sense and are not as strict as laws. If not followed, there may not be any punishment immediately. However, after a certain level of disobedience, a cancellation of licensure or other penalty may be imposed.

Problem 2.11

As an engineer, when you are asked to testify as an expert, your testimony should be based on-

A. What your client's lawyer has recommended
B. Only the points asked by the judge
C. Objective and well-founded observations
D. Only those points that you believe need to be revealed

Solution: C

Problem 2.12

A structural engineer working for a design firm has decided to move to another company few months before he leaves. The engineer –

A. should discuss his plans with his current employer
B. may share with other employees
C. should immediately quit
D. should return all of the pens, pencils, pads of paper, and other equipment he had brought home over the years

Solution: A

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Problem 2.13

Licensees who have knowledge or reason to believe that any person or firm has violated any rules or laws applying to the practice of engineering or surveying -

A. shall report it to the board
B. may report it to appropriate legal authorities
C. shall cooperate with the board and those authorities as may be requested
D. All of the above

Solution: D

Rule #8 of the Licensee's Obligation to the Public

Problem 2.14

Licensees shall notify their employer or client, and such other authorities as may be appropriate, when their professional judgment is overruled under circumstances in which -

A. the health of the public is endangered
B. safety of the public is endangered
C. welfare of the public is endangered
D. all of the above

Solution: D

Rule #3 of the Licensee's Obligation to the Public

Problem 2.15

Licensees shall not reveal facts, data, or information obtained in a professional capacity without the prior consent of the -

A. client on which they serve, except as authorized or required by law or rules
B. employer on which they serve, except as authorized or required by law or rules
C. public body on which they serve, except as authorized or required by law or rules
D. All of the above

Solution: D

Rule #4 of the Licensee's Obligation to Employer and Clients

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Problem 2.16

Which of the following statements is not true about profession?

A. Professions are based on a large knowledge requiring extensive training
B. Professions are self-regulating; they control the training and evaluation processes
C. Professionals have autonomy in the workplace
D. Professionals are expected to utilize their dependent judgment in their professional responsibilities

Solution: D

NCEES FE Ref. Handbook, the first three statements are obviously true. The 4th one will be "... independent judgment in carrying out their professional responsibilities."

Problem 2.17

Which of the following statements is not true about engineers?

A. The expertise possessed by engineers is vitally important to societal welfare
B. Engineers must maintain some level of technical competence
C. High level of technical expertise without adherence to ethical guidelines is as much a threat to public welfare as is professional incompetence
D. Engineers must also be guided by ethical principles

Solution: B

NCEES FE Ref. Handbook: The second option should be "engineers must maintain a high level of technical competence." Although, the term 'some' in place of 'high' may be acceptable, however, the NCEES FE Ref. Handbook uses 'high'. The other three options are good with the NCEES FE Ref. Handbook.

Problems 2.18

Engineering licensees have an obligation to –

A. Public
B. Employers and Clients
C. Other Licensees
D. All of the above

Solution: D

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NCEES FE Ref. Handbook: Engineering Licensees have obligation to the public, his/her employers, his/her clients, and other licensees as well.

Problem 2.19

Which of the following statements is not true?

A. Registration offers protection from misuse by unauthorized groups
B. A patent grants legally protected rights to the owner of the item being patented
C. Copyrights apply to original literary, dramatic, musical, or artistic works
D. Patents protect the appearance of new products

Solution: D

NCEES FE Ref. Handbook: The first statements are true. The 4th one will be - Industrial design rights protect the appearance of new products. This includes shapes, colors, and any other visual features.

Problem 2.20

To be eligible for patent, the item need not be –

A. New
B. Useful
C. Inventive
D. Surprising

Solution: D

NCEES FE Ref. Handbook: To be eligible for patent, the item must be new, useful, and inventive.

Problem 2.21

Why shall the rules of professional conduct be binding upon every licensee and on all firms authorized to offer or perform engineering or surveying services?

A. To safeguard the health, safety, and welfare of the public
B. To maintain integrity and high standards of skill
C. To practice in the engineering and surveying professions
D. All the above

Solution: D

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NCEES FE Ref. Handbook: "To safeguard the health, safety, and welfare of the public and to maintain integrity and high standards of skill and practice in the engineering and surveying professions, the rules of professional conduct provided shall be binding upon every licensee and on all firms authorized to offer or perform engineering or surveying services."

Problem 2.22

Which of the following is not the Licensee's Obligation to the Public?

A. Licensees shall include all relevant and pertinent information in an objective and truthful manner
B. Licensees shall not partner, practice, or offer to practice with any fraudulent or dishonest business
C. Licensees shall make a reasonable effort to inform another dishonest licensee
D. Licensees shall not provide an applicant incomplete information in obtaining licensure

Solution: C

NCEES FE Ref. Handbook: The third statement (option C) is the Licensee's Obligation to Other Licensees. The other three statements are Licensee's Obligation to the Public.

Problem 2.23

Which of the following is not the Licensee's Obligation to Employer and Clients?

A. Licensees shall not reveal facts, data, or information obtained in a professional capacity without the prior consent of the client
B. Licensees shall sign and seal only those surveys that conform to accepted surveying standards
C. Licensees shall not affix their signatures or seals to any plans or documents dealing with subject matter in which they lack competence
D. Licensees shall not solicit or accept a professional contract from a governmental body on which a principal or officer of their organization serves as a member

Solution: B

NCEES FE Ref. Handbook: The statement (Option B) is the Licensee's Obligation to the Public. The other three statements are the Licensee's Obligation to Employer and Clients.

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Problem 2.24

A professional engineer worked to collect the feasibility data on a proposed water treatment plant. He completed the collection and officially submitted it to the employer. A third party, who is not involved in that project, is asking for the data to that engineer. The engineer may reveal the data --

A. with the prior consent of the employer
B. if authorized by the law
C. if required by law
D. All of the above

Solution: D

NCEES FE Ref. Handbook: 4th item of the Licensee's Obligation to Employer and Clients - Licensees shall not reveal facts, data, or information obtained in a professional capacity without the prior consent of the client, employer, or public body on which they serve except as authorized or required by law or rules.

"The NCEES FE Reference Handbook is the only reference material that can be used during the exam. You will be provided with an electronic reference handbook during the exam. For access prior to your exam, you may either purchase a hard copy or download a free electronic copy. Register or log in to MyNCEES to download your free copy of the FE Reference Handbook." Visit http://ncees.org/engineering/fe/ for more information.

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3 MATHEMATICS

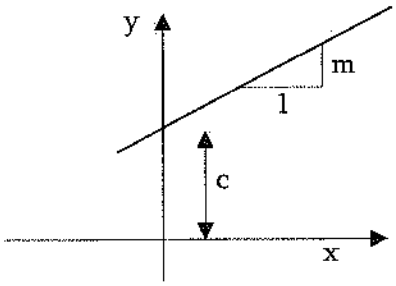
Fundamentals of Engineering (FE) CIVIL Exam Specification mentions, 7-11 questions may appear in the exam based on Mathematics as follows:

A. Analytic geometry
B. Calculus
C. Roots of equations
D. Vector analysis

It is important to remind the students again that the NCEES FE Ref. Handbook is to be used by students of all engineering disciplines. Therefore, the NCEES FE Ref. Guide is written considering all disciplines' students. The topics which are important for Civil Engineers are discussed in this review manual.

Straight Line

The general form of a straight-line equation is expressed as $Ax + By + C = 0$. It is also written in the slope-intercept form as $y = mx + c$.



$$m = \text{slope} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

If the straight-line passes through (x_1, y_1) and (x_2, y_2), then:

The point-slope form is $y - y_1 = m(x - x_1)$, where slope, $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

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The angle between two lines with slopes m_1 and m_2 is, $\alpha = \tan^{-1}\left[\frac{m_2 - m_1}{1 + m_1 m_2}\right]$

Two lines are perpendicular if $m_2 = -\frac{1}{m_1}$

Two lines are parallel if $m_2 = m_1$

The distance between two points A(x_A, y_A) and B A(x_B, y_B) is $d = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2}$

In a three-dimensional space, the distance between two points A(x_A, y_A, z_A), and B(x_B, y_B, z_B) is

$$d = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2 + (z_A - z_B)^2}$$

Problem 3.1

A straight line passes through the points, (2, 3) and (6, 7). The equation of the line is most nearly:

A. $y = \frac{4}{3}x + 5$ B. x – y + 1 =0 C. x + y + 1 =0 D. x - y -1 =0

Solution: B

Slope of the line, $m = \frac{\Delta y}{\Delta x} = \frac{7-3}{6-2} = \frac{4}{4} = 1$

Therefore, the equation of the line is $y - y_1 = m(x - x_1)$

⇨ $y - 3 = 1(x - 2)$

⇨ $x - y + 1 = 0$

Alternative: you can plug in the points on each equation and check.

For example, plugging (2, 3) in option (A): $3 = \frac{4}{3}(2) + 5$; Not Good

plugging (2, 3) in option (B): 2 – 3 + 1 = 0; Good, (B) may be the answer.
plugging (6, 7) in option (B): 6 – 7 +1 = 0; Good, (B) is the answer.

You can check options (C) and (D) if you can manage time during the exam.

Problem 3.2

If the straight lines, $3x - 4y + 1 = 0$ and $kx + 3y + 5 = 0$, are parallel to each other, then the value of 'k' is most nearly:

A. 3/5 B. -9/4 C. ¾ D. 2/5

Solution: B

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Equation of the 1st line,
$3x - 4y + 1 = 0$
$\Rightarrow -4y = -3x - 1$
$\Rightarrow y = 3/4x + 1/4$
Slope of the first line, $m_1 = ¾$

Equation of the 2nd line,
$kx + 3y + 5 = 0$
$\Rightarrow 3y = -kx - 5$
$\Rightarrow y = -k/3x - 5/3$
Slope of the first line, $m_2 = -k/3$

To be parallel, $m_1 = m_2$
$\Rightarrow -k/3 = 3/4$
$\Rightarrow k = -9/4$

Quadratic Equation

The general form of quadratic equation is $ax^2 + bx + c = 0$

The roots of this equation, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Sometimes, the quadratic equation can be solved if it can be written as –

$x^2 + 2xa + a^2 = 0$, then $(x + a)^2 = 0$ or
$x^2 - 2xa + a^2 = 0$, then $(x - a)^2 = 0$

Problem 3.3

The intersection point on the x-axis of the equation, $y = x^2 - 6x + 9$ is_____.

[write the x-coordinate in integer such as 12, or -6, etc.; not 3.5 or -7.3, etc.]

Solution: 3

Intersecting x-axis means, $y = 0$
$x^2 - 6x + 9 = 0$
$\Rightarrow x^2 - 3x - 3x + 9 = 0$
$\Rightarrow (x - 3)(x - 3) = 0$
$\Rightarrow x = 3$

Problem 3.4

Two straight lines $3x + 4y + 7 = 0$ and $6x + y + 7 = 0$ intersect. The intersection point is most nearly:

A. (-1,-1) B. (1,1) C. (-1,1) D. (1,-1)

Solution: A

$3x + 4y + 7 = 0$ (i)

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$6x + y + 7 = 0$ (ii)

Multiplying Eq. (i) by 2 and then subtracting from Eq. (ii) gives-

$6x + y + 7 = 0$ (ii)

$6x + 8y + 14 = 0$ (i)x(2)

After the subtraction, $-7y - 7 = 0$; => $y = -1$

From Eq. (i), $3x + 4(-1) + 7 = 0$; => $x = -1$

Problem 3.5

The solution of the following systems of equations is most nearly:

$$2x + 3y + 10 = 0$$
$$-3x + y - 15 = 0$$

A. (-5, 5) B. (-5, 0) C. (0, -5) D. (5, -5)

Solution: B

For this type of problem, it is easier to solve by placing the options into the equations and check.

For example, place the first option (-5, 5) into the equations and see-

First eq., $2(-5) + 3(5) + 10 \neq 0$, so option (A) is not the solution.

Now, check option (B):

First Equation: $2(-5) + 3(0) + 10 = 0$; okay

Second Equation: $-3(-5) + 0 - 15 = 0$; okay

Therefore, Option (B) is the answer. You can check the 3rd and the 4th options if you get time during the exam.

Alternative Solution:

$2x + 3y + 10 = 0$ (i)

$-3x + y - 15 = 0$ (ii)

After multiplying Eq. (ii) with 3 then subtracting it from Eq. (i), we get,

$11x + 55 = 0$

$\Rightarrow x = -5$

From Eq. (1),

$2(-5) + 3y + 10 = 0$

$\Rightarrow y = 0$

Logarithms

The logarithm of x to the Base b is defined by $\log_b(x) = c$, where $b^c = x$

Special definitions for $b = e$ or $b = 10$ are:

$\ln x$, Base $= e$

$\log x$, Base $= 10$

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To change from one Base to another: $\log_b^x = \frac{\log_a^x}{\log_a^b}$

e.g., $\ln x = \frac{\log_{10}^x}{\log_{10}^e} = 2.302585 \log_{10}^x$

Some Identities

$\log_b bn = n$

$\log x^c = c \log x$; $x^c = \text{antilog}\,(c \log x)$

$\log xy = \log x + \log y$

$\log_b b = 1$; $\log 1 = 0$

$\log x/y = \log x - \log y$

Problem 3.6

ln(x) is most nearly:

A. log(x) B. 2.303 log(x) C. 2.303 ln(x) D. None of the above

Solution: B

Complex numbers

Complex numbers may be designated in rectangular form or polar form. In rectangular form, a complex number is written in terms of its real and imaginary components.

$z = a + jb$, where

a = the real component, b = the imaginary component, and

$j = \sqrt{-1}$ (some disciplines use $i = \sqrt{-1}$)

In polar form $z = c \angle \theta$

where:

$c = \sqrt{a^2 + b^2}$ = absolute value or modulus of the complex number

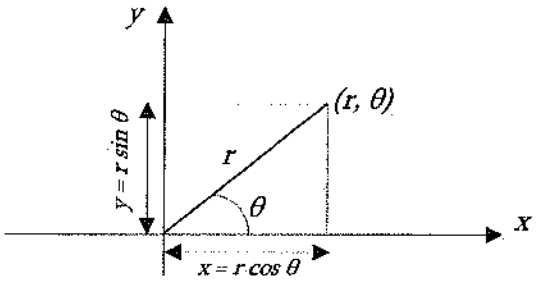
$\theta = \tan^{-1}(b/a)$ = Argument of the complex number

$a = c \cos \theta$, and $b = c \sin \theta$

Polar Coordinate System

Cartesian coordinate system uses (x, y) system. The polar coordinate system uses (r, θ). The relationship between them is shown below:

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$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$r = \sqrt{x^2 + y^2}$$
$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

Problem 3.7

The polar coordinate (r, θ) of a point is $(2, \pi/3)$. The Cartesian coordinate (x, y) is most nearly:

A. 1, 2 B. $\sqrt{2}$, 3 C. $\sqrt{2}$, 3 D. 1, $\sqrt{3}$

Solution: D

$x = r\cos\theta = 2\cos(\pi/3) = 1$; $y = r\sin\theta = 2\sin(\pi/3) = \sqrt{3}$

Problem 3.8

The Cartesian coordinate (x, y) of a point is $(-3, \sqrt{3})$. The polar coordinate (r, θ) of the point is most nearly:

A. $2\sqrt{3}, 5\pi/6$ B. $2\sqrt{3}, 3\pi/6$ C. $\sqrt{2}, \pi/6$ D. $2\sqrt{3}, \pi/2$

Solution: A

$$r = \sqrt{x^2 + y^2} = \sqrt{(-3)^2 + \left(\sqrt{3}\right)^2} = 2\sqrt{3}$$

$$\tan\theta = \frac{y}{x} = \frac{\sqrt{3}}{-3} = \frac{1}{-\sqrt{3}} \quad \Rightarrow \quad \theta = \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) = \frac{5\pi}{6}$$

Therefore, $(r, \theta) = (2\sqrt{3}, 5\pi/6)$

Problem 3.9

The polar form of an equation is $r = 6\cos\theta - 2\sin\theta$. The Cartesian form of the equation is most nearly:

A. $x^2 + y^2 - 6x + 2y = 0$
B. $x^2 + y^2 - 4x + 2y = 0$
C. $x^2 + y^2 - 2x + 6y = 0$
D. $x^2 + 2y^2 - 6x + 2y = 0$

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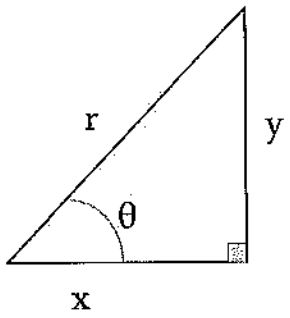
Solution: A

$$r = 6\cos\theta - 2\sin\theta$$
$$\Rightarrow r^2 = 6r\cos\theta - 2r\sin\theta$$
$$\Rightarrow r^2 = 6x - 2y$$

$$r^2 = x^2 + y^2$$
$$x^2 + y^2 = 6x - 2y$$
$$x^2 + y^2 - 6x + 2y = 0$$

Trigonometry

Trigonometry is widely used in civil engineering and surveying. It is therefore very important for the NCEES FE exam as well. Trigonometric functions are defined using a right-angle triangle as follows. Again, to remind you, right angle triangle is the triangle which has one angle of 90°.

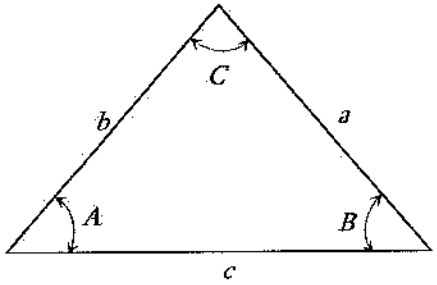


$$\cos\theta = \frac{x}{r} = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\sin\theta = \frac{y}{r} = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\tan\theta = \frac{y}{x} = \frac{\text{opposite}}{\text{adjacent}}$$

Pythagoras law: $r^2 = x^2 + y^2$

Law of Sines/Cosines

Let us assume a triangle shown below, where *a*, *b*, and *c* are sides, and *A*, *B*, and *C* are corresponding opposite angles.



Based on the above triangle, the Sine Rule can be written as:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Based on the above triangle, the Cosine Rule can be written as:

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

Some other Trigonometric Identities listed in the NCEES FE Ref. Handbook are listed here:

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$$\cos\theta = \sin\left(\theta + \frac{\pi}{2}\right) = -\sin\left(\theta - \frac{\pi}{2}\right)$$

$$\sin\theta = \cos\left(\theta - \frac{\pi}{2}\right) = -\cos\left(\theta + \frac{\pi}{2}\right)$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\cot^2\theta + 1 = \operatorname{cosec}^2\theta$$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 1 - 2\sin^2\alpha = 2\cos^2\alpha - 1$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}$$

$$\cot 2\alpha = \frac{\cot^2\alpha - 1}{2\cot\alpha}$$

More trigonometric identities are listed in the NCEEES FE Ref. Handbook. Now, practice the following problems for better understanding.

Problem 3.10

If $\sin\theta = \frac{12}{13}$, the value of $\tan\theta$ is most nearly:

A. 12/5 B. 5/12 C. ±12/5 D. ±5/12

Solution: C

$$\cos^2\theta = 1 - \sin^2\theta = 1 - \left(\frac{12}{13}\right)^2 = \frac{169 - 144}{169} = \frac{25}{169}$$

Now, $\tan^2\theta + 1 = \sec^2\theta \quad \Rightarrow \quad \tan^2\theta = \sec^2\theta - 1 = \frac{1}{\cos^2\theta} - 1 = \frac{169}{25} - 1 = \frac{144}{25}$

$$\tan\theta = \pm\frac{12}{5}$$

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Problem 3.11

$\cos^4\theta - \sin^4\theta = ?$

A. $2\sin^2\theta - 1$ B. $2\cos^2\theta - 1$ C. $2\tan^2\theta - 1$ D. $2\sec^2\theta - 1$

Solution: B

$$\cos^4\theta - \sin^4\theta = (\cos^2\theta - \sin^2\theta)(\cos^2\theta + \cos^2\theta) = (\cos^2\theta - \sin^2\theta)(1) = (\cos^2\theta - \sin^2\theta) = (\cos^2\theta - (1 - \cos^2\theta)) = 2\cos^2\theta - 1$$

Problem 3.12

450° equals:

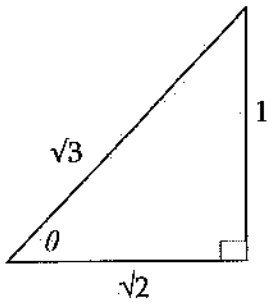
A. $5\pi/4$ B. $5\pi/2$ C. $13\pi/4$ D. $9\pi/3$

Solution: B

$450° = (\pi/180)450 = 5\pi/2$

Problem 3.13

For the triangle shown below, the value of ($\tan\theta + \cot\theta$) is most nearly:



A. $2\sqrt{3}$ B. $2/\sqrt{3}$ C. $3/\sqrt{2}$ D. $\sqrt{3}/2$

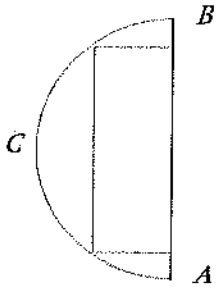
Solution: C

$\tan\theta$ = opposite / Adjacent = $1/\sqrt{2}$
$\cot\theta = \sqrt{2}/1$
$\tan\theta + \cot\theta = 1/\sqrt{2} + \sqrt{2} = 3/\sqrt{2}$

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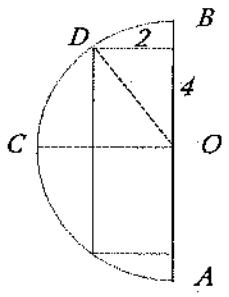
Problem 3.14

In the following figure, *ABC* is a half-circle. Inside the half circle, a rectangle of 8-in by 2-in (maximum dimension) can be drawn. The area (in²) of the half circle is most nearly:



A. 5π B. 10π C. 15π D. 20π

Solution: B

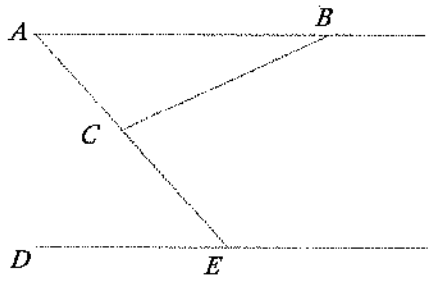


Radius of the circle, $r = \sqrt{4^2 + 2^2} = 4.47$ in

Area of the half circle, $A = \frac{1}{2} \pi r^2 = \frac{1}{2} \pi (4.47)^2 = 10\pi$ in²

Problem 3.15

In the following figure, *AB* and *DE* are parallel. Angle *ABC* = 30° and Angle *CED* =60°. The magnitude of the angle *BAC* is most nearly:



A. 45° B. 30° C. 60° D. 90°

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Solution: C

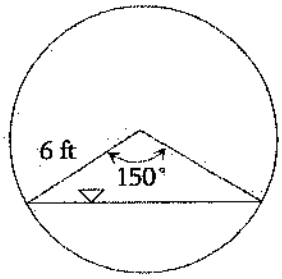
$AB//DE$; AE crosses them. Therefore, Angle BAC = Angle DEA = 60°

Areas and Volumes

The NCEES FE Ref. Handbook listed several shapes and their area and volume calculation equations. Explore the Ref. Handbook so that you can find out the equations you need to calculate the area and volume required. Several examples are given here.

Problem 3.16

The area (ft²) of the water shown below is most nearly:



A. 29.3 B. 32 C. 43 D. 38

Solution: D

$$\phi = 150^\circ = (\pi/180)(150^\circ) = 0.833\pi$$

$$A = \frac{r^2(\phi - \sin\phi)}{2} = \frac{(6)^2(0.833\pi - \sin 150)}{2} = 38.12 \text{ ft}^2$$

Problem 3.17

The volume of a 2-ft radius sphere is most nearly:

A. 23.5 ft³ B. 33.5 ft³ C. 43.5 ft³ D. 53.5 ft³

Solution: B

Volume of sphere, $V = \frac{4}{3}\pi r^3 = 33.51 \text{ ft}^3$

Problem 3.18

A cubical block of metal weighs 5 pounds. How much will another cube of the same metal weigh if its sides are three times as long?

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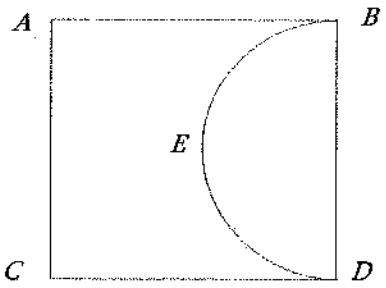
A. 15 lbs B. 45 lbs C. 135 lbs D. 405 lbs

Solution: C

Let the side of first cube = a So, volume of first cube = a^3
Therefore, $a^3 = 5$ lbs
The volume of second cube = $(3a)^3 = 27a^3$
Weight of the second cube = $27a^3 = 27$ (5 lbs) = 135 lbs

Problem 3.19

In the following figure, ABCD is a square and BED is a half-circle. The area of the ABCD square is 200 in^2. The area (in^2) of the half-circle is most nearly:



A. 25π
B. 50π
C. 100π
D. 200π

Solution: A

Side of the square = $\sqrt{200}$ = 14.14 in
So, the diameter of the circle = 14.14 in
Area of half circle = $\frac{1}{2}\ \pi r^2 = \frac{1}{2}\ \pi\ (7.07)^2 = 25\pi$ in^2

Problem 3.20

A water tank has 50-cm length, 30-cm width, and 55-cm height. If 20 gallons of water is poured into the tank, the rise (cm) of water is most nearly:

A. 48.2 B. 46.5 C. 50.5 D. 60.5

Solution: C

20 gallons = 20 (3.785) liter = 20(3.785)(1,000) cm^3 = 75,700 cm^3

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Let the water height be 'h' cm

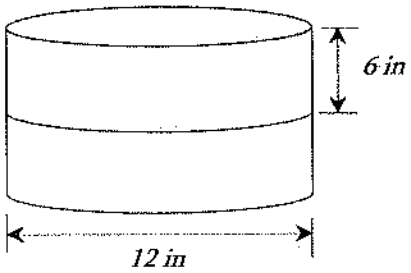
⇨ Length x width x height = 75,700 cm^3

⇨ 50 cm x 30 cm x h = 75,700

⇨ h = 50.5 cm

Problem 3.21

Two equal sized cylindrical blocks (diameter: 12 in, height: 6 in) were bonded together as shown below. If these two cylindrical blocks are separated, the increase in surface area is most nearly:



A. 14.33% B. 12.5% C. 33.3% D. 50%

Solution: C

Bonded block has 1 curved and 2 circular surfaces.

Bonded surface area, $A_b = 2\pi rH + 2\pi r^2 = 2\pi(6)(12) + 2\pi(6)^2 = 216\pi$

Unbonded blocks have 2 curved and 4 circular surfaces.

Unbonded surface area, $A_b = 2(2\pi rh) + 4\pi r^2 = 2(2)\pi(6)(6) + 4\pi(6)^2 = 288\pi$

$$\%\ \text{Increase} = \frac{288\pi - 216\pi}{216\pi}(100\%) = 33.33\%$$

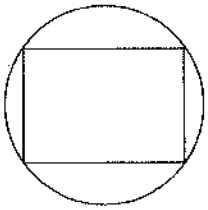
Problem 3.22

A 5-inch by 12-inch rectangle is inscribed inside a circle. The circumference (in.) of the circle is most nearly:

A. 5π B. 12π C. 13π D. 17π

Solution: C

The diagonal of the rectangle will be the diameter of the circle.



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Diagonal of the rectangle = diameter of the circle = $\sqrt{12^2+5^2}=13$
Circumference of the circle = $\pi D = 13\ \pi$ inch

Problem 3.23

A circular seal is increased to fit a design paper. The new diameter is 100% larger than the original diameter. The increase in area of the seal is most nearly:

A. 75% B. 100% C. 200% D. 300%

Solution: D

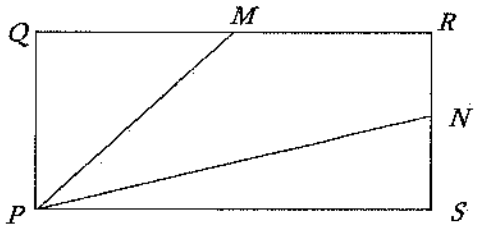
Let the diameter of original seal = d
Area of the original seal = $\pi d^2/4$
The diameter of the new seal = d + 100% of $d = 2d$
Area of new seal= $\pi(2d)^2/4 = \pi d^2$
Increase in area = $\pi d^2 - \pi d^2/4 = \frac{3}{4}(\pi d^2)$

$$\% \text{ increase in area} = \frac{\frac{3}{4}\pi d^2}{\frac{\pi d^2}{4}}(100\%) = 300\%$$

Problem 3.24

$PQRS$ is a rectangle with $PS = QR = 6$ inch and $PQ = RS = 4$ inch, as shown below. M and N are the mid points of sides QR and RS, respectively. The area (in^2) of the quadrilateral $PMRN$ is most nearly:

A. 12.0
B. 13.5
C. 11.5
D. Cannot be determined with this information



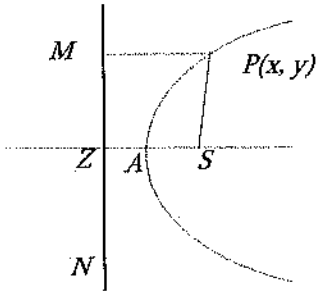
Solution: A

Area of $PMRN$ = Area of $PQRS$ – Area of PQM – Area of PNS
Area of $PQRS$ = 6 (4) = 24 in^2
Area of PQM = ½ (4) (6/2) = 6 in^2
Area of PNS = ½ (6) (4/2) = 6 in^2
Area of $PMRN$ = 24 in^2 - 6 in^2 - 6 in^2 = 12 in^2

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Conic Sections

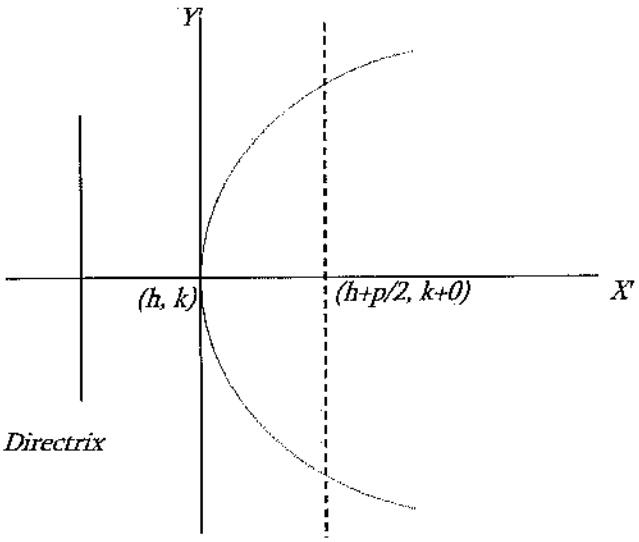
If in the Cartesian plane, the ratio of distances of any point from a fixed point and a fixed line is constant; then the locus of that point is called conic. In the following figure, *P(x, y)* is a moving point; the fixed point, *S*, and the line, *MZN*.



The fixed point, S, is called the focus; the constant ratio is called the eccentricity (e); the fixed straight line is called directrix of the conic which is denoted by *MZN* here. For different values of eccentricities (*e*), the shapes of the locus are different, such as:

i) Circle, if $e = 0$
ii) Parabola, if $e = 1$. The distances of every point on the parabola from the focus and from the directrix are equal.
iii) Ellipse, if $0 < e < 1$. The ratio of distances of every point on ellipse from the focus and from the directrix lies within 0 and 1.
iv) Hyperbola, if $e > 1$. The ratio of distances of every point on ellipse from the focus and from the directrix is greater than 1.

Let us see the details of Parabola ($e = 1$) in the following figure.



Standard Equation:

$(y - k)^2 = 2p(x - h)$

If vertex is at (0, 0) or h = k = 0

Standard Equation:

$y^2 = 2px$

Focus = (p/2, 0)
Eq. of directrix: x = -p/2

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The general form of the conic section equation is $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

If $B^2 - 4AC < 0$, an ellipse is defined.
If $B^2 - 4AC > 0$, a hyperbola is defined.
If $B^2 - 4AC = 0$, the conic is a parabola.
If $A = C$ and $B = 0$, a circle is defined.
If $A = B = C = 0$, a straight line is defined.

In this way, the NCEES FE Ref. Handbook, v. 9.5, listed the general equations, eccentricity equations, foci coordinates, equation of directrix for ellipse, and hyperbola as well. Circle can be defined by its center and radius. If the center of a circle is located at (a, b) and radius is c, then the standard form of the equation is –

$$(x-a)^2 + (y-b)^2 = c^2$$

Equation of a circle is also written as:

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

where:

center, $(-g, -f)$

radius = $\sqrt{g^2 + f^2 - c}$

Problem 3.25

The coordinate of the focus of the parabola $5y^2 = 12x$ is most nearly:

A. (6/5, 0) B. (3/5, 0) C. (12/5, 0) D. (5, 12)

Solution: B

$5y^2 = 12x \quad \Rightarrow \quad y^2 = \frac{12}{5}x \quad \Rightarrow \quad y^2 = 2\left(\frac{6}{5}\right)x$

Coordinate of the focus, $(p/2, 0)$ is (3/5, 0)

Problem 3.26

The equation of the directrix of the parabola $5y^2 = 12x$ is most nearly:

A. $3x + 5 = 0$ B. $5x + 3 = 0$ C. $10x + 3 = 0$ D. $5x + 6 = 0$

Solution: B

$5y^2 = 12x \quad \Rightarrow \quad y^2 = \frac{12}{5}x \quad \Rightarrow \quad y^2 = 2\left(\frac{6}{5}\right)x$

Eq. of directrix, $x = -p/2$ or $x = -3/5$ or $5x + 3 = 0$

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Problem 3.27

The eccentricity of the parabola $5y^2 = 12x$ is most nearly:

A. 1 B. >1 C. <1 D. 0

Solution: A

The eccentricity of a parabola is always 1.0.

Problem 3.28

The coordinate of the foci of the ellipse $25x^2 + 16y^2 = 400$ is most nearly:

A. (0, 3) B. (0, ±3) C. (±3, 0) D. (3, 0)

Solution: B

$25x^2+16y^2=400 \quad \Rightarrow \quad \frac{25x^2}{400}+\frac{16y^2}{400}=1 \quad \Rightarrow \quad \frac{x^2}{16}+\frac{y^2}{25}=1$

$\Rightarrow \quad \frac{x^2}{4^2}+\frac{y^2}{5^2}=1$

Comparing with $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $a = 4$ and $b = 5$. $b > a$; so, the ellipse is along the y-axis.

Coordinates of foci, $(0, \pm be)$

Eccentricity, $e=\sqrt{1-\left(\frac{a^2}{b^2}\right)}=\sqrt{1-\frac{16}{25}}=\sqrt{\frac{9}{25}}=\frac{3}{5}$

Coordinates of foci, $(0, \pm be)$ is $(0, \pm 3)$.

Problem 3.29

The equation of the directrix of the ellipse $25x^2 + 16y^2 = 400$ is most nearly:

A. $3y + 15 = 0$ B. $5y \pm 15 = 0$ C. $3y \pm 25 = 0$ D. None of the above

Solution: C

$25x^2 + 16y^2 = 400$

$\Rightarrow \quad \frac{25x^2}{400}+\frac{16y^2}{400}=1$

$\Rightarrow \quad \frac{x^2}{16}+\frac{y^2}{25}=1$

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$\Rightarrow \quad \frac{x^2}{4^2}+\frac{y^2}{5^2}=1$

Comparing with $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $a=4$ and $b=5$. $b>a$; so, the ellipse is along the y-axis.

Coordinates of foci, $(0, \pm be)$

Eccentricity, $e=\sqrt{1-\left(\frac{a^2}{b^2}\right)}=\sqrt{1-\frac{16}{25}}=\sqrt{\frac{9}{25}}=\frac{3}{5}$

Equation of directrix, $y=\pm\frac{b}{e}=\pm\frac{5}{3/5}=\pm\frac{25}{3}$

$\Rightarrow \quad 3y\pm 25=0$

Problem 3.30

The eccentricity of the hyperbola $\frac{x^2}{144}-\frac{y^2}{25}=1$ is most nearly:

A. 12/13 B. 13/12 C. 5/3 D. 3/5

Solution: B

Comparing with, $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, $a=12$ and $b=5$, $a>b$.

Eccentricity, $e=\sqrt{1+\left(\frac{b^2}{a^2}\right)}=\sqrt{1+\left(\frac{25}{144}\right)}=\frac{13}{12}$

Problem 3.31

The coordinates of the foci of the hyperbola $\frac{x^2}{144}-\frac{y^2}{25}=1$ is most nearly:

A. (±13, 0) B. (±12, 0) C. (0, ±13) D. (0, ±12)

Solution: A

Comparing with, $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, $a=12$ and $b=5$.

Eccentricity, $e=\sqrt{1+\left(\frac{b^2}{a^2}\right)}=\sqrt{1+\left(\frac{25}{144}\right)}=\frac{13}{12}$

Foci $(\pm ae, 0)=\left(\pm 12\left(\frac{13}{12}\right), 0\right)=(\pm 13, 0)$

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Problem 3.32

The equation of circle whose center is (4, -8) and touches the *y*-axis is most nearly:

A. $(x-4)^2+(y+8)^2=16$
B. $(x-4)^2+(y+8)^2=4$
C. $(x+4)^2+(y-8)^2=16$
D. $(x-4)^2+(y+8)^2=16^2$

Solution: A

Equation of the circle with the center (*a*, *b*) and radius, *c*, is $(x-a)^2+(y-b)^2=c^2$
If it touches the *y*-axis, the radius *c* = *x*-axis value of the center = 4

Problem 3.33

The equation of circle which passes through (3, 0) and (-4, 1), and whose center lies on the *y*-axis is most nearly:

A. $(x)^2+(y-4)^2=25$
B. $(x)^2+(y+4)^2=25$
C. $(x)^2+(y-4)^2=5$
D. $(x-4)^2+(y)^2=25$

Solution: A

Equation of circle with center (*a*, *b*) and radius, c is $(x-a)^2+(y-b)^2=c^2$
If the center lies on the y-axis, $a=0$. Therefore, $(x)^2+(y-b)^2=c^2$
⇨ $x^2+y^2-2by+b^2-c^2=0$
It passes through (3, 0), therefore, $9+0-0+b^2-c^2=0 \Rightarrow b^2-c^2=-9$
It passes through (-4, 1), therefore, $16+1-2b+b^2-c^2=0 \Rightarrow 17-2b+(-9)=0$
⇨ $b=4$
Therefore, $c^2=b^2+9=25$
$(x)^2+(y-4)^2=25$

Differential Calculus

The Derivative

For any function $y=f(x)$ the derivative, $\frac{dy}{dx}=y'=y_1$

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Test for a Maximum

$y = f(x)$ has a maximum value at $x = a$, if $f'(a) = 0$ and $f''(a) < 0$

Test for a Minimum

$y = f(x)$ has a minimum value at $x = a$, if $f'(a) = 0$ and $f''(a) > 0$

Test for a Point of Inflection

$y = f(x)$ has a point of inflection at $x = a$ if $f''(a) = 0$, and if $f''(a)$ changes sign as x increases through $x = a$.

L'Hospital's Rule (L'Hôpital's Rule)

If the fractional function $\frac{f(x)}{g(x)}$ assumes one of the indeterminate forms $\frac{0}{0}$ or $\frac{\infty}{\infty}$ (where α is finite or infinite) then, $\lim_{x \to \alpha} \frac{f(x)}{g(x)}$ is equal to first of the expressions, $\lim_{x \to \alpha} \frac{f'(x)}{g'(x)}$, $\lim_{x \to \alpha} \frac{f''(x)}{g''(x)}$, $\lim_{x \to \alpha} \frac{f'''(x)}{g'''(x)}$, which is not indeterminate, provided such a first indicated limit exists.

Derivatives

Some of the common derivatives are listed in the NCEES FE Ref. Handbook. These derivatives can be readily used during the examination.

Problem 3.34

The derivative of $x^3 \ln x$ is most nearly:

A. $3x \ln x + x$
B. $3x^2 \ln x + x^2$
C. $3x^2 \log x + x$
D. $3x \ln x + x^2$

Solution: B

$$y = x^3 \ln x$$

$$\frac{dy}{dx} = x^3\left(\frac{d}{dx}\ln x\right) + \ln x\left(\frac{d}{dx}x^3\right) = x^3\left(\frac{1}{x}\right) + \ln x\ (3x^2) = x^2 + 3x^2 \ln x$$

Problem 3.35

The minimum value of $x^3 - 5x^2 + 3x + 2$ is most nearly:

A. 7 B. 3 C. -7 D. -12

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Solution: C

$$y = x^3 - 5x^2 + 3x + 2$$

$$\frac{dy}{dx} = 3x^2 - 10x + 3$$

For minimum or maximum, $\frac{dy}{dx} = 0$

$3x^2 - 10x + 3 = 0$

$\Rightarrow \quad 3x^2 - 9x - x + 3 = 0$

$\Rightarrow \quad 3x(x-3) - 1(x-3) = 0$

$\Rightarrow \quad (3x-1)(x-3) = 0$

$\Rightarrow \quad x = 3 \text{ or } 1/3$

$$\frac{d^2y}{dx^2} = 6x - 10$$

$$\frac{d^2y}{dx^2} \text{ at } x = 3 \text{ is} = 6(3) - 10 = +8 > 0$$

So, at $x = 3$, the minimum value exists. The minimum value is $-3^3 - 5(3)^2 + 3(3) + 2 = -7$

Problem 3.36

The maximum value of $x^3 - 5x^2 + 3x + 2$ is most nearly:

A. 7 B. 2.48 C. -7 D. -12

Solution: B

$$y = x^3 - 5x^2 + 3x + 2$$

$$\frac{dy}{dx} = 3x^2 - 10x + 3$$

For minimum or maximum, $\frac{dy}{dx} = 0$

$3x^2 - 10x + 3 = 0$

$\Rightarrow \quad 3x^2 - 9x - x + 3 = 0$

$\Rightarrow \quad 3x(x-3) - 1(x-3) = 0$

$\Rightarrow \quad (3x-1)(x-3) = 0$

$\Rightarrow \quad x = 3 \text{ or } 1/3$

$$\frac{d^2y}{dx^2} = 6x - 10$$

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$\frac{d^2y}{dx^2}$ *at* $x = 3$ *is* $= 6(3) - 10 = +8 > 0$

$\frac{d^2y}{dx^2}$ *at* $x = \frac{1}{3}$ *is* $= 6(\frac{1}{3}) - 10 = -8 < 0$

So, at x = 1/3, the maximum value exists. The maximum value is -

$$\left(\frac{1}{3}\right)^3 - 5\left(\frac{1}{3}\right)^2 + 3\left(\frac{1}{3}\right) + 2 = 2.48$$

Problem 3.37

The derivative of $(x^2 + 3)(2x^2 - 1)$ is most nearly:

A. $8x^3 + 10x$ B. $2x^3 + 10x$ C. $8x^3 + 16x$ D. $6x^3 + 10x$

Solution: A

$y = (x^2 + 3)(2x^2 - 1)$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx} = \left(x^2 + 3\right)\frac{d}{dx}\left(2x^2 - 1\right) + \left(2x^2 - 1\right)\frac{d}{dx}\left(x^2 + 3\right)$$
$$= \left(x^2 + 3\right)(4x - 0) + \left(2x^2 - 1\right)(2x + 0) = 4x^3 + 12x + 4x^3 - 2x$$
$$= 8x^3 + 10x$$

Problem 3.38

The derivative of $\sqrt{x} + \sqrt[4]{x} + \frac{3}{x}$ is most nearly:

A. $\frac{1}{2\sqrt{x}} + \frac{1}{4\sqrt[4]{x^3}} - \frac{3}{x^2}$

B. $\frac{1}{2\sqrt{x}} + \frac{1}{4\sqrt[4]{x}} + \frac{3}{x^2}$

C. $\frac{1}{2\sqrt{x}} + \frac{1}{\sqrt[4]{x^3}} - \frac{2}{x^2}$

D. $\frac{1}{2\sqrt{x}} - \frac{1}{3\sqrt[4]{x^3}} - \frac{3}{x^2}$

Solution: A

Follow: if $y = x^n$, then $\frac{dy}{dx} = nx^{n-1}$

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Problem 3.39

The value of $\frac{dy}{dx}$ if $\frac{8}{y} = \frac{12}{x+y}$ is most nearly:

A. 2
B. $2x + 2$
C. $12x$
D. Cannot be determined

Solution: A

$$\frac{8}{y} = \frac{12}{x+y} \quad \Rightarrow \quad 8(x+y) = 12y \quad \Rightarrow \quad y = 2x$$

$$\text{So, } \frac{dy}{dx} = \frac{d}{dx}(2x) = 2$$

Problem 3.40

The derivative of $(\sin x)^2$ is most nearly:

A. $\sin 2x$ B. $2 \sin x$ C. $2 \cos x$ D. $-2 (\sin x) \cos x$

Solution: A

$y = (\sin x)^2$

$$\frac{dy}{dx} = \frac{d}{dx}(\sin x)^2 = 2\sin x \frac{d}{dx}(\sin x) = 2\sin x(\cos x) = \sin 2x$$

Problem 3.41

The derivative of $\ln(\tan 5x)$ is most nearly:

A. $5\frac{\sec^2 5x}{\tan 5x}$ B. $\frac{5}{\tan 5x}$ C. $5\frac{\sec 5x}{\tan 5x}$ D. -2 (sin x) cos x

Solution: A

$$y = \ln(\tan 5x)$$

$$\frac{dy}{dx} = \frac{d}{dx}\ln(\tan 5x) = \frac{1}{\tan 5x}\frac{d}{dx}(\tan 5x) = \frac{1}{\tan 5x}\sec^2 5x \frac{d}{dx}(5x)$$

$$= \frac{1}{\tan 5x}\sec^2 5x(5) = 5\frac{\sec^2 5x}{\tan 5x}$$

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Problem 3.42

The derivative of $\ln(\ln x)$ is most nearly:

A. $\frac{1}{x \ln x}$ B. $\frac{1}{\ln 2x}$ C. $\frac{1}{x \ln x^2}$ D. None of the above

Solution: A

Problem 3.43

The derivative of $\frac{d^2y}{dx^2}$, if $y = \tan x$, is most nearly:

A. $2y(y^2 + 1)$ B. $y(y^2 + 1)$ C. $2(y + 1)$ D. $\tan^2 x$

Solution: A

$$y = \tan x \quad \Rightarrow \quad \frac{dy}{dx} = \sec^2 x$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{d}{dx}\sec^2 x = (2\sec x)\frac{d}{dx}\sec x = 2\sec x(\sec x \tan x) = 2\sec^2 x \tan x$$

$$= 2\tan x(\sec^2 x) = 2\tan x(1 + \tan^2 x) = 2y(1 + y^2)$$

Problem 3.44

The x-coordinate of the points at which the tangent to the curve, $y = 4x^3 + 3x^2 - 6x + 1$ are parallel to the x-axis is most nearly:

A. -1 B. 0.5 C. Both of them D. None of them

Solution: C

Parallel to the x-axis means, $\frac{dy}{dx} = 0$

$$y = 4x^3 + 3x^2 - 6x + 1$$

$$\frac{dy}{dx} = 12x^2 + 6x - 6 + 0$$

$$12x^2 + 6x - 6 = 0$$

$$6(2x^2 + x - 1) = 0$$

$$2x^2 + x - 1 = 0$$

$$2x^2 + 2x - x - 1 = 0$$

$$2x(x + 1) - 1(x + 1) = 0$$

$$(2x - 1)(x + 1) = 0$$

$$x = 0.5, \text{ and/or } -1$$

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Problem 3.45

The slope of the tangent at point (-1, 0) to the curve, $y = 3x^2 + 2x - 1$ is most nearly:

A. -1 B. -4 C. 4 D. -6.23

Solution: B

Tangent, $\frac{dy}{dx} = \frac{d}{dx}\left(3x^2 + 2x - 1\right) = 6x + 2 - 0 = 6x + 2$

Tangent at (-1, 0) = 6(-1) + 2 = - 6 + 2 = - 4

Problem 3.46

The point of inflection of the curve, $y = 2x^3 - 3x^2 - 6$ is most nearly:

A. 0 B. 1 C. 0.5 D. 0.25

Solution: C

$y = 2x^3 - 3x^2 - 6$
$y' = dy/dx = 6x^2 - 6x$
$y'' = 12x - 6$
Point of inflection occurs when the double derivative is zero.
$y'' = 12x - 6 = 0$
$x = 0.5$

Indefinite Integrals

Some of the common integrals are listed in the NCEES FE Ref. Handbook; a few of them are listed here as well. These integrals can be readily used during the examination.

1. $\int d\,f(x) = f(x)$
2. $\int dx = x$
3. $\int a\,f(x) = a\int f(x)dx$
4. $\int [u(x) \pm v(x)]dx = \int u(x)dx \pm \int f(x)dx$
5. $\int x^m dx = \frac{x^{m+1}}{m+1} \qquad (m \neq -1)$
6. $\int \frac{dx}{ax+b} = \frac{1}{a}\ln|ax+b|$

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7. $\int \frac{dx}{\sqrt{x}} = 2\sqrt{x}$
8. $\int a^x dx = \frac{a^x}{\ln a}$
9. $\int \sin x dx = -\cos x$
10. $\int \cos x dx = \sin x$
11. $\int \sin^2 x dx = \frac{x^2}{2} - \frac{\sin 2x}{4}$

Problem 3.47

$\int \sin^3 2x \ dx = ?$ Note: $4\sin^3 A = 3\sin A - \sin 3A$

A. $-\frac{3}{8}\cos 2x + \frac{1}{24}\cos 6x + C$
B. $-\frac{3}{8}\cos 2x + \frac{1}{4}\cos 6x + C$
C. $\frac{3}{8}\cos 2x + \frac{1}{24}\cos 6x + C$
D. $-\frac{3}{8}\cos 2x - \frac{1}{24}\cos 6x + C$

Solution: A

$$\int \sin^3 2x \ dx = \frac{1}{4}\int 4\sin^3 2x \, dx = \frac{1}{4}\int (3\sin 2x - \sin 6x) dx = \frac{1}{4}\left(\frac{-3\cos 2x}{2} + \frac{\cos 6x}{6}\right) + C$$
$$= -\frac{3}{8}\cos 2x + \frac{1}{24}\cos 6x + C$$

Problem 3.48

$\int \left(3\cos x - 5\sec^2 x\right) dx = ?$

A. 5 sin x – 3 tan x + C
B. 3 sin x + 5 tan x + C
C. 6 sin x – 5 tan x + C
D. 3 sin x – 5 tan x + C

Solution: D

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$$\int (3\cos x - 5\sec^2 x)\,dx = \int 3\cos x\,dx - \int 5\sec^2 x\,dx$$
$$= 3\int \cos x\,dx - 5\int \sec^2 x\,dx$$
$$= 3\sin x - 5\tan x + C$$

Problem 3.49

$$\int_0^{\pi/2} \sin^2 x\,dx = ?$$

A. π B. π/2 C. π/3 D. π/4

Solution: D

$$\int_0^{\pi/2} \sin^2 x\,dx = \frac{1}{2}\int_0^{\pi/2} 2\sin^2 x\,dx = \frac{1}{2}\int_0^{\pi/2} (1-\cos 2x)\,dx = \frac{1}{2}\left[x - \frac{1}{2}\sin 2x\right]_0^{\pi/2}$$
$$= \frac{1}{2}\left[\frac{\pi}{2} - \frac{1}{2}\sin\pi\right] = \frac{\pi}{4}$$ [This integral is also listed in the Handbook]

Problem 3.50

The value of $\int_1^2 \left(\frac{(x^2-1)^2}{x^2}\right)dx$ is most nearly:

A. 2/3 B. ¾ C. 5/6 D. 1/3

Solution: C

$$\int_1^2 \left(\frac{(x^2-1)^2}{x^2}\right)dx = \int_1^2 \left(\frac{x^4-2x^2+1}{x^2}\right)dx = \int_1^2 x^2\,dx - 2\int_1^2 dx + \int_1^2 x^{-2}\,dx$$
$$= \left[\frac{x^3}{3}\right]_1^2 - 2[x]_1^2 - \left[\frac{1}{x}\right]_1^2 = \frac{8}{3} - \frac{1}{3} - 2(2) + 2(1) - \frac{1}{2} + \frac{1}{1} = \frac{5}{6}$$

Problem 3.51

The value of $\int_0^1 xe^x\,dx$ is most nearly:

A. 1 B. ¾ C. 0.325 D. 3

Solution: A

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$$\int xe^x dx = x\int e^x dx - \int\left(\frac{d}{dx}x\int e^x dx\right)dx = xe^x - \int\left(1.e^x\right)dx = xe^x - \int e^x dx = xe^x - e^x + C$$

$$\left[xe^x - e^x + C\right]_0^1 = \left(1.e^1 - e^1 + C\right) - \left(0.e^0 - e^0 + C\right) = 1.0$$

Problem 3.52

The area bounded by the parabola, $y^2 = 4x$ and the line, $y = 2x$ is most nearly:

A. 1.0 square unit
B. 3/4 square unit
C. 0.125 square unit
D. 1/3 square unit

Solution: D

Bounded area is calculated as $A = \int_{x_1}^{x_2}(y_2 - y_1)dx$

For the current case, let, $y_1 = \sqrt{4x} = 2\sqrt{x}$

$y_2 = 2x$

To determine the limit, we need to find the x-coordinate of the intersection points.

$y_1 = y_2$
⇨ $2\sqrt{x} = 2x$
⇨ $4x = 4x^2$ (squaring both sides)
⇨ $4x^2 - 4x = 0$
⇨ $4x(x - 1) = 0$
⇨ $x = 0, 1$

Bounded area $A = \int_0^1 (2x - 2\sqrt{x})\,dx = \left[x^2 - \frac{2}{1.5}x^{1.5}\right]_0^1 = 1 - \frac{2}{1.5}(1) = -0.33$

Neglect the negative sign. If we assumed y_1 and y_2 the opposite way, the sign would have been positive.

Problem 3.53

The area bounded by the parabola, $y = 4x^2$ and the line, $y = 4$ is most nearly:

A. 5.33 square unit
B. 3.4 square unit
C. 2.325 square unit
D. 1.3 square unit

Solution: A

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Bounded area, $A = \int_{x_1}^{x_2} (y_2 - y_1)\,dx$ Or, Bounded area $A = \int_{y_1}^{y_2} (x_2 - x_1)\,dy$

Centroids and Moment of Inertia

This topic is discussed in the Statics Section

Progressions and Series

Arithmetic Progression

To determine whether a given finite sequence of numbers is an arithmetic progression, subtract each number from the following number. If the differences are equal, the series is arithmetic.

1. The first term is *a*.
2. The common difference is *d*.
3. The number of terms is *n*.
4. The last or *n*th term is *l*.
5. The sum of *n* terms is *S*.

$l = a + (n-1)d$

$S = \frac{n}{2}(a+l) = \frac{n}{2}[2a + (n-1)d]$

Geometric Progression

To determine whether a given finite sequence is a geometric progression (G.P.), divide each number after the first by the preceding number. If the quotients are equal, the series is geometric:

1. The first term is *a*.
2. The common ratio is *r*.
3. The number of terms is *n*.
4. The last or *n*th term is *l*.
5. The sum of *n* terms is *S*.

$l = ar^{n-1}$

$S = \frac{a(1-r^n)}{1-r}$

Problem 3.54

For the following arithmetic progression, the sum of the 1st 10 terms is most nearly:

$3 + 6 + 9 + \ldots\ldots\ldots\ldots$

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A. 156 B. 165 C. 195 D. 145

Solution: B

1st term, $a = 3$
Common difference, d = 2nd term − 1st term = 6 − 3 = 3
Sum of 10-term, $S = \frac{n}{2}[2a + (n-1)d] = \frac{10}{2}[2(3) + (10-1)3] = 165$

Problem 3.55

For the following arithmetic progression, the value of the 20th term is most nearly:

$$3 + 6 + 9 + \ldots\ldots\ldots\ldots$$

A. 57 B. 55 C. 60 D. 600

Solution: C

1st term, $a = 3$
Common difference, d = 2nd term − 1st term = 6 − 3 = 3
20-th term, $l = a + (n-1)\, d = 3 + (20 - 1)\, 3 = 60$

Problem 3.56

For the following geometric progression, the value of the 30th term is most nearly:

$$2 + 4 + 8 + 16 + \ldots\ldots\ldots\ldots$$

A. 2^{30} B. 2^{29} C. 2^{31} D. None of the above

Solution: A

1st term, $a = 2$
Common ratio, r = 2nd term / 1st term = 4/2 = 2
30-th term = $ar^{n-1} = 2(2)^{30-1} = 2^{30}$

Problem 3.57

For the following geometric progression, the sum of the first 10 terms is most nearly:

$$2 + 4 + 8 + 16 + \ldots\ldots\ldots\ldots$$

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A. 1,024 B. 1,042 C. 2,046 D. 2,064

Solution: C

1st term, $a = 2$
Common ratio, r = 2nd term / 1st term = 4/2 = 2
Sum of 10 terms, $S = \frac{a\left(1-r^n\right)}{1-r} = \frac{2\left(1-2^{10}\right)}{1-2} = 2,046$

Problem 3.58

For the following geometric progression, the sum of the first 5 terms is most nearly:

59,049 + 19,683 + 6,561 + ...

A. 88,200 B. 38,572 C. 9,000 D. 102,531

Solution: A

1st term, a = 59049
Common ratio, r = 2nd term / 1st term = 19,683/59,049 = 0.3333
Sum of 5 terms, $S = \frac{a\left(1-r^n\right)}{1-r} = \frac{59,049\left(1-0.3333^5\right)}{1-0.3333} = 88,200$

[Note: some rounding errors may occur for these types of fraction problems. This should not affect the answer. Choose the nearest option]

Matrices

A matrix is an ordered rectangular array of numbers with m rows and n columns. The element a_{ij} refers to row i and column j.

Multiplication of Two Matrices

$$A = \begin{bmatrix} A & B \\ C & D \\ E & F \end{bmatrix}$$, $A_{3,2}$ is a 3-row, 2-column matrix

$$B = \begin{bmatrix} H & I \\ J & K \end{bmatrix}$$, $B_{2,2}$ is a 2-row, 2-column matrix

For multiplication to be possible, the number of columns in A must equal the number of rows in B. Multiplying matrix A by matrix B occurs as follows:

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$$C = \begin{bmatrix} A & B \\ C & D \\ E & F \end{bmatrix} \begin{bmatrix} H & I \\ J & K \end{bmatrix} = \begin{bmatrix} (A.H + B.J) & (A.I + B.K) \\ (C.H + D.J) & (C.I + D.K) \\ (E.H + F.J) & (E.I + F.K) \end{bmatrix}$$

Matrix multiplication is not commutative.

Addition

$$\begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix} + \begin{bmatrix} G & H & I \\ J & K & L \end{bmatrix} = \begin{bmatrix} A+G & B+H & C+I \\ D+J & E+K & F+L \end{bmatrix}$$

Identity Matrix

The matrix, $I = a_{ij}$ is square $n \times n$ matrix with 1's on the diagonal and 0's everywhere else.

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Matrix Transpose

Rows become columns. Columns become rows, $A = \begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix}$, then $A^T = \begin{bmatrix} A & D \\ B & E \\ C & F \end{bmatrix}$

Inverse [ ]$^{-1}$

The inverse of B of a square $n \times n$ matrix, A is, $B = A^{-1} = \dfrac{adj(A)}{|A|}$

where:

$adj(A)$ = adjoint of A (obtained by replacing A^T elements with their cofactors)

|A|= determinant of A.

$[A][A]^{-1} = [A]^{-1}[A] = [I]$ where I is the identity matrix.

Determinants

For a second-order determinant: $\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1$

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For a third-order determinant:

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2 - a_3b_2c_1 - a_2b_1c_3 - a_1b_3c_2$$

Problem 3.59

The magnitude of the matrix $\begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 5 \\ 7 & 2 & 1 \end{bmatrix}$ is most nearly:

[write in integer such as 12, or -6, etc.; not 3.5 or -7.3, etc.]

Solution: 38

Magnitude = 2(2 - 10) - 3(1 - 35) + 4(2 -14) = -16 + 102 - 48 = 38

Problem 3.60

The inverse of the matrix, $\begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$ is most nearly:

A. $\begin{bmatrix} -2 & 3 \\ 3 & -5 \end{bmatrix}$ B. $\begin{bmatrix} -2 & 3 \\ -3 & 5 \end{bmatrix}$ C. $\begin{bmatrix} 5 & -3 \\ -3 & 2 \end{bmatrix}$ D. $\begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix}$

Solution: D

$$\text{If, } [A] = \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} \text{ then, } A^{-1} = \frac{\text{adj}(A)}{|A|} = \frac{\begin{bmatrix} b_2 & -a_2 \\ -b_1 & a_1 \end{bmatrix}}{|A|} = \frac{\begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix}}{10-9} = \begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix}$$

Problem 3.61

Which of the following statements is not true?

A. Matrix multiplication is not commutative
B. For A x B, the columns in A must equal the number of rows in B
C. In matrix transpose, rows become rows
D. $[a]\ [a]^{-1} = [I]$ where $[I]$ is a diagonal matrix

Solution: C

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See the NCEES FE Ref Handbook, Mathematics Section.

Problem 3.62

The magnitude of matrix '*A*' is '*X*'. The magnitude of matrix '*B*' is most nearly:

$$A=\begin{bmatrix}2 & 4 & 3\\ 1 & 2 & 6\\ 12 & 25 & 20\end{bmatrix} \quad ; \quad B=\begin{bmatrix}4 & 8 & 6\\ 1 & 2 & 6\\ 12 & 25 & 20\end{bmatrix}$$

A. $2X$ B. X^2 C. $X/2$ D. $X/3$

Solution: A

The first row of [B] is twice of that of [A]; all other values are same. The value of [B] is twice of that of [A].

Problem 3.63

If $A=\begin{vmatrix}0 & 1\\ 1 & 2\end{vmatrix}$ and $B=\begin{vmatrix}1\\ 2\end{vmatrix}$, then AB is most nearly:

A. $\begin{vmatrix}2\\ 5\end{vmatrix}$ B. $\begin{vmatrix}1\\ 5\end{vmatrix}$ C. $\begin{vmatrix}0\\ 5\end{vmatrix}$ D. $\begin{vmatrix}5\\ 2\end{vmatrix}$

Solution: A

$$AB=\begin{vmatrix}0x1+1x2\\ 1x1+2x2\end{vmatrix}=\begin{vmatrix}2\\ 5\end{vmatrix}$$

Problem 3.64

If $A=\begin{vmatrix}8 & 4 & -1\\ 0 & 1 & 3\\ 5 & 4 & 8\end{vmatrix}$ and $B=\begin{vmatrix}-4 & 6 & 2\\ 1 & 3 & 7\\ 5 & 4 & 1\end{vmatrix}$, then $3A - 5B$ is most nearly:

A. $\begin{vmatrix}44 & -18 & -13\\ -5 & -12 & -26\\ -10 & -8 & -19\end{vmatrix}$ B. $\begin{vmatrix}44 & -18 & -13\\ -5 & -12 & -26\\ -10 & -8 & 19\end{vmatrix}$

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C. $\begin{vmatrix} 44 & -18 & -13 \\ -5 & 6 & -26 \\ -10 & -8 & -19 \end{vmatrix}$ D. $\begin{vmatrix} -44 & -18 & -13 \\ -5 & -12 & -26 \\ -10 & -8 & -19 \end{vmatrix}$

Solution: B

Problem 3.65

If $A = \begin{vmatrix} 5 & 4 \\ 6 & 5 \end{vmatrix}$, then A^{-1} is most nearly:

A. $\begin{vmatrix} 5 & -4 \\ -6 & 4 \end{vmatrix}$ B. $\begin{vmatrix} 5 & -4 \\ -6 & -4 \end{vmatrix}$ C. $\begin{vmatrix} 5 & -4 \\ 6 & 5 \end{vmatrix}$ D. $\begin{vmatrix} 5 & -4 \\ -6 & 5 \end{vmatrix}$

Solution: D

Problem 3.66

If $A = \begin{vmatrix} 2 & 5 \\ 3 & 10 \end{vmatrix}$, then AA^{-1} is most nearly:

A. $\begin{vmatrix} 5 & -4 \\ -6 & 4 \end{vmatrix}$ B. $\begin{vmatrix} 5 & -4 \\ -6 & -4 \end{vmatrix}$ C. $\begin{vmatrix} 5 & -4 \\ 6 & 5 \end{vmatrix}$ D. [I]

Solution: D

Problem 3.67

The magnitude of $A = \begin{vmatrix} 16 & 5 & 9 \\ 12 & 4 & 7 \\ 17 & 6 & 10 \end{vmatrix}$ is most nearly:

A. 1 B. -1 C. 23 D. 122

Solution: B

Vectors

The general form of vector A is $\vec{A} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$

***vectors are interchangeably represented by arrows or bolding in this text.*

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Addition and subtraction:

$$\vec{A}+\vec{B}=(a_x+b_x)\vec{i}+(a_y+b_y)\vec{j}+(a_z+b_z)\vec{k}$$
$$\vec{A}-\vec{B}=(a_x-b_x)\vec{i}+(a_y-b_y)\vec{j}+(a_z-b_z)\vec{k}$$

The dot product is a scalar product and represents the projection of B onto A times $|A|$. It is given by:

$$\vec{A}.\vec{B}=(a_xb_x)+(a_yb_y)+(a_zb_z)=|\vec{A}||\vec{B}|\cos\theta=\vec{B}.\vec{A}$$

The cross product is a vector product of magnitude $|B||A|\sin\theta$ which is perpendicular to the plane containing A and B. The product is –

$$\vec{A}\,\mathrm{x}\,\vec{B}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ a_x & a_y & a_z\\ b_x & b_y & b_z\end{vmatrix}=\vec{n}|\vec{A}||\vec{B}|\sin\theta=-\vec{B}\,\mathrm{x}\,\vec{A}$$

where: n = unit vector perpendicular to the plane of A and B. The sense of A x B is determined by the right-hand rule.

Problem 3.68

The dot product of the two vectors: 8i - 23j - 17k and i + j + k is most nearly:

A. -32 B. 18 C. 32 D. None of the above

Solution: A

A.B. = (8i - 23j - 17k).(i + j + k) = 8(1) + (-23) (1) + (-17)(1) = -32

Problem 3.69

The magnitude of the resultant of the following set of vectors is most nearly:

P = 7i + 3j + 2k
Q = -2i - 4j + k
R = -14i + 22j + 2k

A. 42.2 B. 23.4 C. 32.4 D. 34.2

Solution: B

From the NCEES FE Ref Handbook, Mathematics Section,
Resultant = P + Q + R = (7 – 2 - 14) i +(3 – 4 + 22) j + (2 + 1 + 2) k = -9 i + 21 j + 5 k

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Magnitude of the resultant $= \sqrt{(-9)^2 + (21)^2 + (5)^2} = 23.4$

Problem 3.70

Two vectors, are given below. The angle between these two vectors is most nearly:

A = 2i + 3j + 5k
B = 5i – j + k

A. 42° B. 58° C. 68° D. 78°

Solution: C

$$\theta = \cos^{-1}\left(\frac{\vec{A}.\vec{B}}{\left|\vec{A}\right|\left|\vec{B}\right|}\right) = \cos^{-1}\left(\frac{10-3+5}{\sqrt{2^2+3^3+5^2}\sqrt{5^2+(-1)^2+1^2}}\right) = \cos^{-1}(0.375) = 68^\circ$$

Problem 3.71

What is the vector product of the following two vectors?

A = 2i + 3j + 5k
B = 5i – j + k

A. 8i + 23j - 17k B. 2i + 23j - 13kC. 7i + 23j - 13k D. 2i + 3j - 13k

Solution: A

$$\vec{A}x\vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & 5 \\ 5 & -1 & 1 \end{vmatrix} = \text{i}\,(3+5) - \text{j}(2-25) + \text{k}(-2-15) = 8\,\text{i} + 23\,\text{j} - 17\,\text{k}$$

Problem 3.72

The dot product of two vectors is –

A. Scalar B. Vector C. Can be any one of the above D. None of the above

Solution: A

The dot product of two vectors is scalar.

Problem 3.73

The cross product of two vectors is –

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A. Scalar B. Vector C. Can be any one of the above D. None of the above

Solution: B

The cross product of two vectors is vector.

Problem 3.74

For the two vectors, A = 3 i – 7 j + 3 k and B = 5 i + 3 j + 2 k, which of the following statements is true?

A. They are parallel
B. They are perpendicular
C. The angle between them is 45°
D. None of the above

Solution: B

Let us determine the angle between them. If θ = 0, they are parallel. If θ = 90, they are perpendicular. Now, A.B = |A||B|cosθ = 15 – 21 + 6 = 0
So, $\theta = \cos^{-1}(0) = 90°$

Problem 3.75

Two vectors are given: A = i + 2 j + 2 k and B = 4 i + 8 j – k. The projection of vector B along vector A is most nearly:

A. 6 B. 3 C. 18 D. 7

Solution: A

A.B = 4 + 16 – 2 = 18

$|\vec{A}| = \sqrt{1^2 + 2^2 + 2^2} = 3$

Projection of vector B along vector A = $|\vec{B}|\cos\theta = \frac{|\vec{A}||\vec{B}|\cos\theta}{|\vec{A}|} = \frac{\vec{A}.\vec{B}}{|\vec{A}|} = \frac{18}{3} = 6$

Problem 3.76

Two vectors perpendicular to each other are given: A = 2 i + a j + k and B = 4 i - 2 j – k. The value of '*a*' is most nearly:

A. 6 B. 3.5 C. 3.12 D. 4.5

Solution: B

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If the angle between them, $\theta = 90$, then they are perpendicular.
Or, A.B = AB cosθ = 8 – 2*a* - 1= 0
⇨ 7 – 2*a* = 0
⇨ *a* = 3.5

Problem 3.77

What is most nearly the component of B = 5 i - 3 j + 2 k along the vector A = 2 i + j – 2 k?

A. $\left(\frac{1}{3}\right)\left(2\vec{i}+\vec{j}-2\vec{k}\right)$
B. $\left(\frac{1}{3}\right)\left(2\vec{i}-\vec{j}+2\vec{k}\right)$
C. $\left(\frac{1}{3}\right)\left(\vec{i}+\vec{j}-2\vec{k}\right)$
D. $\left(\frac{1}{9}\right)\left(2\vec{i}+\vec{j}-2\vec{k}\right)$

Solution: A

Unit vector along A, $\vec{a}=\frac{\vec{A}}{|\vec{A}|}=\frac{2\vec{i}+\vec{j}-2\vec{k}}{\sqrt{(2)^2+(1)^2+(-2)^2}}=\left(\frac{1}{3}\right)\left(2\vec{i}+\vec{j}-2\vec{k}\right)$

The component of B along the vector A =

$$|\vec{B}|\cos\theta\ \vec{a}=\frac{\vec{A}.\vec{B}}{|\vec{A}|}\vec{a}=\frac{10-3-4}{\sqrt{(2)^2+(1)^2+(-2)^2}}\left(\frac{1}{3}\right)\left(2\vec{i}+\vec{j}-2\vec{k}\right)=\left(\frac{1}{3}\right)\left(2\vec{i}+\vec{j}-2\vec{k}\right)$$

Problem 3.78

What is most nearly the value of '*a*' such that vector B = *a* i – 2 j + k is perpendicular to the vector A = 2*a* i - *a* j – 4 k ?

A. 1
B. -2
C. Both of them
D. None of them

Solution: C

Problem 3.79

For the two vectors, A = 9 i + j - 6 k and B = 4 i - 6 j + 5 k, which of the following is true?

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A. They are parallel
B. They are perpendicular
C. The angle between them is 45°
D. None of the above

Solution: B

Miscellaneous Practice Problems

Problem 3.80

If an object travels at ten feet per second, how many feet does it travel in two hours?

A. 7,200 B. 3,600 C. 72,000 D. 96,000

Solution: C

$$S = 10\frac{\text{ft}}{\text{s}}(2\,\text{hr})\left(3,600\,\frac{\text{s}}{\text{hr}}\right) = 72,000\ \text{ft}$$

Problem 3.81

In a class of 80 students 40 are taking Geotechnical Engineering, 20 are taking Structural Engineering as majors. Of the students taking Geotechnical or Structural Engineering, 10 are taking both majors. The number of students are not enrolled in either major is most nearly:

A. 20 B. 10 C. 40 D. 30

Solution: D

$(A \cup B) = A + B - (A \cap B)$
$(A \cup B) = 40 + 20 - 10$
$(A \cup B) = 50$
$(A \cup B)' = 80 - 50 = 30$

Problem 3.82

If $(2x + 5)(3x - 2) = ax^2 + bx + c$. The value of $|a - b + c|$ is most nearly:

A. -15 B. 15 C. 27 D. 5

Solution: B

$(2x + 5)(3x - 2) = 6x^2 + 15x - 4x - 10 = 6x^2 + 11x - 10$
Comparing with $ax^2 + bx + c$, $a = 6$; $b = 11$, and $c = -10$

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Therefore, $a - b + c = 6 - 11 - 10 = -15$
Finally, $|a - b + c| = |-15| = 15$

Problem 3.83

In a point of a curve, the first derivative is zero and the second derivative is -2. Which of the following is true for the point?

A. The minimum point of the curve
B. The maximum point of the curve
C. It is a point of contra-flexure
D. It is an inflection point

Solution: B

Problem 3.84

The weight of a tank of water and the tank itself is 300 lbs. When the tank is 25% empty, the weight of water and the tank itself is 250 lbs. The weight (lbs) of the empty tank is most nearly:

A. 50 B. 75 C. 100 D. 125

Solution: C

25% water is 50 lbs; so 100% water is 200 lbs. The rest 100 lbs is the empty tank.

Problem 3.85

A front loader is to dig 1,000 m^3 of soil in a day. In every attempt, it extracts 2 m^3 of soil, however, 0.1 m^3 of soil is accidently fallen off. Most nearly, how many times the front loader is required to attempt to finish its job?

A. 524 B. 525 C. 526 D. 527

Solution: D

Effective removal of soil in single attempt = 2 – 0.1 = 1.9 m^3
No. of attempt required = 1,000 m^3 / 1.9 m^3 = 526.3 ≈ 527

Problem 3.86

The original price of a shirt is $75. During the Christmas event, the price is lowered by 15%. During the checkout, the cashier can give further discount on the reduced price of 5%. The price ($) if a customer gets both discounts is most nearly:

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A. 10.69 B. 3.19 C. 63.75 D. 60.56

Solution: D

Original Price = $ 75
First time reduced price = $ 75 (85%) = $ 63.75
Checkout further reduced price = $ 63.75 (95%) = $60.56

Problem 3.87

If $f(x)=\dfrac{3x+2}{2x+1}$, then $f\left(\dfrac{3x+2}{2x+1}\right)$ is most nearly:

A. $\left(\dfrac{3x+2}{2x+1}\right)^2$ B. $\dfrac{3\left(\dfrac{3x+2}{2x+1}\right)+2}{2\left(\dfrac{3x+2}{2x+1}\right)+1}$ C. $\dfrac{3x^2+2}{2x^2+1}$ D. None of the above

Solution: B

Problem 3.88

If $n \geq 0$, then, $\dfrac{n^2-1}{n-1}$ is which one of the following?

A. Always positive
B. Always negative
C. Can be positive or negative
D. Can be zero

Solution: A

$\dfrac{n^2-1}{n-1}=\dfrac{(n+1)(n-1)}{(n-1)}=n+1$, As, $n \geq 0$, it is always positive and cannot be zero or negative.

If you cannot understand a problem or have any question regarding a problem or any other issue regarding the FE Civil Exam, please feel free to email the author at books.mrislam@gmail.com.

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4 Engineering Probability and Statistics

Fundamentals of Engineering (FE) CIVIL Exam Specification mentions, 4–6 questions may appear in the exam based on Engineering Probability and Statistics as follows:

A. Measures of central tendencies and dispersions (e.g., mean, mode, standard deviation)
B. Estimation for a single mean (e.g., point, confidence intervals)
C. Regression and curve fitting
D. Expected value (weighted average) in decision making

Dispersion, Mean, Median, and Mode Values

If $X_1, X_2, \ldots, X_n$ represent the values of a random sample of n items or observations, the arithmetic mean of these items or observations, denoted $\overline{X}$, is defined as:

$$\overline{X} = \left(\frac{1}{n}\right)(X_1 + X_2 + \ldots\ldots + X_n) = \left(\frac{1}{n}\right)\sum_{i=1}^{n} X_i$$

The weighted arithmetic mean is written as: $\overline{X_w} = \dfrac{\sum w_i X_i}{\sum w_i}$

where:

X_i = the value of the i-th observation, and
w_i = the weight applied to X_i

The variance of the population is the arithmetic mean of the squared deviations from the population mean. If μ is the arithmetic mean of a discrete population size, N, the population variance (σ^2) is defined by:

$$\sigma^2 = \frac{1}{N}\left[\sum_{i=1}^{N}(X_i - \mu)^2\right]$$

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Standard deviation of the population is: $\sigma_{population} = \sqrt{\frac{1}{N}\left[\sum_{i=1}^{N}(X_i-\mu)^2\right]}$

The sample variance is: $s^2 = \frac{1}{n-1}\left[\sum_{i=1}^{n}(X_i-\overline{X})^2\right]$

The sample standard deviation is: $s = \sqrt{\left(\frac{1}{n-1}\right)\sum(X_i-\overline{X})^2}$

The sample coefficient of variation: $CV = \frac{s}{\overline{X}}$

Sample Geometric Mean: $\sqrt[n]{X_1X_2X_3....X_n}$

The sample root- mean-square value: $\sqrt{\left(\frac{1}{n}\right)\sum X_i^2}$

When n is even, the median is the average of $\frac{n}{2}$th and $\left(\frac{n}{2}+1\right)$th items.

When n is odd, the median is the average of $\left(\frac{n+1}{2}\right)$th item.

Mode is defined as the value with the highest frequency in the sample, or population.
The sample range is the largest sample value minus the smallest sample value.

Problem 4.1

The mean of the data set 13, 17, 12, 10, 18, 22 is most nearly:

A. 14.7 B. 12.9 C. 16.6 D. 15.3

Solution: D

$$\overline{X} = \left(\frac{1}{n}\right)(X_1+X_2+.....+X_n) = \left(\frac{1}{n}\right)\sum_{i=1}^{n}X_i = \frac{13+17+12+10+18+22}{6} = 15.3$$

Problem 4.2

The standard deviation of the data set 6, 17, 12, 10, 18 is most nearly:

A. 2.95 B. 4.45 C. 5.96 D. 3.33

Solution: B

$$\overline{X} = \left(\frac{1}{n}\right)(X_1+X_2+.....+X_n) = \left(\frac{1}{n}\right)\sum_{i=1}^{n}X_i = \frac{6+17+12+10+18}{5} = 12.6$$

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$$\sigma = \sqrt{\left(\frac{1}{n}\right)\sum (X_i - \mu)^2} = \sqrt{\left(\frac{1}{5}\right)\begin{bmatrix}(6-12.6)^2 + (17-12.6)^2 + (12-12.6)^2 \\ + (10-12.6)^2 + (18-12.6)^2\end{bmatrix}} = 4.45$$

Problem 4.3

The population variance of the date set 13, 17, 12, 10, 8 is most nearly:

A. 3.63 B. 12.3 C. 5.7 D. 9.2

Solution: D

Arithmetic mean, $\mu = \left(\frac{1}{n}\right)(X_1 + X_2 + \ldots\ldots + X_n) = \left(\frac{1}{n}\right)\sum_{i=1}^{n} X_i = \frac{13+17+12+10+8}{5} = 12$

$$\sigma^2 = \frac{1}{N}\left[\sum_{i=1}^{N}(X_i - \mu)^2\right] = \frac{1}{5}\begin{bmatrix}(13-12)^2 + (17-12)^2 + (12-12)^2 + (10-12)^2 \\ + (8-12)^2\end{bmatrix} = 9.2$$

Problem 4.4

The sample variance of the date set 13, 17,12, 0, 18 is most nearly:

A. 7.17 B. 40.2 C. 51.5 D. 122

Solution: C

Arithmetic mean, $\overline{X} = \left(\frac{1}{n}\right)(X_1 + X_2 + \ldots\ldots + X_n) = \left(\frac{1}{n}\right)\sum_{i=1}^{n} X_i = \frac{13+17+12+0+18}{5} = 12$

$$s^2 = \frac{1}{n-1}\left[\sum_{i=1}^{N}(X_i - \overline{X})^2\right] = \frac{1}{4}\begin{bmatrix}(13-12)^2 + (17-12)^2 + (12-12)^2 + (0-12)^2 \\ + (18-12)^2\end{bmatrix} = 51.5$$

Problem 4.5

The sample standard deviation of the date set 13, 17, 12, 18 is most nearly:

A. 2.94 B. 3.91 C. 3.33 D. 2.55

Solution: A

Arithmetic mean, $\overline{X} = \left(\frac{1}{n}\right)(X_1 + X_2 + \ldots\ldots + X_n) = \left(\frac{1}{n}\right)\sum_{i=1}^{n} X_i = \frac{13+17+12+18}{4} = 15$

$$s = \sqrt{\left(\frac{1}{n-1}\right)\sum (X_i - \overline{X})} = \sqrt{\left(\frac{1}{3}\right)\left[(13-15)^2 + (17-15)^2 + (12-15)^2 + (18-15)^2\right]} = 2.94$$

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Problem 4.6

The sample co-efficient of variance of the date set 1.3, 1.7, 1.2, 10 is most nearly:

A. 1.05 B. 1.21 C. 0.36 D. 2.12

Solution: B

Arithmetic mean, $\overline{X} = \left(\frac{1}{n}\right)(X_1 + X_2 + ...+ X_n) = \left(\frac{1}{n}\right)\sum_{i=1}^{n} X_i = \frac{1.3+1.7+1.2+10}{4} = 3.55$

$$s = \sqrt{\left(\frac{1}{n-1}\right)\sum\left(X_i - \overline{X}\right)^2} = \sqrt{\left(\frac{1}{3}\right)\left[(1.3-3.55)^2 + (1.7-3.55)^2 + (1.2-3.55)^2 + (10-3.55)^2\right]} = 4.3$$

Sample coefficient of variance, $CV = \frac{s}{\overline{X}} = \frac{4.3}{3.55} = 1.21$

Problem 4.7

The sample geometric mean of the date set 0.11, 0.17, 0.23, 0.01 is most nearly:

A. 0.101 B. 0.08 C. 0.13 D. 1.7

Solution: B

Sample Geometric Mean, $= \sqrt[n]{X_1 X_2 X_3 .... X_n} = \sqrt[4]{(0.11)(0.17)(0.23)(0.01)} = 0.08$

Problem 4.8

The sample root- mean-square value of the date set 113, 117, 112, 110 is most nearly:

A. 137 B. 113 C. 107 D. 116.8

Solution: B

Sample root- mean-square value, $= \sqrt{\left(\frac{1}{n}\right)\sum X_i^2} = \sqrt{\frac{1}{4}\left(113^2 + 117^2 + 112^2 + 110^2\right)} = 113$

Problem 4.9

In a class of 50, 17 students scored 85%, 13 students scored 72%, and the rest of students scored 66%. The average/mean score in that class is most nearly:

A. 66% B. 71% C. 74% D. 78%

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Solution: C

$$\overline{X_w} = \frac{\sum w_i X_i}{\sum w_i} = \frac{17(85) + 13(72) + 20(66)}{50} = 74.02$$

Problem 4.10

Which of the following parameters does not measure the central tendency of a data set?

A. Average/mean
B. Median value
C. Mode
D. Skewness

Solution: D

Problem 4.11

The mode of the data set 7, 8, 9, 3, 4, 7, 8, 10, 11, 4, 3, 7, 9, 7, 6 is most nearly:

A. 7 B. 4 C. 8 D. 9

Solution: A

Occurrence of 3 = 2
Occurrence of 4 = 2
Occurrence of 6 = 1
Occurrence of 7 = 4 (controls)
Occurrence of 8 = 2
Occurrence of 9 = 2
Occurrence of 10 = 1
Occurrence of 11 = 1

Problem 4.12

The median of the data set 2, 17, 8, 7, 9, 3, 4, 10 is most nearly:

A. 4 B. 7 C. 8 D. 7.5 E. I don't know it

Solution: D

The increasing order of the data: 2, 3, 4, 7, 8, 9, 10, 17

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Here, n is even. So, the median is the average of $\frac{n}{2}$th and $\left(\frac{n}{2}+1\right)$th items.

$\frac{n}{2}$th item $=7$ and $\left(\frac{n}{2}+1\right)$th items $=8$

The average of $\frac{n}{2}$th and $\left(\frac{n}{2}+1\right)$th items is $\frac{7+8}{2}=7.5$

Problem 4.13

The median of the following data set 2, 17, 8, 7, 9, 3, 4 is most nearly:

A. 3 B. 4 C. 7 D. 9

Solution: C

The increasing order of the data is 2, 3, 4, 7, 8, 9, 17. As, n is odd. So, the median is the $\left(\frac{n+1}{2}\right)=\frac{7+1}{2}=4$th item, 4th item $=7$

Problem 4.14

The sample range of the data set 8, 7, 9, 3, 4, 37, 39 is most nearly:

A. 39 B. 36 C. 63 D. 7

Solution: B

Sample Range = Largest value – Smallest value = 39 – 3 = 36

Laws of Probability

Property 1. General Character of Probability

The probability, $P(E)$ of an event, E is a real number in the range of 0 to 1. The probability of an impossible event is 0 and that of an event certain to occur is 1.

Property 2. Law of Total Probability

$P(A+B)=P(A)+P(B)-P(A,B)$

$P(A+B)$ = the probability that either A or B occur alone or that both occur together
$P(A)$ = the probability that A occurs
$P(B)$ = the probability that B occurs

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$P(A, B)$ = the probability that both A and B occur simultaneously

Property 3. Law of Compound or Joint Probability

If neither $P(A)$ nor $P(B)$ is zero,

$$P(A, B) = P(A)P(B \mid A) = P(B)P(A \mid B)$$

where

$P(B \mid A)$ = the probability that B occurs given the fact that A has occurred
$P(A \mid B)$ = the probability that A occurs given the fact that B has occurred

If either $P(A)$ or $P(B)$ is zero, then $P(A, B) = 0$.

Problem 4.15

The probability of raining today is 20%. The probability of not raining today is most nearly:

A. 50%
B. 20%
C. 80%
D. 100%

Solution: C

The summation of probability of occurring and not occurring of an event is 1.0 (100%).

Problem 4.16

The probability of occurring A is 0.1. The probability of occurring B is 0.15. The probability of occurring A and B together is 0.05. The probability of occurring A or B is most nearly:

A. 0.25 B. 0.2 C. 0.3 D. None

Solution: B

Property 2(low of total probability)
$P(A+B) = P(A) + P(B) - P(A,B) = 0.1 + 0.15 - 0.05 = 0.2$

Problem 4.17

Box I contains 3 red and 2 blue marbles while Box II contains 2 red and 8 blue marbles. A fair coin is tossed. If the coin turns up heads, a marble is chosen from Box I. If the coin turns up tails, a marble is chosen from Box II. The probability that a red marble is chosen is most nearly:

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A. 3/5
B. 2/10
C. 2/5
D. 5/15

Solution: C

Let, R = red; I = Box I; II = Box II. Since a red marble can result from either I or II.

$$P(R) = P(I)P(R \mid I) + P(II)P(R \mid II) = \left(\frac{1}{2}\right)\left(\frac{3}{3+2}\right) + \left(\frac{1}{2}\right)\left(\frac{2}{2+8}\right) = \frac{2}{5}$$

Problem 4.18

A coin is flipped twice. The probability that the 1st time it head and the 2nd time it tail is most nearly:

A. ½
B. 1/3
C. ¼
D. ¾

Solution: C

The probability that 1st time it is head = ½
The probability that 2nd time it is tail = ½
The probability that 1st time it is head and 2nd time it is tail = (½) (½) = ¼.

Alternative

	H	*T*
H	*HH*	*HT*
T	*TH*	*TT*

Favorable option = 1
Total option = 4
The probability that 1st time it is head and 2nd time it is tail = ¼.

Problem 4.19

A dice is flipped twice. The probability that the 1st time score is smaller than that of the 2nd time is most nearly:

A. 18/36 B. 15/36 C. 21/36 D. None

Solution: B

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	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Favorable option = 15; Total option = 36; So, probability = 15/36.

Problem 4.20

A dice is flipped twice. The probability that the 1st time score is equal or greater than that of the 2nd time is most nearly:

A. 18/36
B. 15/36
C. 21/36
D. None

Solution: C

	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Total option = 36; Favorable option = 21; So, probability = 21/36.

Problem 4.21

From a set of 52 cards, two cards have been picked up without replacement. The probability that the first card is "A" [A = Ace] or the second card is "K" [K = King] is most nearly:

A. 2/52 B. (4/52)(4/51) C. (4/52) + (4/51) D. None

Solution: C

There are four types of cards in a set. Thus, there are 4 As and 4 Ks in a set of 52 cards.
The probability of first card 'A' = 4/52
The probability of second card 'K' = 4/51

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The probability that the first card is "A" or the second card is "K" = (4/52)+(4/51)

Problem 4.22

Two cards have been picked up (one after another without replacing) from a set of 52 cards. The probability that both of them are Queens (Qs) is most nearly:

A. 0.003846 B. 0.00452 C. 0.1538 D. None of the above

Solution: B

P(First time Q) = 4/52
P(Second time Q) = 3/51
Both time Qs = (4/52) x (3/51) = 0.00452

Problem 4.23

A box contains 7 red, 9 black and 6 white balls. One ball is drawn randomly. The probability of the ball being red or white is most nearly:

A. 42/484 B. 13/22 C. 7/22 D. 6/22

Solution: B

The box contains 7 red, 9 black, and 6 white balls.
P(red) = 7/22, P(black) = 9/22, P(white) = 6/22
P (red or white) = 7/22 + 6/22 = 13/22

Problem 4.24

A box contains 5 red and 10 white balls. Two balls are drawn randomly, one after another without replacing. The probability of the balls being different colors is most nearly:

A. 10/21
B. 50/95
C. 10/15
D. 5/15

Solution: A

The box contains 5 red and 10 white balls.
P (different colors) = (1st one red and 2nd one white) or (1st one white and 2nd one red)
= (5/15 x 10/14) + (10/15 x 5/14) =10/21.

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Problem 4.25

A box contains 5 red and 10 white balls. Two balls are drawn randomly, one after another without replacing. The probability of the balls being the same color is most nearly:

A. 10/21
B. 50/95
C. 10/15
D. 11/21

Solution: D

The box contains 5 red and 10 white balls.
P (same colors) = (1st one red and 2nd one red) or (1st one white and 2nd one white)
= (5/15 x 4/14) + (10/15 x 9/14) =11/21.

Problem 4.26

A uniform coin is successively tossed three times. The probability that every toss has a tail is most nearly:

A. 1/8 B. 2/9 C. 2/8 C. 1/16

Solution: A

Set = {H, T} x {H, T} x {H, T}
= {H, T} {HH, HT, TH, TT}
= {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}
Total options = 8
Favorable option = 1
P(every toss has tail) = 1/8.

Problem 4.27

A uniform coin is successively tossed three times. The probability of having two or more tails with the condition of getting tail first is most nearly:

A. 1/8 B. 2/9 C. 4/8 D. 3/8

Solution: D

Set = {H, T} x {H, T} x {H, T}
= {H, T} {HH, HT, TH, TT}
= {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

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Total options = 8
Favorable option = 3 [THT, TTH, TTT]
P(two or more tails with first time tail) = 3/8.

Problem 4.28

A uniform coin is successively tossed three times. The probability of having two or more tails in any order is most nearly:

A. 1/8 B. 2/8 C. 4/8 D. 3/8

Solution: C

Set = {H, T} x {H, T} x {H, T}
= {H, T} {HH, HT, TH, TT}
= {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}
Total options = 8
Favorable option = 4 [HTT, THT, TTH, TTT]
P(two or more tails in any order) = 4/8.

Problem 4.29

A uniform coin is successively tossed three times. The probability of having tail and head successively with first time tail is most nearly:

A. 1/8 B. 2/8 C. 4/8 D. 3/8

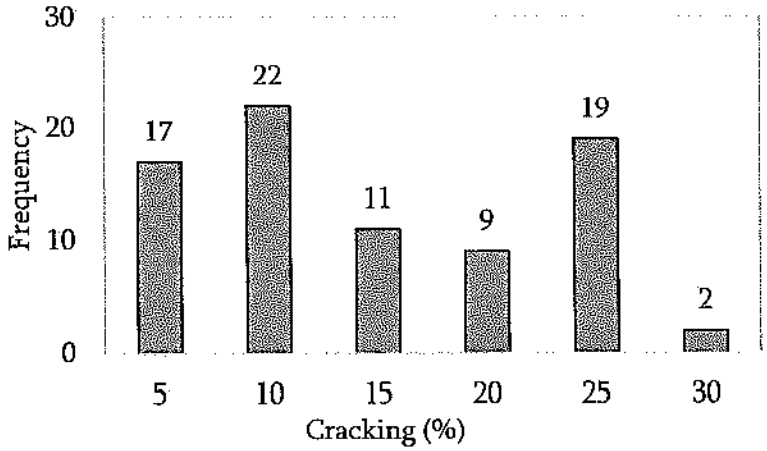
Solution: A

Set = {H, T} x {H, T} x {H, T}
= {H, T} {HH, HT, TH, TT}
= {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}
Total options = 8
Favorable option = 1 {THT}
P(tail and head successively) = 1/8

Problem 4.30

Field survey has been conducted to determine the amount of fatigue cracking in many of pavements. The data is presented using histogram as shown below. 'Cracking (%)' means the percentage of lane that has fatigue cracking.

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The number of pavement where the survey has been conducted is most nearly:

A. 80
B. 22
C. 17
D. 82

Solution: A

Sum of frequency = 17 + 22 + 11 + 9 + 19 + 2 = 80

Linear Regression and Goodness of Fit

Least Squares

$$\hat{y} = \hat{a} + \hat{b}x$$

where:

$$\hat{b} = \frac{S_{xy}}{S_{xx}}$$

$$\hat{a} = \bar{y} - \hat{b}\bar{x}$$

$$S_{xy} = \sum_{i=1}^{n} x_i y_i - \left(\frac{1}{n}\right)\left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)$$

$$S_{xx} = \sum_{i=1}^{n} x_i^2 - \left(\frac{1}{n}\right)\left(\sum_{i=1}^{n} x_i\right)^2$$

$$\bar{y} = \left(\frac{1}{n}\right)\left(\sum_{i=1}^{n} y_i\right)$$

$$\bar{x} = \left(\frac{1}{n}\right)\left(\sum_{i=1}^{n} x_i\right)$$

n = Sample size

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S_{xx} = Sum of squares of x
S_{yy} = Sum of squares of y
S_{xy} = Sum of x-y products

Residual

$$e_i = y_i - \hat{y} = y_i - \left(\hat{a} + \hat{b}x_i\right)$$

Standard Error of Estimate, $\left(S_e^2\right)$: $S_e^2 = \dfrac{S_{xx}S_{yy} - S_{xy}^2}{S_{xx}(n-2)} = MSE$, where:

$$S_{yy} = \sum_{i=1}^{n} y_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} y_i\right)^2$$

Confidence Interval for Intercept (â): $\hat{a} \pm t_{\frac{\alpha}{2},n-2}\sqrt{\left(\dfrac{1}{n} + \dfrac{\bar{x}^2}{S_{xx}}\right)MSE}$

Confidence Interval for Slope (b̂): $\hat{b} \pm t_{\frac{\alpha}{2},n-2}\sqrt{\dfrac{MSE}{S_{xx}}}$

Sample Correlation Coefficient (R): $R = \dfrac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$

Coefficient of Determination (R^2): $R^2 = \dfrac{S_{xy}^2}{S_{xx}S_{yy}}$

Problem 4.31

A best fit straight line, $\hat{y} = \hat{a} + \hat{b}x$ can be fitted using the following data points:

x	y	x^2	xy	y^2
1	1	1	1	1
3	2	9	6	4
4	4	16	16	16
6	4	36	24	16
8	5	64	40	25
9	7	81	63	49
11	8	121	88	64
14	9	196	126	81
$\sum x = 56$	$\sum y = 40$	$\sum x^2 = 524$	$\sum xy = 364$	$\sum y^2 = 256$

What are most nearly the values of $\hat{a}$, and $\hat{b}$ respectively?

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A. 0.636, 0.55
B. 0.55, 0.636
C. 0.879, 0.45
D. None of the above

Solution: B

$$S_{xy} = \sum_{i=1}^{n} x_i y_i - \left(\frac{1}{n}\right)\left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right) = 364 - \frac{1}{8}(56)(40) = 84$$

$$S_{xx} = \sum_{i=1}^{n} x_i^2 - \left(\frac{1}{n}\right)\left(\sum_{i=1}^{n} x_i\right)^2 = 524 - \frac{1}{8}(56)^2 = 132$$

$$\hat{b} = \frac{S_{xy}}{S_{xx}} = \frac{84}{132} = 0.636$$

$$\bar{x} = \frac{56}{8} = 7\,;\ \bar{y} = \frac{40}{8} = 5$$

$$\hat{a} = \bar{y} - \hat{b}\bar{x} = 5 - 0.636(7) = 0.55$$

Problem 4.32

For the above problem, Problem 4.31, the coefficient of determination is most nearly:

A. 0.87
B. 0.95
C. 0.76
D. None of the above

Solution: B

$$S_{xy} = \sum_{i=1}^{n} x_i y_i - \left(\frac{1}{n}\right)\left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right) = 364 - \frac{1}{8}(56)(40) = 84$$

$$S_{xx} = \sum_{i=1}^{n} x_i^2 - \left(\frac{1}{n}\right)\left(\sum_{i=1}^{n} x_i\right)^2 = 524 - \frac{1}{8}(56)^2 = 132$$

$$S_{yy} = \sum_{i=1}^{n} y_i^2 - \left(\frac{1}{n}\right)\left(\sum_{i=1}^{n} y_i\right)^2 = 256 - \frac{1}{8}(40)^2 = 56$$

$$R^2 = \frac{S_{xy}^2}{S_{xx} S_{yy}} = \frac{84^2}{(132)(56)} = 0.95$$

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Problem 4.33

For Problem 4.31, what is most nearly the 95% confidence interval of the slope of the regression line predicting the *y* based *x*? Given,

$$S_{xy} = 84$$
$$S_{xx} = 132$$
$$S_{yy} = 56$$

A. (2.34, 1.99)
B. (1.49, 1.22)
C. (0.99, 0.77)
D. (0.77, 0.50)

Solution: D

$$MSE = \frac{S_{xx}S_{yy} - S_{xy}^2}{S_{xx}(n-2)} = \frac{(132)(56) - (84)^2}{132(6)} = 0.42$$

$$\hat{b} = \frac{S_{xy}}{S_{xx}} = \frac{84}{132} = 0.636$$

$t_{\frac{\alpha}{2},n-2} = t_{\frac{0.05}{2},6} = 2.447$ from the Student's *t*-Distribution table in the NCEES Ref. Handbook

Confidence Interval for Slope (b):

$$\hat{b} \pm t_{\frac{\alpha}{2},n-2}\sqrt{\frac{MSE}{S_{xx}}} = 0.636 \pm (2.447)\sqrt{\frac{0.42}{132}} = 0.636 \pm 0.139 = (0.77, 0.497)$$

Remember, sometimes you may not need to solve the problem. The answer can be picked up without calculation. For example, "A 10 inch long rod…….. Which is following is the Poisson's Ratio of it?

A. 0.67 B. 0.52 C. 0.87 D. 0.41

Answer is D, as Poisson's Ratio is between 0 and -0.5. You do not need to do the calculation here. Negative sign is commonly omitted.

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5 STATICS

Fundamentals of Engineering (FE) CIVIL Exam Specification mentions, 7-11 questions may appear in the exam based on Statics as follows:

A. Resultants of force systems
B. Equivalent force systems
C. Equilibrium of rigid bodies
D. Frames and trusses
E. Centroid of area
F. Area moments of inertia
G. Static friction

Please be advised that Statics is a very important topic as a few other areas, such as Mechanics of Materials, Structural Analysis, Fluid Mechanics, etc. are dependent on Statics. Therefore, a very clear knowledge of Statics is expected. The details of Statics are discussed herein with numerical problems.

Resultant Force – Vector Method

A force is a vector quantity. It is defined when its (1) magnitude, (2) point of application, and (3) direction are known. Vector method works well for three-dimensional force systems.

The vector form of a force is given as: $\vec{F} = F_x\vec{i} + F_y\vec{j}$

In three-dimensional system, $\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$

where: $F_x = F\cos\theta_x$ and $\cos\theta_x = \frac{F_x}{F}$, similar for y and z directions.

The magnitude of the force can be determined as: $F = \sqrt{F_x^2 + F_y^2 + F_z^2}$

If there are several forces, $\vec{F}_1$, $\vec{F}_2$ etc. then, their resultant is

$\vec{R} = (F_{1x} + F_{2x} + \ldots)\vec{i} + (F_{1y} + F_{2y} + \ldots)\vec{j} + (F_{1z} + F_{2z} + \ldots)\vec{k}$

Force resultant (R) can be determined as:

$R = \sqrt{(F_{1x} + F_{2x} + \ldots)^2 + (F_{1y} + F_{2y} + \ldots)^2 + (F_{1z} + F_{2z} + \ldots)^2}$

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Resultant Force – Rectangular Component Method

The resultant, F, of n forces with components $F_{x,i}$ and $F_{y,i}$ has the magnitude of –

$$F = \sqrt{\left(\sum_{i=1}^{n} F_{x,i}\right)^2 + \left(\sum_{i=1}^{n} F_{y,i}\right)^2}$$

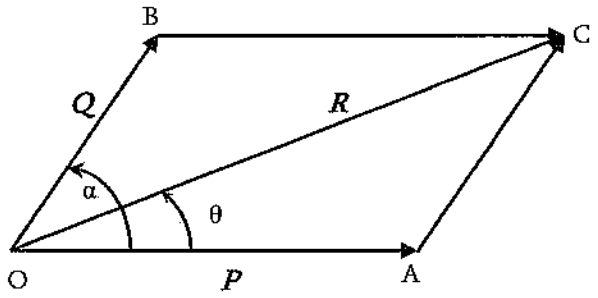
The resultant direction with respect to the x-axis is $\theta = \tan^{-1}\left(\frac{\left(\sum_{i=1}^{n} F_{y,i}\right)}{\left(\sum_{i=1}^{n} F_{x,i}\right)}\right)$

Rectangular component method works well for a two-dimensional force system for any number of forces. However, if the number of forces is two, then the Parallelogram Method is easier.

Resultant Force – Parallelogram Method

To determine the resultant of a two-force system, Parallelogram can be used more reasonably. It may be easier/quicker than any other method for a two-force system. Parallelogram can be described as follows:

Let us assume that two forces, $\boldsymbol{P}$ and $\boldsymbol{Q}$ are acting at O in a plane along OA and OB, respectively, with an angle of α. If a parallelogram $OACB$ can be drawn such that OC is the diagonal. Then, the diagonal OC can be represented as the resultant ($\boldsymbol{R}$) of the two forces, $\boldsymbol{P}$ and $\boldsymbol{Q}$ which are acting at O in a plane along OA and OB, respectively.



Then, the resultant (R) can be expressed as:

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$

If the angle between the force (P) and the resultant (R) is θ, then it can be expressed as:

$$\theta = \tan^{-1}\left(\frac{Q\sin\alpha}{P + Q\cos\alpha}\right)$$

Please note that the force system in which the lines of action of the applied forces meet at one point is called a concurrent force system.

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Problem 5.1

The component of a 230 N force along the *x*-axis is 200 N. The angle between the *x*-axis and the 230 N force is most nearly:

A. 30° B. 40° C. 20° D. 33°

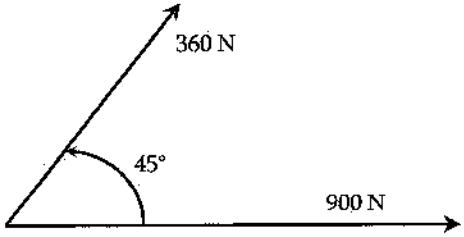
Solution: A

$$F_x = F\cos\theta$$

$$\theta = \cos^{-1}\left(\frac{F_x}{F}\right) = \cos^{-1}\left(\frac{200\text{N}}{230\text{ N}}\right) = 30^\circ$$

Problem 5.2

Two concurrent forces are acting at a point as shown below. The magnitude and the direction of it (with respect to 900 N force), respectively, of the resultant are most nearly:



A) 1,109 N, 14.43°
B) 1,213 N, 14.43°
C) 1,182 N, 12.43°
D) 1,324 N, 21.43°

Solution: C

Using the parallelogram law, $R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$

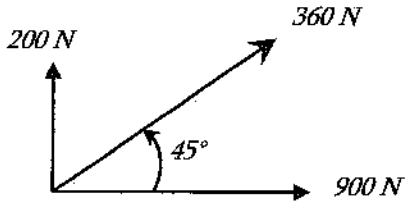
$$R = \sqrt{900^2 + 360^2 + 2(900)(360)\cos 45} = 1{,}182\text{ N}$$

$$\theta = \tan^{-1}\left(\frac{Q\sin\alpha}{P + Q\cos\alpha}\right) = \tan^{-1}\left(\frac{360\sin 45}{900 + 360\cos 45}\right) = 12.43^\circ$$

Problem 5.3

Three coplanar forces are acting on a point as shown below. What is most nearly the magnitude and direction (with respect to 900 N force) of the resultant? The 900 N and the 200 N forces are acting along the horizontal and the vertical directions, respectively.

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A) 1,110 N, 14°
B) 1,213 N, 14.43°
C) 1,182 N, 12.43°
D) 1,241 N, 21.5°

Solution: D

We would use the Parallelogram Law, if it had two forces.

Rectangular Component Method:

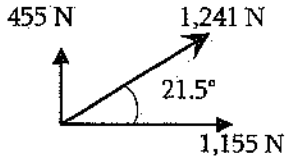
x-axis is positive in the right direction; *y*-axis is positive in the vertical upward direction.

$$\sum F_x = F_1\cos\theta_1 + F_2\cos\theta_2 + F_3\cos\theta_3 = 900\cos 0 + 360\cos 45 + 200\cos 90 = 1{,}155\,N$$

$$\sum F_y = F_1\sin\theta_1 + F_2\sin\theta_2 + F_3\sin\theta_3 = 900\sin 0 + 360\sin 45 + 200\sin 90 = 455\,N$$

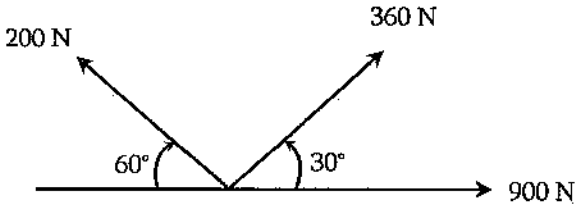
Resultant, $R = \sqrt{\left(\sum F_x\right)^2 + \left(\sum F_y\right)^2} = \sqrt{1{,}155^2 + 455^2} = 1{,}241\,N$

Direction of resultant with F_x, $\theta = \tan^{-1}\left|\dfrac{\left(\sum F_y\right)}{\left(\sum F_x\right)}\right| = \tan^{-1}\left|\dfrac{455}{1{,}155}\right| = 21.5°$ (1st Quadrant)



Problem 5.4

Three coplanar forces are acting on a point as shown below. The magnitude and the direction (with respect to 900 N force), respectively, of the resultant are most nearly:



A) 1,110 N, 14°
B) 1,213 N, 14.43°
C) 1,167 N, 17.6°
D) 1,340 N, 20°

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Solution: C

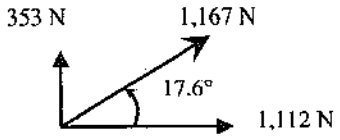
x-axis is positive in the right direction; y-axis is positive in the vertical upward direction.

$$\sum F_x = F_1\cos\theta_1 + F_2\cos\theta_2 + F_3\cos\theta_3 = 900\cos 0 + 360\cos 30 - 200\cos 60 = 1{,}112\ \text{N}$$

$$\sum F_y = F_1\sin\theta_1 + F_2\sin\theta_2 + F_3\sin\theta_3 = 900\sin 0 + 360\sin 30 + 200\sin 60 = 353\ \text{N}$$

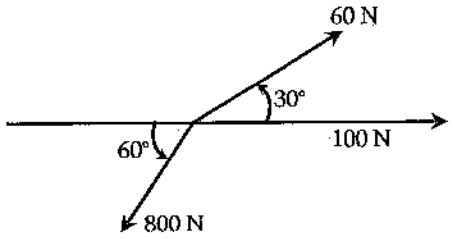
Resultant, $R = \sqrt{\left(\sum F_x\right)^2 + \left(\sum F_y\right)^2} = \sqrt{1{,}112^2 + 353^2} = 1{,}167\ N$

Direction of resultant with 900 N, $\theta = \tan^{-1}\left|\dfrac{\sum F_y}{\sum F_x}\right| = \tan^{-1}\left|\dfrac{353}{1{,}112}\right| = 17.6^\circ$



Problem 5.5

Three coplanar forces are acting on a point as shown below. The magnitude and the direction (with respect to 100 N force), respectively, of the resultant are most nearly:



A) 1,111 N, 140°
B) 708 N, 250°
C) 1,112 N, 170.6°
D) 1,340 N, 250°

Solution: B

x-axis is positive in the right direction; y-axis is positive in the vertical upward direction.

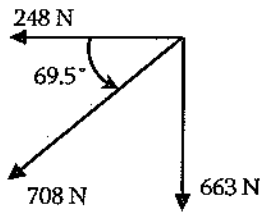
$$\sum F_x = F_1\cos\theta_1 + F_2\cos\theta_2 + F_3\cos\theta_3 = 100\cos 0 + 60\cos 30 - 800\cos 60 = -248N$$

$$\sum F_y = F_1\sin\theta_1 + F_2\sin\theta_2 + F_3\sin\theta_3 = 100\sin 0 + 60\sin 30 - 800\sin 60 = -663N$$

Resultant, $R = \sqrt{\left(\sum F_x\right)^2 + \left(\sum F_y\right)^2} = \sqrt{(-248)^2 + (-663)^2} = 708\ \text{N}$

Direction of resultant with 100 N, $\theta = \tan^{-1}\left|\dfrac{\sum F_y}{\sum F_x}\right| = \tan^{-1}\left|\dfrac{-663}{-248}\right| = 69.5^\circ = 249.5^\circ$

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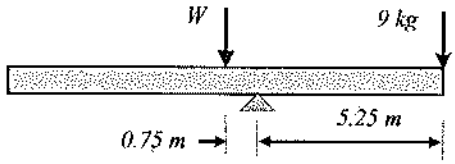
Problem 5.6

A weight of 9 kg is hanging from one end of a uniform rod of 12-m length. If the rod rests horizontally on a pole at 5.25 m from that end, the weight of the rod is most nearly:

A. 65 kg B. 61 kg C. 63 kg D. 47.25 kg

Solution: C

The weight (W) acts at the center. Consider the summation of moment at the pole, then, $W = 9$ kg (5.25 m)/0.75 m = 63 kg.

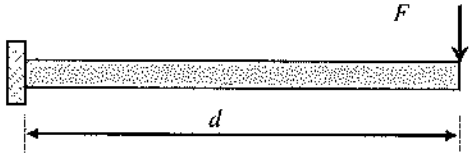


Equilibrium Requirements

From Newton's Third law of motion, we know that every action has its own and opposite reaction. That means, if you apply a force to a wall it will react with an equal amount of force on you. Therefore, action is equal to reaction. This is called equilibrium. If we sum up this action and its reactions in a direction, it will be zero, as these two are numerically equal but the signs are opposite. Thus, we can write the following three equations for two dimensional conditions:

$\sum F_x = 0$ (the summation of force along the x-direction is zero)
$\sum F_y = 0$ (the summation of force along the y-direction is zero)
$\sum M = 0$ (the summation of point of a body is zero)

For three-dimensional conditions, we need to add z-axis. Moment is defined as the force times the perpendicular distance. If a force, F acts at d distance from a point, A, the moment at point A can be determined as $M = F^*d$. The direction in counter-clockwise is considered positive.



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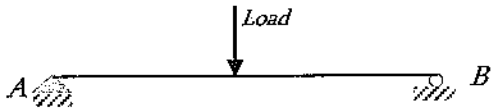
Free body diagram is essential to apply the equilibrium equations. Free body diagrams are one of the most important materials for you to know especially how to draw and use it in statics and other subjects. This diagram shows all external forces acting on the body. We do not show the internal reactions. Free body diagram is key to the equations of equilibrium, which are used to solve for the unknowns (usually forces or angles).

Procedure to draw the free-body diagram:

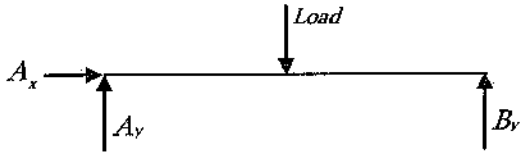
1. Imagine the particle to be isolated from its surrounding supports or restrained.
2. Show all the forces (active and reactive) that act on the particle.
   Active forces: They want to move the particle.
   Reactive forces: They tend to resist the motion.
3. Identify each force and show all known magnitudes and directions. Show all unknown magnitudes and / or directions as variables.

Let us see an example:

The following beam is supported by two supports: the left one is pinned and the right one is roller. A point load is acting on it as shown below.



To draw the free body diagram of the beam the restraints on it (the two supports) must be removed and the all forces and reactions are to be shown. Therefore, the free body of the beam can be drawn as:



Here, A_x and A_y are the reactions at A by the pin support along the x- and y-directions respectively; B_y is the reaction along the y-direction at B. Note that pin support restrains translation in both orthogonal directions. Roller support resists movement in one direction. There are broadly three kinds of supports: roller, pin, and fixed. Roller support resists translation of a member in one direction. Pin support resists translation in both orthogonal directions. Fixed support resists translation in both orthogonal directions and the rotation.

Problem 5.7

A couple consists of which type of forces?

A. Equal, opposite, and nonparallel

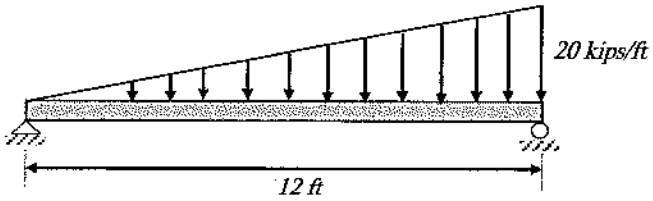
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B. Unequal, opposite, and parallel
C. Equal, and parallel
D. Equal, opposite, and parallel
E. I don't know it

Solution: D

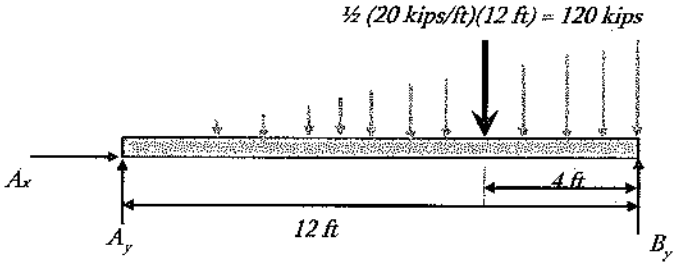
Problem 5.8

The reactions at the left support of the following beam are most nearly:



A. 0, 80 kips B. 0, 60 kips C. 0, 70 kips D. 0, 40 kips

Solution: D

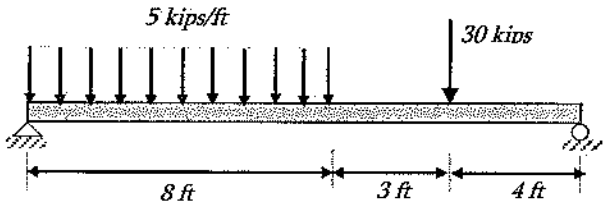


$\sum F_x = 0$
$\Rightarrow A_x = 0$

$\sum M_B = 0$
$\Rightarrow$ 120 kips (4 ft) – A_y (12 ft) = 0
$\Rightarrow A_y = 40$ kips

Problem 5.9

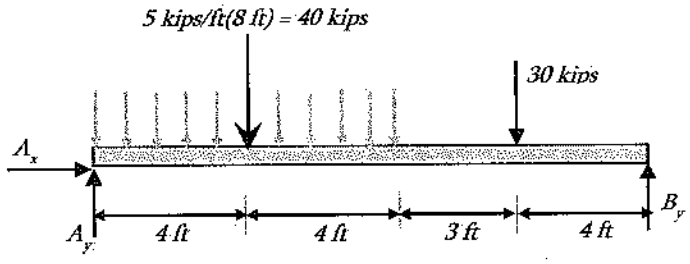
The reaction (kips) at the right support of the following beam is most nearly:



A. 25.67 B. 32.67 C. 36.67 D. 40

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Solution: B



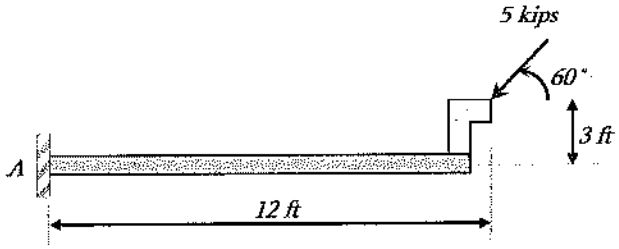
$\sum M_A = 0$

⇨ 40 k (4 ft) + 30 k (11 ft) – B_y (15 ft) = 0

⇨ B_y = 32.67 kips

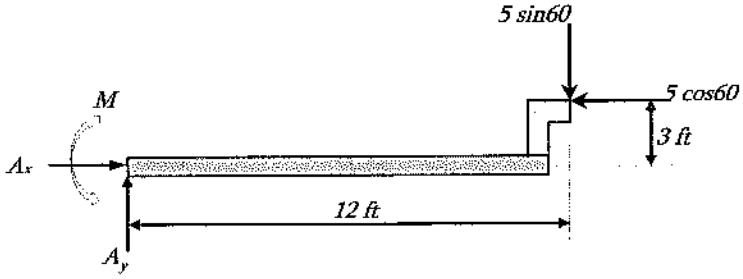
Problem 5.10

The moment (kips-ft) developed at the fixed support, A of the following beam is most nearly:



A. 44.5
B. 38.2
C. 48.5
D. 52.5

Solution: A



$\sum M_A = 0$

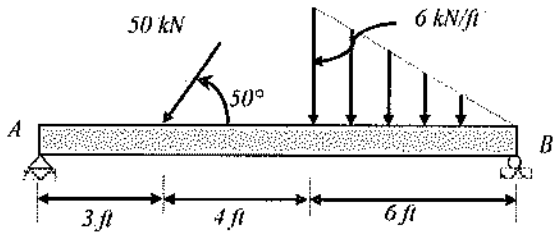
⇨ - M + 5 sin 60 (12 ft) – 5 cos 60 (3 ft) = 0

⇨ M = 44.46 kips-ft

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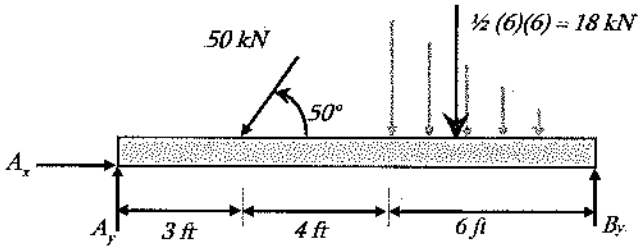
Problem 5.11

The reactions developed at the left support, A of the following beam are most nearly:



A. (32, 35) kN
B. (52, 32) kN
C. (23, 53) kN
D. (22, 35) kN

Solution: A



$$\sum F_x = 0 \quad \Rightarrow \quad A_x - 50\cos 50 = 0 \quad \Rightarrow \quad A_x = 32.1 \text{ kN } (\rightarrow)$$

$$\sum M_B = 0$$

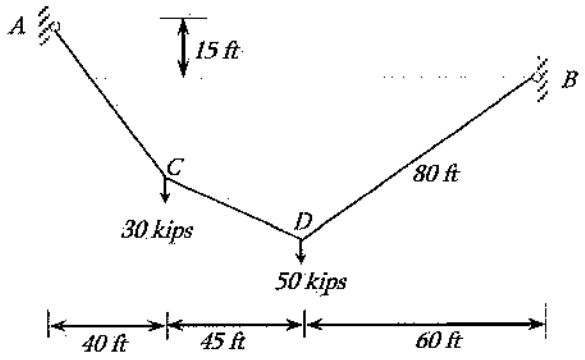
$\Rightarrow$ A_y (13 ft) − 50 sin 50 (10 ft) − 18 kN (4 ft) = 0

$\Rightarrow$ A_y = 35 kN (↑)

Problem 5.12

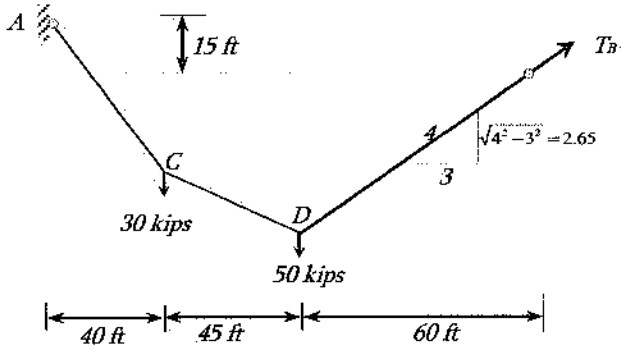
What is the reaction at B, the right support of the following cable system? The length of the BD cable is 80 ft. The difference in elevation between A and B is 15 ft.

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A. 45.8 kips B. 50.8 kips C. 55.8 kips D. 40.5 kips

Solution: B



$\sum M_A = 0$

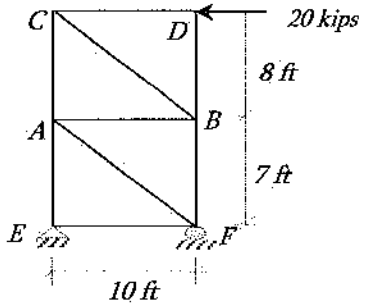
⇨ 30 k (40 ft) + 50 k (85 ft) – T_B (2.65/4) (145 ft) – T_B (3/4) (15 ft) = 0

⇨ 5,450 – 107.31 T_B = 0

⇨ T_B = 50.8 kips

Problem 5.13

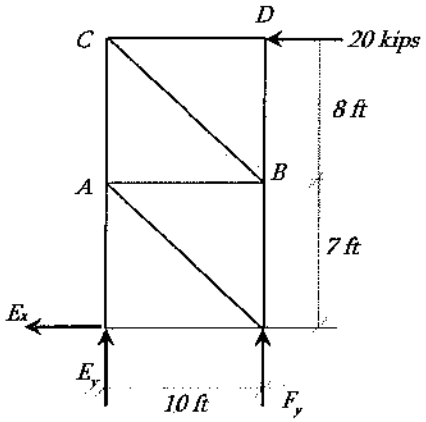
The vertical reaction (kips) at *F*, the right support of the following truss system is most nearly:



A. 30 (up)
B. 30 (down)
C. 20 (left)
D. None of the above

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Solution: B



$\sum M_E = 0$

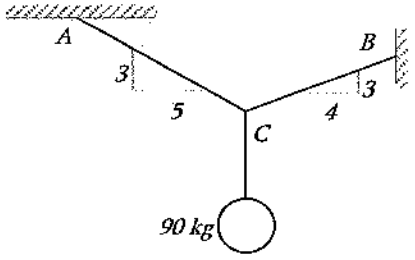
⇨ 20 k (15 ft) + F_y (10 ft) = 0

⇨ F_y = - 30 kips

⇨ F_y = 30 kips (down)

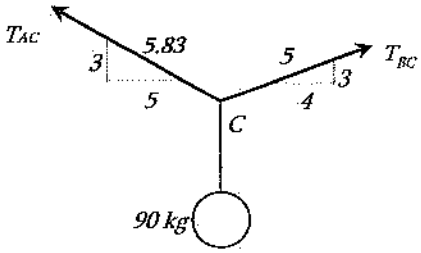
Problem 5.14

A 90-kg man is hanging using a cable system as shown below. The tension at the cable BC is most nearly:



A. 63 kg B. 73 kg C. 83 kg D. 90 kg

Solution: C



$\sqrt{3^2+4^2} = 5$

$\sqrt{3^2+5^2} = 5.83$

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$$\sum F_x = 0 \ (\leftarrow +ve)$$

$$\Rightarrow T_{AC}\left(\frac{5}{5.83}\right) - T_{BC}\left(\frac{4}{5}\right) = 0$$

$$\Rightarrow 0.857T_{AC} = 0.8T_{BC}$$

$$\Rightarrow T_{AC} = 0.933T_{BC}$$

$$\sum F_y = 0 \ (\uparrow +ve)$$

$$\Rightarrow T_{AC}\left(\frac{3}{5.83}\right) + T_{BC}\left(\frac{3}{5}\right) - 90 = 0$$

$$\Rightarrow 0.515T_{AC} + 0.6T_{BC} = 90$$

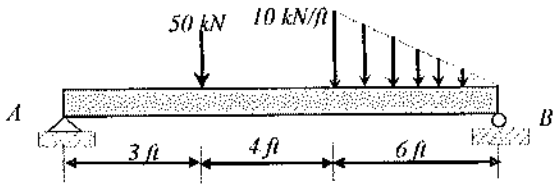
$$\Rightarrow 0.515(0.933T_{BC}) + 0.6T_{BC} = 90$$

$$\Rightarrow 1.08T_{BC} = 90$$

$$\Rightarrow T_{BC} = 83.33 \text{ kg}$$

Problem 5.15

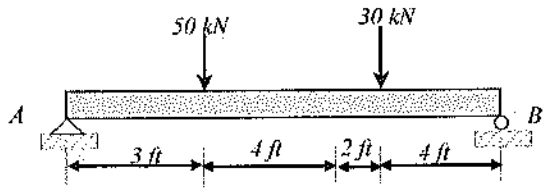
A structural beam has the following loading configuration. The resultant of the load and its acting point with respect to the left support are most nearly:



A. 80 kN, 5.25 ft
B. 110 kN, 5.25 ft
C. 80 kN, 2.25 ft
D. 110 kN, 4.25 ft

Solution: A

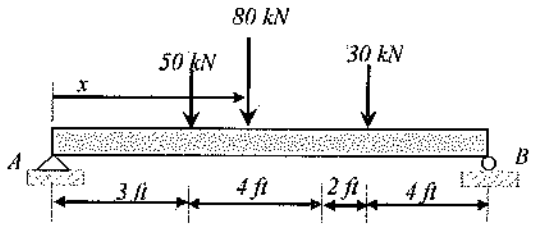
The equivalent load of the triangular loading is ½(10 kip/ft)(6 ft) = 30 kN. The loading configuration can be shown as follows:



Both the loads are parallel. Therefore, the resultant is 30 + 50 = 80 kN.

To determine the acting point of the resultant with respect to the left support, we need to take moment about the left support. Again to remind you, the moment created by the load group with respect to a point is equal to the moment created by the resultant with respect to that point. Let us assume that the resultant acts at '*x*' distance from the left support, then –

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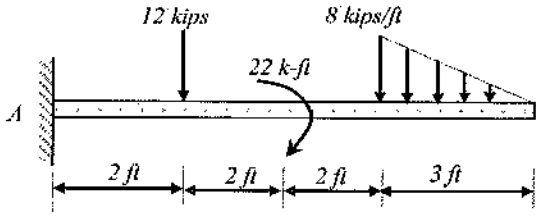
80 kN (x) = 50 kN (3 ft) + 30 kN (9 ft)

⇨ x = 5.25 ft

The equivalent load is 80 kN and it acts at 5.25 ft from the left support

Problem 5.16

The following cantilever beam is supporting three types of loads. What is the equivalent concentrated load and its point of action with respect to A, which will cause the same reactions at the support?



A. 42 kN and it acts at 5.42 ft from the support
B. 32 kN and it acts at 5.42 ft from the support
C. 24 kN and it acts at 7.42 ft from the support
D. 24 kN and it acts at 5.42 ft from the support

Solution: D

$\sum F_y$ = 12 kips + ½ (8 kips/ft) (3 ft) = 24 kips

Moment at A caused by the resultant and by the load group will be equal.

24 kips (x) = 12 kips (2 ft) + 22 k-ft + ½ (8)(3) (7 ft)

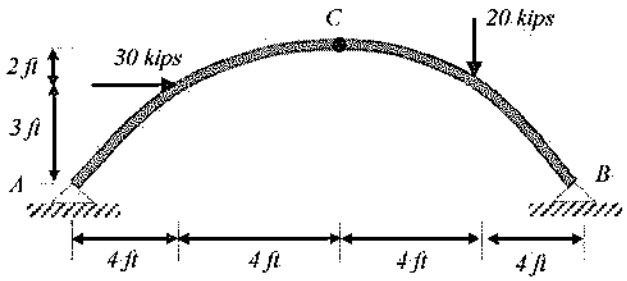
x = 5.42 ft

The equivalent load is 24 kN and it acts at 5.42 ft from the support

Problem 5.17

A three-hinged arch is supporting two loads of 20 kips, and 30 kips as shown below. The supports, A and B are pinned supports, and C is pin joint. The reactions (C_x, and C_y) at the interior hinge, C regardless of directions are most nearly:

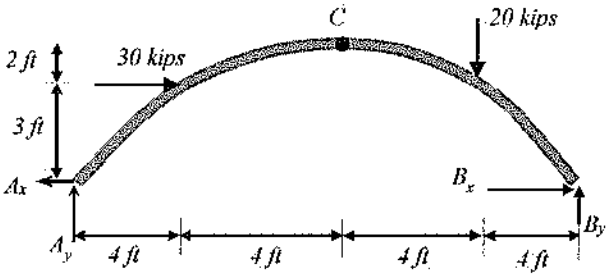
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A. 12 kips, 0.63 kips
B. 17 kips, 1.63 kips
C. 17 kips, 2.63 kips
D. 17 kips, 0.63 kips

Solution: D

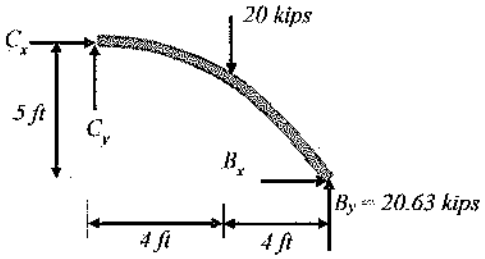
Step 1. Consider the whole structure



$\sum M_A = 0$ (clock-wise moment positive)
30 kips (3 ft) + 20 kips (12 ft) – B_y (16 ft) = 0
$B_y = 20.63$ kips

Step 2. Cut at C and consider the right part:

$\sum M_C = 0$ (clock-wise moment +ve)
20 kips (4 ft) – B_x (5 ft) – 20.63 kips (8 ft) = 0
$B_x = -17$ kips



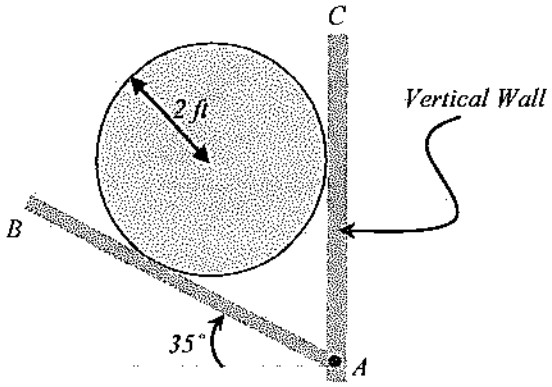
$\sum F_x = 0$ (→ +ve)
$C_x + B_x = 0$
$C_x = -B_x = -(-17) = 17$ kips
$\sum F_y = 0$ (↑ +ve)
$C_y + B_y - 20 = 0$
$C_y = 20 - B_y = 20 - 20.63 = -0.63$ kips

Problem 5.18

A 20-kips sphere is supported by two rigid planes shown below. The plane, AB is inclined with the horizon at an angle of 35° and the other plane AC is vertical. The joint of AB and AC are pin

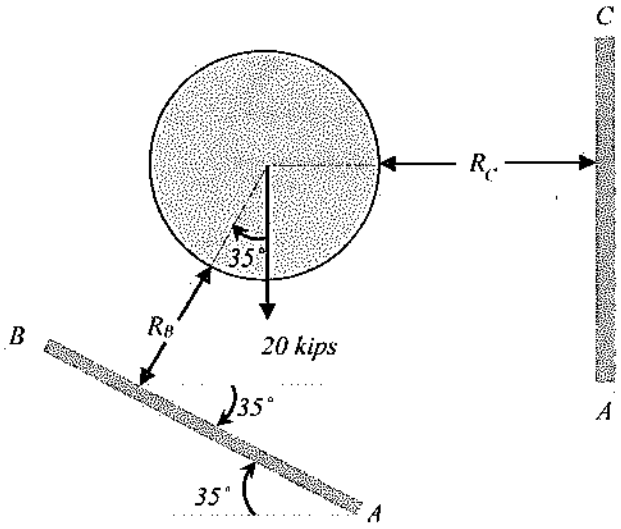
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connected. If the contact surfaces are frictionless, what is the forces exerted by the spherical body on the planes AB and AC respectively?



A. 2.5 kips, 6.3 kips
B. 24.42 kips, 14 kips
C. 14 kips, 24.42 kips
D. 18.42 kips, 8.6 kips

Solution: B



Let us apply, $\sum F_y = 0$ in the free-body diagram of the spherical body only. The reason for choosing $\sum F_y = 0$ is that there is only one unknown force (R_B) in y-direction; in x-direction, there are two unknown forces (R_C and component of R_B).

$\sum F_y = 0 \quad (\uparrow +ve)$
$R_B \cos 35 - 20 \text{ kips} = 0$
$R_B = 24.42 \text{ kips}$

$\sum F_x = 0 \quad (\rightarrow +ve)$
$R_B \sin 35 - R_C = 0$

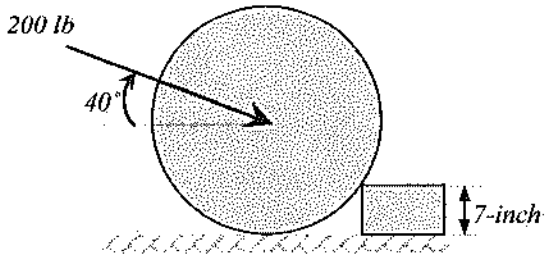
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$Rc = 14$ kips

Problem 5.19

A 30-inch wheel is to be rolled over an obstacle of 7-inch high as shown below. The wheel load is 300 lbs. If the pushing force of 200 lb is applied as shown below at an angle of 40° with the ground, the factor of safety against overturning of the wheel over the obstacle is most nearly:

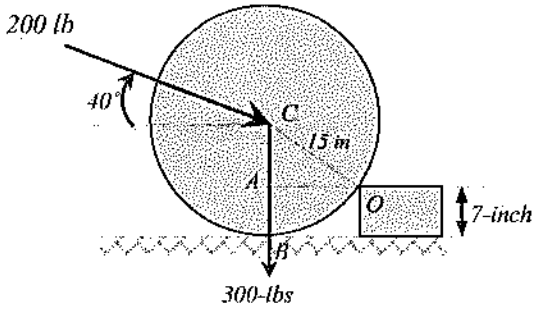
$$\text{Factor of Safety} = \frac{\text{Resisting Moment}}{\text{Overturning Moment}}$$



A. 2.22
B. 3.33
C. 4.44
D. 5.55

Solution: C

Step 1. Draw the free-body diagram



Step 2. Apply equilibrium equations.

Radius, $OC = 15$ in.

$CA = BC - AB = 15 - 7 = 8$ in.

$OA = \sqrt{OC^2 - AC^2} = \sqrt{15^2 - 8^2} = 12.69$ in.

Overturning Moment about $O = 200 \cos 40\,(8\text{ in}) = 1{,}226$ lb-in

Resisting Moment about $O = 200 \sin 40(12.69\text{ in}) + 300(12.69\text{ in}) = 5{,}438$ lb-in

$$\text{Factor of Safety} = \frac{\text{Resisting Moment}}{\text{Overturning Moment}} = \frac{5{,}438}{1{,}226} = 4.44$$

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Centroids of Areas

The centroid (or the center of gravity) of an object is the point where the masses of all grains of the object orients. For example, the centroid of a circular disk acts at the center; the centroid of a square/rectangle acts at the intersection of the diagonals. The determination of centroid is essential for determining the centroid of distributed loads, and to determine the moment of inertia for composite sections.

$$\bar{x} = \frac{\sum x\Delta A}{\sum \Delta A} = \frac{x_1 A_1 + x_2 A_2 + \ldots}{A_1 + A_2 + \ldots}$$
$$\bar{y} = \frac{\sum y\Delta A}{\sum \Delta A} = \frac{y_1 A_1 + y_2 A_2 + \ldots}{A_1 + A_2 + \ldots}$$

For composite section, it is to be divided into several regular parts for which the centroid is known.

Moment of Inertia

Moment of inertia (I) is a geometric property of a section which controls the deflection and stress carrying capacity of a section. The higher the moment of inertia, the lower the possible deflection and actual stress level in beam. It also helps columns to resist buckling. Although calculus is the most accurate method of determining the moment of inertia, use the following equations for simplicity:

Moment of inertia about x-axis: $I_x = \sum y^2 \Delta A$
Moment of inertia about y-axis: $I_y = \sum x^2 \Delta A$

From the above equation, the I-value is mathematically second moment of an area. The above equations can be used for I-values for regular sections (square, rectangle, circular, and so on) or summation of several regular sections. For irregular section, calculus is the only method to do it. However, in civil engineering related structures, most of the sections are either regular or the summation of several regular sections.

Parallel Axis Theorem for Moment of Inertia

Moments of inertia of a composite section is typically determined at the centroid of the section. Therefore, the I-value determined for a smaller part of a composite section is needed to transfer it the centroidal parallel axis. It can be conducted by using the parallel axis theorem, which is expressed as follows:

$$I_x = \sum I_{xc} + \sum A d_y^2$$

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I_x = Moment of inertia about the major centroidal x-axis; (in.^{4} or mm^4)
I_{xc} = Moment of inertia about its own centroidal x-axis; (in.^{4} or mm^4)
A = Area of the component; (in.^{2} or mm^2)
d_y = Perpendicular distance between the major centroidal x-axis and the parallel axis that passes through the centroid of the component; (in. or mm)

Similarly, for y-axis, $I_y = \sum I_{yc} + \sum A d_x^2$

For a rotating member, such as the shaft of motor, the polar moment of inertia is required to determine the stress. Polar moment of inertia (J) can be written as:

$$J = I_x + I_y$$

Radius of Gyration

The radius of gyration expresses the relationship between the area of a cross-section and a centroidal moment of inertia. It is a shape factor that measures a column's resistance to buckling about an axis. The radius of gyration of a cross-section (area) is defined as that distance from its moment of inertia axis, at which the entire area could be considered as being concentrated without changing its moment of inertia. Radius of gyration is calculated as:

Radius of gyration about x-axis, $r_x = \sqrt{\frac{I_x}{A}}$

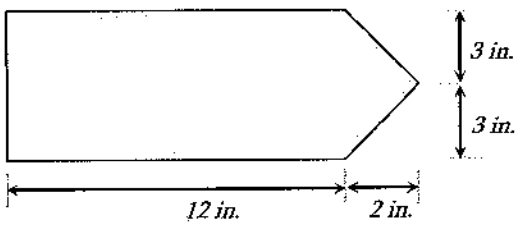
Radius of gyration about y-axis, $r_y = \sqrt{\frac{I_y}{A}}$

Polar radius of gyration about z-axis, $r_j = \sqrt{\frac{J}{A}} = \sqrt{\frac{I_x + I_y}{A}}$

The NCEES FE Ref. Handbook lists centroid, moment of inertia, and other parameters of several regular sections. This table is very important. Read the table carefully and make sure you can use all the equations if required.

Problem 5.20

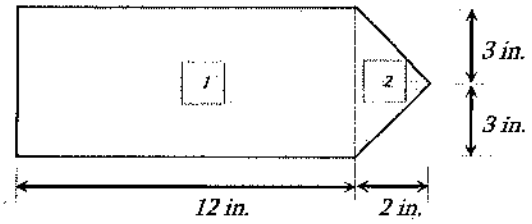
The centroid of the following section is most nearly:



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A. (6, 3) inch
B. (6.5, 3) inch
C. (7, 3) inch
D. (7.5, 3) inch

Solution: B



Then, the centroid $(\bar{x}, \bar{y})$ can be calculated as:

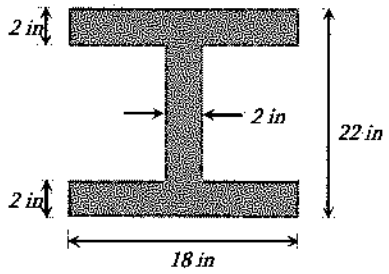
$$\bar{x} = \frac{\sum x\Delta A}{\sum \Delta A} = \frac{x_1 A_1 + x_2 A_2}{A_1 + A_2} = \frac{(6)(12x6) + \left(12 + \frac{1}{3}(2)\right)\left(\frac{1}{2}x6x2\right)}{12x6 + \frac{1}{2}(6)(2)} = 6.51 \text{ in}$$

$$\bar{y} = \frac{\sum y\Delta A}{\sum \Delta A} = \frac{y_1 A_1 + y_2 A_2}{A_1 + A_2} = \frac{(3)(12x6) + (3)\left(\frac{1}{2}x6x2\right)}{(12x6) + \frac{1}{2}(6)(2)} = 3 \text{ in}$$

(By visual inspection, it can be seen that $\bar{y} = 3$ inch)

Problem 5.21

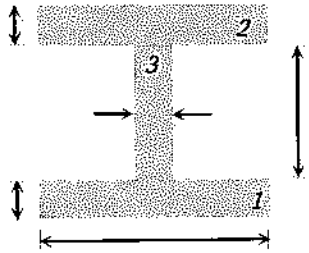
The moment of inertia of the following symmetric W-section with respect to the centroidal *x*-axis is most nearly:



A. 8,196 in^4 B. 7,196 in^4 C. 9,196 in^4 D. 10,196 in^4

Solution: A

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To determine the I-value about the centroidal x-axis, first the centroid needs to be determined. This is a both way symmetric section. So, the centroid is located at the intersection of both symmetry lines, i.e., at (9, 11) inch with respect to any corner. Then, the section is divided into smaller regular parts, 1, 2, and 3.

I-value about x-axis:

Part 1: $I_{x(1)} = \dfrac{bh^3}{12} = \dfrac{18(2)^3}{12} = 12 \text{ in}^4$

This I-value is about the local $x(1)$ axis; it must be transferred to the centroidal x-axis. To do it, $A_1 d_{y1}^2$ must be added to it.

$A_1 d_{y1}^2 = (18x2)(10)^2 = 3,600 \text{ in}^4$

Part 2: Like part 1.

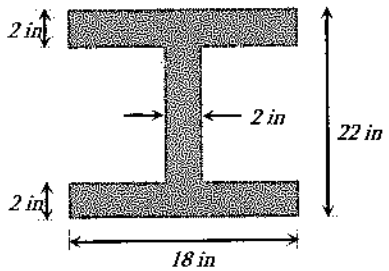
Part 3: $I_{x(3)} = \dfrac{bh^3}{12} = \dfrac{2(18)^3}{12} = 972 \text{ in}^4$; $x(3)$ axis is already at the major centroid. Therefore, no transformation is needed.

The combined I-value, $I_x = 2(I_{x(1)} + A_1 d_{y1}^2) + I_{x(3)} = 2(12 + 3,600) + 972 = 8,196 \text{ in}^4$

As parts 1 and 2 are symmetric. Therefore, 2 was multiplied with the value of part 1.

Problem 5.22

The moment of inertia of the following symmetric W-section with respect to the centroidal y-axis is most nearly:



A. 1,956 in^4
B. 2,196 in^4
C. 1,796 in^4
D. 2,396 in^4

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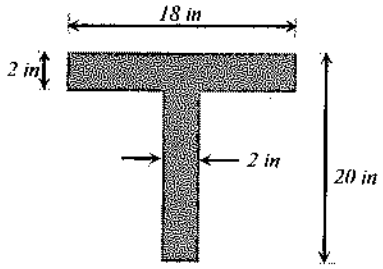
Solution: A

The y-axes of all these three parts coincide the centroidal *y*-axis.

$$I_y = I_{y1} + I_{y2} + I_{y3} = \frac{2(18)^3}{12} + \frac{2(18)^3}{12} + \frac{18(2)^3}{12} = 1{,}956 \text{ in}^4$$

Problem 5.23

The moment of inertia of the following symmetric T-section with respect to the centroidal *x*-axis is most nearly:



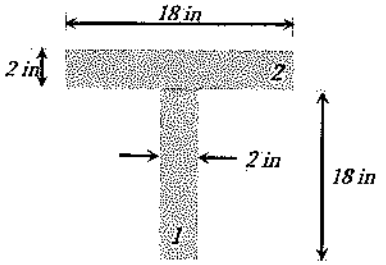
A. 2,284 in^4
B. 2,874 in^4
C. 1,278 in^4
D. 2,784 in^4

Solution: D

To determine the *I*-value about centroidal *x*-axis, first the centroid needs to be determined.

$$\bar{y} = \frac{y_1 A_1 + y_2 A_2}{A_1 + A_2} = \frac{9(2x18) + 19(18x2)}{(2x18) + (18x2)} = \frac{324 + 684}{36 + 36} = \frac{1{,}008}{72} = 14 \text{ in.}$$

Then the section is divided into smaller regular parts, 1, and 2.



Part 1: $I_{x(1)} = \frac{bh^3}{12} = \frac{2(18)^3}{12} = 972 \text{ in}^4$

This I-value is about the local *x*(1) axis; it must be transferred to the centroidal *x*-axis. To do it, $A_1 d_{y1}^2$ must be added to it.

$A_1 d_{y1}^2 = (18x2)(14-9)^2 = 900 \text{ in}^4$

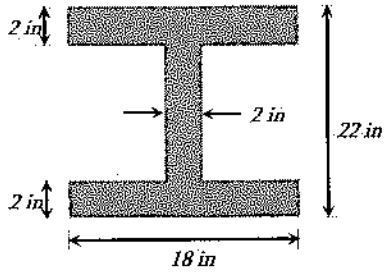
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Part 2: $I_{x(2)} = \dfrac{bh^3}{12} = \dfrac{18(2)^3}{12} = 12 \text{ in}^4$; It must be transferred to the centroidal x-axis. To do it, $A_2 d_{y2}^2$ must be added to it. $A_2 d_{y2}^2 = (18\text{x}2)(14-19)^2 = 900 \text{ in}^4$

The combined I-value, $I_x = I_{x(1)} + A_1 d_{y1}^2 + I_{x(2)} + A_2 d_{y2}^2 = 972 + 900 + 12 + 900 = 2,784 \text{ in}^4$

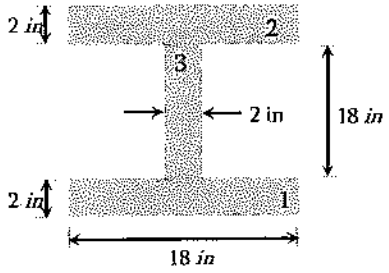
Problem 5.24

The radius of gyration of the following symmetric W-section with respect to the centroidal *x*-axis is most nearly:



A. 7.8 in B. 8.7 in C. 6.9 in D. 9.8 in

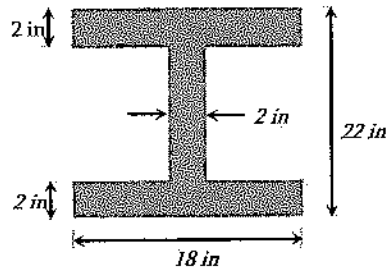
Solution: B



$$I_x = 2(I_{x(1)} + A_1 d_{y1}^2) + I_{x(3)} = 2(12 + 3,600) + 972$$
$$= 8,196 \text{ in}^4$$
$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{8196}{108}} = 8.7 \text{ in.}$$

Problem 5.25

The radius of gyration of the following symmetric W-section with respect to the centroidal *y*-axis is most nearly:



A. 4.3 in B. 6.3 in C. 8.3 in D. 3.3 in

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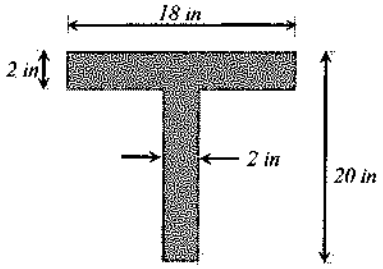
Solution: A

$$I_y = I_{y1} + I_{y2} + I_{y3} = \frac{2(18)^3}{12} + \frac{2(18)^3}{12} + \frac{18(2)^3}{12} = 1{,}956 \text{ in}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{1{,}956}{108}} = 4.26 \text{ in.}$$

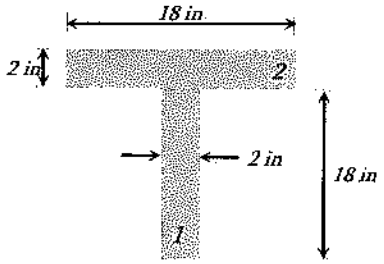
Problem 5.26

The radius of gyration of the following section with respect to the centroidal x-axis is most nearly:



A. 2.26 in B. 4.22 in C. 6.22 in D. 8.22 in

Solution: C



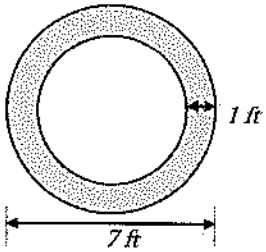
$$\bar{y} = \frac{y_1 A_1 + y_2 A_2}{A_1 + A_2} = \frac{9(2x18) + 19(18x2)}{(2x18) + (18x2)} = \frac{324 + 684}{36 + 36} = \frac{1{,}008}{72} = 14 \text{ in.}$$

$$I_x = I_{x(1)} + A_1 d_{y1}^2 + I_{x(2)} + A_2 d_{y2}^2 = 972 + 900 + 12 + 900 = 2{,}784 \text{ in}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{2784}{72}} = 6.22 \text{ in.}$$

Problem 5.27

What is the radius of gyration of the following hollow section about the centroid x-axis as shown? The outer diameter of the section is 7 ft and wall thickness is 1 ft.



A. 2.15 ft B. 3.15 ft C. 4.15 ft D. None of the above

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Solution: A

Moment of inertia about x-axis, $I_x = \frac{\pi r_o^4}{4} - \frac{\pi r_i^4}{4} = \frac{\pi}{4}(3.5^4 - 2.5^4) = 87.2 \text{ ft}^4$

Radius of gyration, $r = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{87.2}{\pi(3.5^2 - 2.5^2)}} = 2.15 \text{ ft}$

Problem 5.28

Where is the center of a parallelogram located?

A. Equal distance from all four sides
B. At half the distance of the long side
C. At the intersection of the diagonals
D. None of the above

Solution: C

The mass center of a parallelogram is located at the intersection of the diagonals. If you support a parallelogram at the intersection of the diagonals, it will be stable.

Problem 5.29

Which technical term represent the relationship between the area of a cross-section and a centroidal moment of inertia?

A. moment of inertia
B. radius of gyration
C. width to thickness ratio
D. length to width ratio

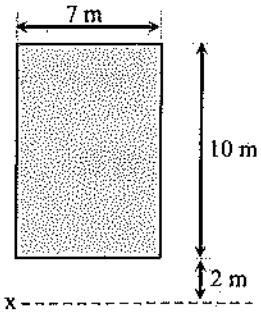
Solution: B

$$r = \sqrt{\frac{I}{A}}$$

Problem 5.30

What is most nearly the moment of inertia (m^4) of the following rectangle about the *x*-axis, 2 m below the base of the rectangular section?

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A. 583 B. 4,013 C. 2,354 D. 5,621

Solution: B

The *I*-value of the rectangular section about its own centroidal x-axis is –

$$I = \frac{bh^3}{12} = \frac{7(10)^3}{12} = 583 \text{ m}^4$$

This *I*-value is to be transferred to the *x*-axis which is 2 m below the base of the rectangular section. Therefore, Ad^2 must be added to it as follows –

$$I_x = I + Ad^2 = 583 + (7x10)(7)^2 = 4,013 \text{ m}^4$$

Friction

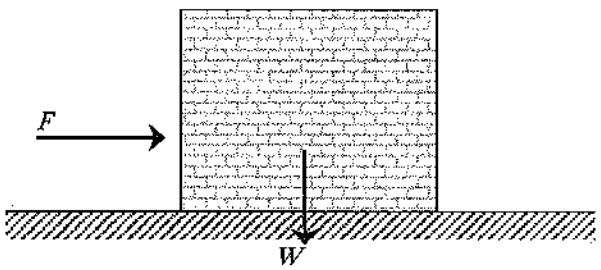
The largest frictional force is called the limiting friction. Any further increase in applied forces will cause motion.

$$F \le \mu_s N$$

F = frictional force
μ_s = coefficient of static friction, and
N = normal force between surfaces in contact

Problem 5.31

A 1 m x 1 m x 1 m block is resting on a horizontal surface as shown below. The density of the block is 2,000 kg/m³. If a force of F = 5 kN is applied, will the block move if coefficient of static friction is 0.3?



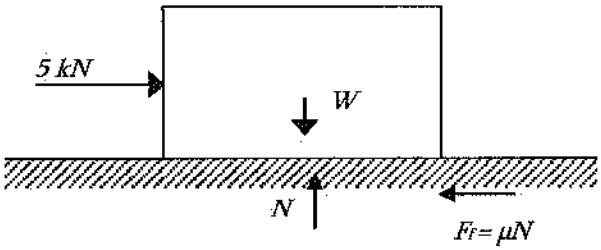
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A. Yes
B. No
C. More data is needed to determine

Solution: B

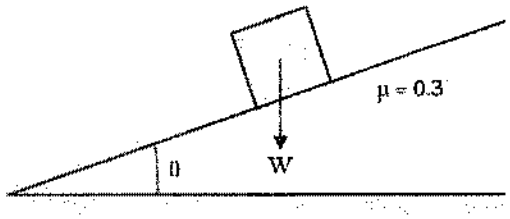
Normal reaction to the surface, $N = W = \rho g V = (2{,}000\ \text{kg/m}^3)(9.81)(1\ \text{m}^3) = 19{,}620\ \text{N}$
Maximum frictional force, $F_f = \mu N = 0.3\ (19{,}620\ \text{N}) = 5{,}886\ \text{N} = 5.9\ \text{kN}$
Driving force, $F = 5\ \text{kN} < F_f$

As $F_f > F$, the block will not move.



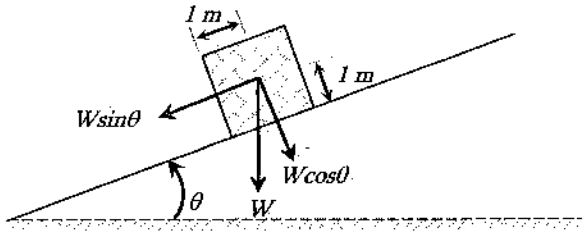
Problem 5.32

A 2 m x 2 m x 2 m block is resting on an inclined surface as shown below. The density of the block is 2,000 kg/m³. Most nearly, at what inclination (θ), will the block intend to tip over?



A. 30° B. 40° C. 45° D. 60°

Solution: C



Taking a moment about the tipping point, which is the lower bottom point of the block:
Resistive moment: $W(\cos\theta)(1\,\text{m})$
Over turning moment: $W(\sin\theta)(1\,\text{m})$

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At the moment of intending to tip, Resistive moment = Over turning moment

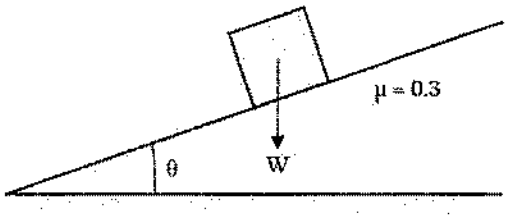
⇨ $W(\cos\theta)(1.0\,\text{m}) = W\sin\theta(1.0\ \text{m})$

⇨ $\tan\theta = 1$

⇨ $\theta = 45°$

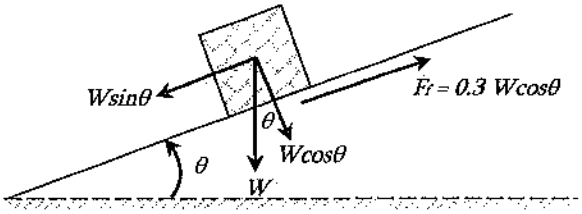
Problem 5.33

A 1 m x 1 m x 1 m block is resting on an inclined surface as shown below. Most nearly at what inclination (θ), will the block intend to slide?



A. 14° B. 17° C. 21° D. 30°

Solution: B



As shown above, Normal reaction to the inclined surface, N = Wcosθ

Friction force, $F_f = \mu N = \mu W\cos\theta = 0.3\text{W}\cos\theta$

Driving force, $F = W\sin\theta$

At the moment of sliding, Friction force=Driving force

⇨ $0.3\,W\cos\theta = W\sin\theta$

⇨ $\tan\theta = 0.3$

⇨ $\theta = 16.7°$

Belt Friction

If a belt is used to pull an object over a pulley with no friction, the belt force at the two ends are different. The force (F_1) at the tight end can be expressed as follows:

$$F_1 = F_2 e^{\mu\theta}$$

where

F_1 = force being applied in the direction of impending motion

F_2 = force applied to resist impending motion

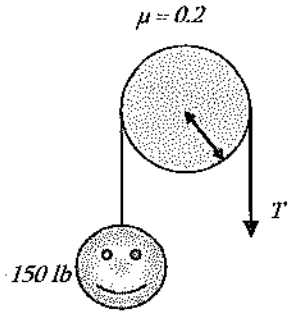
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μ = coefficient of static friction, and
θ = the total angle of contact between the surfaces expressed in radian
As, $F_1 > F_2$ the above equation is also written as: $T_{high} = T_{low} e^{\mu\theta}$

For a frictionless pulley, forces at the both sides of the pulley are equal, $F_1 = F_2$.

Problem 5.34

The minimum force, T to lift the 150-lb man is most nearly:



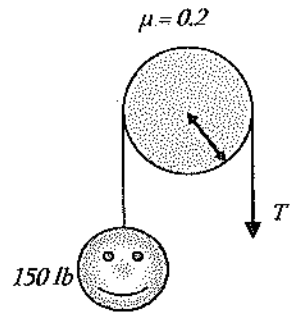
A. 150 lb B. 80 lb C. 280 lb D. 100 lb

Solution: C

To lift the man, 'T' will be higher than the man's weight.
$T_{low} = 150$ lb
$T_{high} = T_{low} e^{\mu\theta} = 150e^{(0.2)(\pi)} = 150(1.87) = 281$ lb

Problem 5.35

The minimum force, T to hold the 150-lb man in position is most nearly:



A. 150 lbs B. 80 lbs C. 280 lbs D. 100 lbs

Solution: B

To hold the man, 'T' will be lower than the man's weight.
$T_{high} = 150$ lb

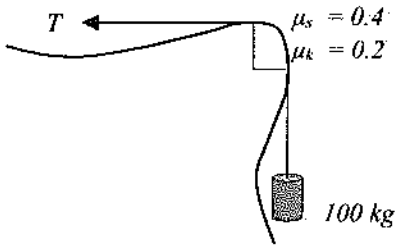
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$T_{high} = T_{low} e^{\mu\theta} = T_{low} e^{(0.2)(\pi)} = 150$ lb

$T_{low} = 150/1.87 = 80$ lb

Problem 5.36

A weight of 100 kg is to be lowered over the quarter-circled rock cliff with a constant velocity. The frictional coefficients are also shown. The force required to do this operation is most nearly:



A. 716 N
B. 1344 N
C. 981 N
D. None of the above

Solution: A

$\mu = \mu_k = 0.2$ (as the body will be in motion)

$\theta = 90° = \pi/2$

$T = T_{low}$ as the body is to go down

$T_{high} = T_{low} e^{\mu\theta}$

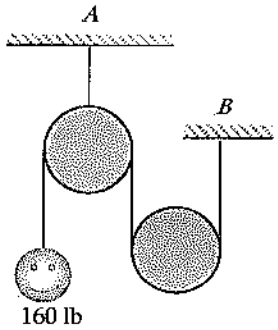
$(100\ kg)(9.81) = T_{low} e^{(0.2)\left(\frac{\pi}{2}\right)}$

981 N = T_{low} (1.37)

$T_{low} = 716$ N

Problem 5.37

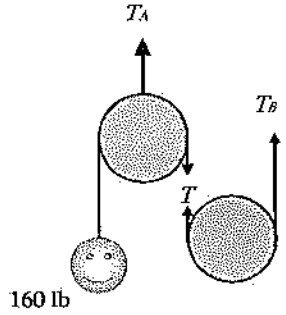
For the following frictionless pulley system, what is the tension at the hangers *A* and *B* respectively?



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A. 160, 160 lbs
B. 160, 320 lbs
C. 320, 160 lbs
D. 320, 320 lbs

Solution: C



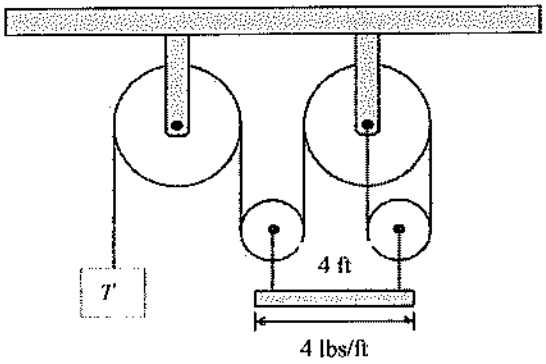
Using the principle, 'Same Cable Same Force'
From the left pulley: $T = 160$ lb
From the right pulley: $T_B = T = 160$ lb

Applying $\sum F_y = 0$ in the left pulley
$T_A = T + 160 = 320$ lb

Remember: same cable, same force in a frictionless pulley.

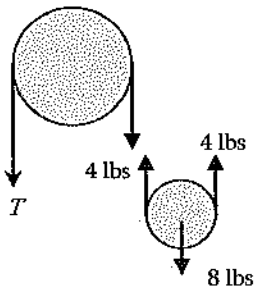
Problem 5.38

A uniform plank of length of 4 ft has mass per unit length 4 lbs/ft and is held up in the air horizontally by light inextensible strings tied to each end of the plank. Each of these strings is attached to a small light pulley which is connect to two large pulleys by a single string as shown in the figure. If there is no friction, what is the T required to keep the system in equilibrium?



A. 4 lbs B. 8 lbs C. 16 lbs D. 24 lbs

Solution: A



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Statically Determinate Truss

A truss is a structure consisted of pin-connected two-force members. Two-force member means that it sustains either tension or compression, not moment. The self-weight of the member is either neglected or assumed, acting at the ends only. As pin-connected members, the members cannot sustain moment. Some of the members in trusses are zero force members. This means the member is not designed to carry any load. However, this member is provided for safety if the load conditions change and to provide additional stability. There are two rules to finding the zero force members:

> **Rule 1.** If two non-collinear members form an unloaded joint, both are zero-force members.
> **Rule 2.** If three members form an unloaded joint of which two are collinear, then the third member is a zero-force member.

A two-force body in static equilibrium has two applied forces that are equal in magnitude, opposite in direction, and collinear.

Method of Joints

In the method of joints, each joint is considered an equilibrium problem, which involves a concurrent force system. A free body diagram of each joint is constructed and two force equations of equilibrium are written to solve for the two unknown member forces. *Let us review the steps for the joint method:*

> **Step 1.** Isolate a joint from the truss that contains no more than two unknowns.
> **Step 2.** Draw a free body diagram of the joint and solve for the two unknown member forces by satisfying the horizontal and vertical conditions of equilibrium.
> **Step 3.** Select another joint (usually adjacent to the joint just completed) that contains no more than two unknowns and solve the two conditions of equilibrium.
> **Step 4.** Continue this process until all member forces are found.
> **Step 5.** External support reactions may not be required to find out.

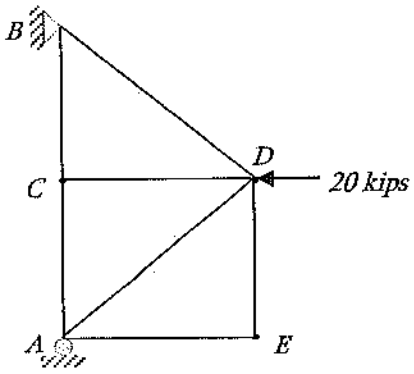
Method of Sections

The second method of analysis is called the section method. The method of sections is a particularly useful analysis technique when only a few selected member forces in a truss are desired. In the method of sections, an imaginary section cut is passed through the truss, cutting members, which includes at least one of the members of interest. The truss is now cut into two parts and a free body diagram is constructed of either section. Since all forces in the free body diagram are not concurrent at a common point, this constitutes a rigid body type problem, and three equations of equilibrium may be written. These three equations of equilibrium can solve the only three unknown member forces.

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Problem 5.39

The zero-force member(s) of the following truss is(are):



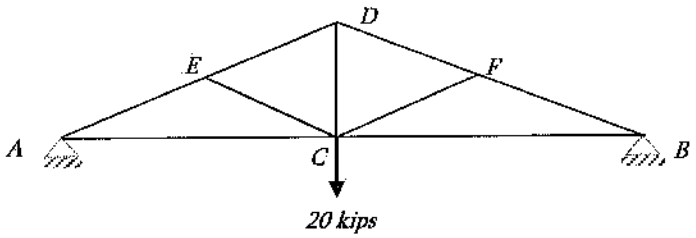
A. *AE* B. *ED* C. *CD* D. All of the above

Solution: D

AE and *ED* are zero force members by Rule 1.
CD is a zero-force member by Rule 2.

Problem 5.40

The zero-force member of the following truss is:



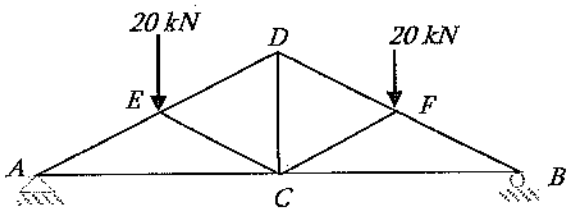
A. *AE* B. *BC* C. *CE* D. None of the above

Solution: C

CE and *CF* are zero force members by Rule 2.

Problem 5.41

The zero-force member(s) of the following truss is(are):



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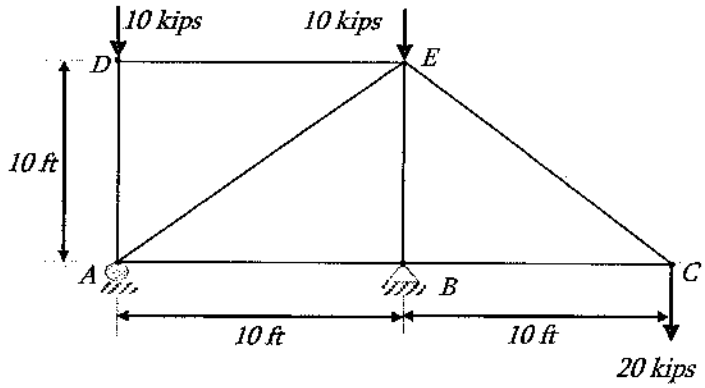
A. *AE*
B. *BC*
C. *CE*
D. None of the above

Solution: D

None of the members is a zero-force member. Joints *A, B, E* and *F* have loads on these. Therefore, members associated with these joints might have forces. Joint *D* has three members with no load; however, no two members are linear. So, the associated three members may not be zero-force member. Joint *C* has five members; no zero-force member can be found from this joint.

Problem 5.42

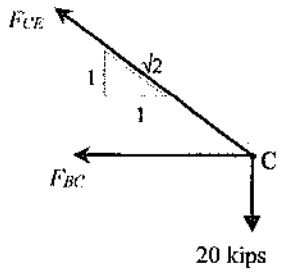
What is most nearly the member force at *CE* of the following plane truss?



A. -20 kips B. 14 kips C. -28 kips D. 28 kips

Solution: D

Considering joint C:



$$\sum F_y = 0 \ (\uparrow +ve)$$

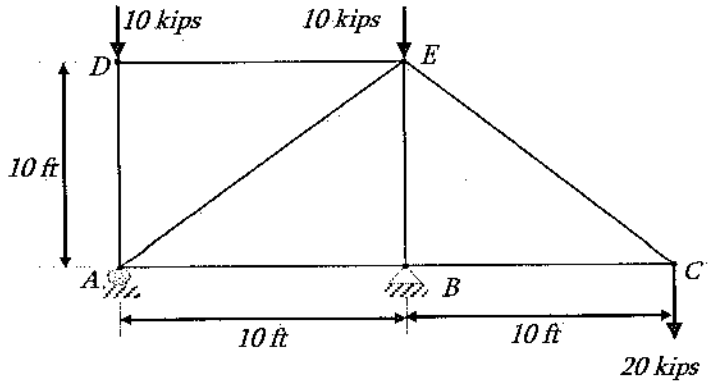
$$\Rightarrow \ F_{CE}\left(\frac{1}{\sqrt{2}}\right) - 20 = 0$$

$$\Rightarrow \ F_{CE} = 28.3 \text{ kips (Tension)}$$

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Problem 5.43

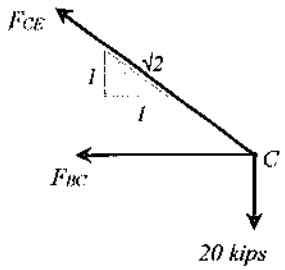
The member force at *CB* of the following plane truss is most nearly:



A. 40 kips B. -20 kips C. -40 kips D. 28 kips

Solution: B

Considering joint C:



$\sum F_y = 0\ (\uparrow +ve)$

$\Rightarrow \quad F_{CE}\left(\frac{1}{\sqrt{2}}\right) - 20 = 0$

$\Rightarrow \quad F_{CE} = 28.3$ kips (Tension)

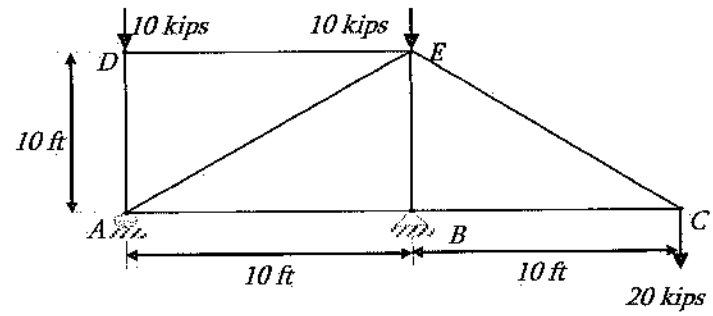
$\sum F_x = 0\ (\rightarrow +ve)$

$\Rightarrow \quad -F_{BC} - F_{CE}\left(\frac{1}{\sqrt{2}}\right) = 0$

$\Rightarrow \quad F_{BC} = -28.3/\sqrt{2} = -20$ kips

Problem 5.44

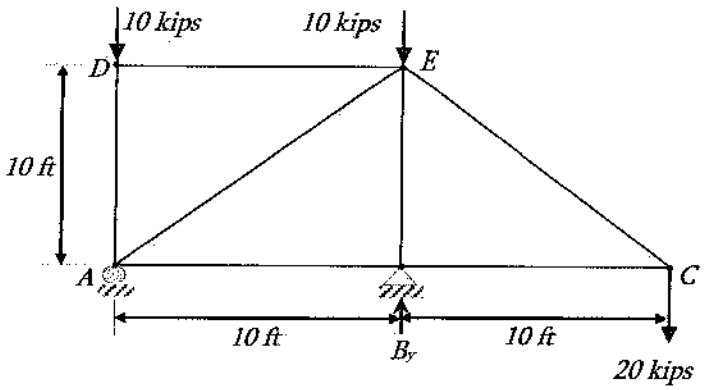
What is most nearly the member force at *BE* of the following plane truss?



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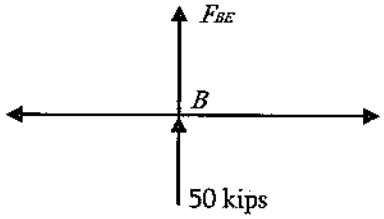
A. 40 kips (T)
B. 50 kips (T)
C. 50 kips (C)
D. 28 kips (C)

Solution: C



$\sum M_A = 0$
10 kips (10 ft) + 20 kips (20 ft) – B_y (10) = 0
B_y = 50 kips

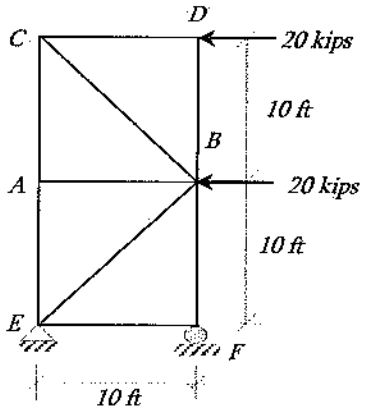
Considering joint B:



$\sum F_y = 0 (\uparrow +ve)$
$F_{BE} + 50 = 0$
F_{BE} = - 50 kips

Problem 5.45

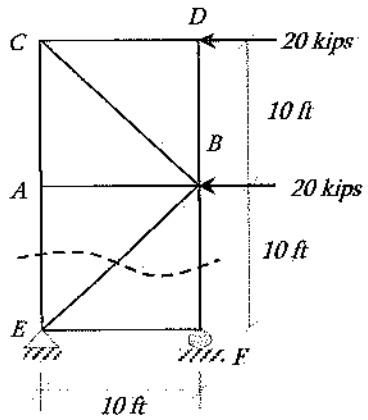
The member force (kips) at *BE* of the following truss is most nearly:



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A. 56.6 (T) B. 56.6 (C) C. 28.6 (T) D. 33.2 (C)

Solution: B



Consider the upper part of the cut section:

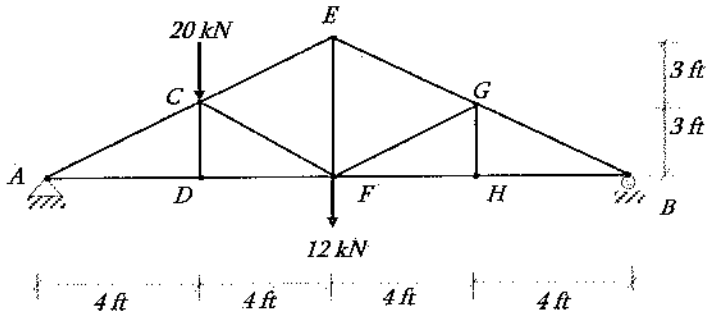
$$\sum F_x = 0 \quad (\leftarrow +ve)$$

$$\Rightarrow \quad F_{BE}\left(\frac{1}{\sqrt{2}}\right) + 20 + 20 = 0$$

$$\Rightarrow \quad F_{BE} = -40\sqrt{2} = -56.6 \text{ kips}$$

Problem 5.46

The following truss is supporting two loads. The member forces at *CD*, and *CF* are, respectively –

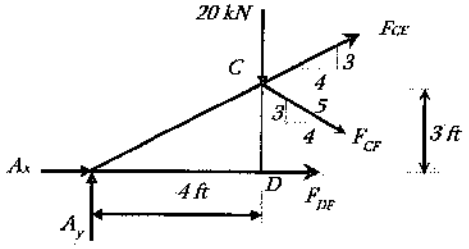


A. 6.5 kN (T), 16.67 kN (T)
B. 0 kN, 16.67 kN (C)
C. 0 kN, 16.67 kN (T)
D. 16.67 kN (C), 0 kN

Solution: B

First, considering the joint – D, and applying $\sum F_y = 0$, it is found that *CD* is a zero-force member. Next, consider the following section:

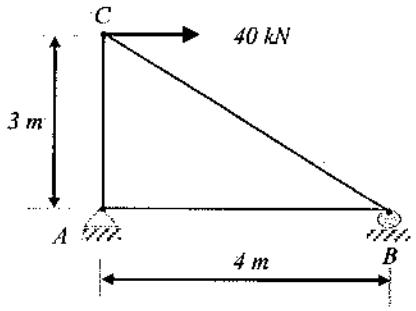
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$\sum M$ at $A = 0$ (assuming clock-wise moment is positive)
20 kN (4 ft) + F_{CF} (3/5) (4 ft) + F_{CF} (4/5) (3 ft) = 0
$F_{CF} = -16.67$ kN

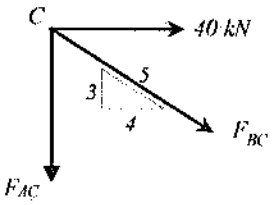
Problem 5.47

The force (kN) in member AC of the following truss is most nearly:



A. - 50 B. 20 C. 30 D. 40

Solution: C



$\sum F_x = 0$

$$F_{BC}\left(\frac{4}{5}\right) + 40 = 0$$

$$F_{BC} = -50 \text{ kN}$$

$\sum F_y = 0 \quad (\uparrow +ve)$

$$F_{BC}\left(\frac{3}{5}\right) + F_{AC} = 0$$

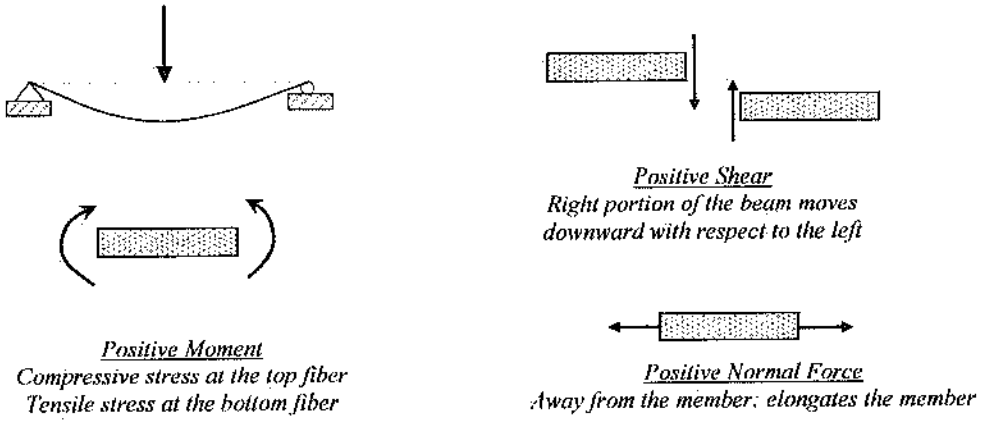
$$F_{AC} = -(-50)\left(\frac{3}{5}\right) \text{kN} = 30 \text{ kN}$$

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Internal Reactions

Internal reactions develop inside the member upon applying external loading. The internal reactions are internal moment, shear force, and normal (axial) force. It is important to determine the internal moments, shear force and axial force at any point in a beam structure. These values can then be used design the beam. In this section, the internal loads (moment, shear and axial) will be calculated for a specific location. If left part is considered the counter-clockwise moment is considered positive. On the other hand, if right part is considered the clockwise moment is considered positive. Similarly, downward V in the left part and upward V in the right part are positive. Normal force is always assumed positive (tension) away from the member. The shearing force and bending moment sign conventions can also be described as follows:

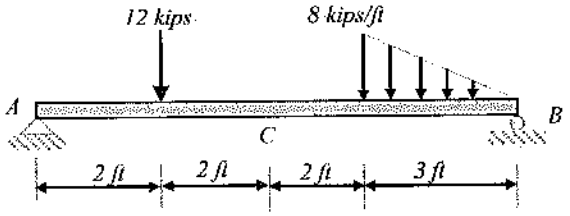
- The bending moment is positive if it produces bending of the beam concave upward (compression in top fibers and tension in bottom fibers).
- The shearing force is positive if the right portion of the beam downward with respect to the left.



Sign-convention for internal reactions

Problem 5.48

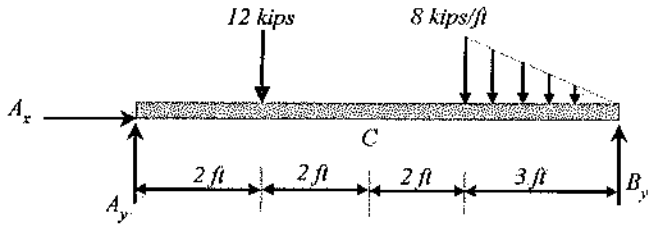
For the following beam, the developed internal moment (kips.ft) at C is most nearly:



A. 0
B. 12
C. 18
D. 24

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Solution: D



$\sum M$ at $B = 0$ (assuming counter-clock-wise moment is positive)

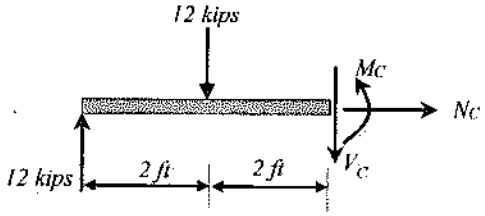
½ (8 kips/ft)(3 ft) (2 ft) + 12 kips (7 ft) – A_y (9 ft) = 0

A_y = 12 kips

$\sum F_x = 0 \quad (\rightarrow +ve)$

A_x = 0 as no lateral load

Cut at C and consider the left part:



$\sum M$ at $C = 0$ (assuming clock-wise moment is positive)

$-M_C$ – 12 kips (2 ft) + 12 kips (4 ft) = 0

M_C = 24 kips-ft

Remember to take two calculators for the NCEES FE exam. One might not work during the exam. NCEES allows few specific models of calculators. Check ncees.org website to learn which calculators are allowed.

- *FE exam is held in January, February, April, May, July, August, October, and November*
- *FE exam allows an examinee to take the exam once during a 2-month window*
- *FE exam does not allow to take the FE exam more than 3 times during a twelve-month period*

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6 DYNAMICS

Fundamentals of Engineering (FE) CIVIL Exam Specification mentions, 4–6 questions may appear in the exam based on Dynamics as follows:

A. Kinematics (e.g., particles and rigid bodies)
B. Mass moments of inertia
C. Force acceleration (e.g., particles and rigid bodies)
D. Impulse momentum (e.g., particles and rigid bodies)
E. Work, energy, and power (e.g., particles and rigid bodies)

The above-mentioned contents are categorized as follows.

Section 1: Kinematics of Particles
Section 2: Kinematics of Rigid Body
Section 3: Kinetics of Particles
Section 4: Kinetics of Rigid Body

Section 1: Kinematics of Particles

Kinematics is one of the major branches of Dynamics. It deals with the state of motion of a particle without the consideration of mass and force of the particle. The motion can be rectilinear, curvilinear, circular and so on.

Particle Rectilinear Motion

The velocity of a particle can be determined by dividing the displacement by total time. This velocity is nothing but the average velocity. However, the motion of the particle with time may be uniform or non-uniform. If the motion of a particle is non-uniform, the moving path can be divided into a number of elements. Each of these elements, dr can be divided by the time, dt taken to travel this distance (element) to determine the velocity. Then, the velocity of the particle during the travel of this element can be expressed as:

$$v = \frac{dr}{dt} \text{ or } v = \frac{ds}{dt} \text{ if displacement is expressed by } s \text{ instead of } r.$$

Acceleration is the rate of change of velocity, and can be expressed as:

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$$a = \frac{dv}{dt} = \frac{d^2r}{dt^2}$$

v = the instantaneous velocity

a = the instantaneous acceleration

t = time

The displacement (r), velocity (v), and acceleration (a) can be represented in vector form as follows:

$$r = xi + yj + zk$$

$$v = \dot{x}i + \dot{y}j + \dot{z}k$$

$$a = \ddot{x}i + \ddot{y}j + \ddot{z}k$$

$$\dot{x} = \frac{dx}{dt} = v_x \text{ and } \ddot{x} = \frac{d^2x}{dt^2} = a_x$$

In summary, the expressions for acceleration and velocity for uniform and non-uniform motion can be expressed as follows:

Variable, a	Constant, $a = a_c$
$a = \frac{dv}{dt}$	$v = v_o + a_c t$
$v = \frac{ds}{dt}$	$s = s_o + v_o t + \frac{1}{2} a_c t^2$
$ads = vdv$	$v^2 = v_o^2 + 2a_c(s - s_o)$

Problem 6.1

A particle's motion is represented by $s(t) = 22t + t^2 - 5t^3$ where s is in m, and t is in second. The particle's acceleration (m/s²) after 2 sec is most nearly:

A. 60 B. -60 C. 58 D. -58

Solution: D

$$v = \frac{ds}{dt} = \frac{d}{dt}\left(22t + t^2 - 5t^3\right) = 22 + 2t - 15t^2$$

$$a = \frac{dv}{dt} = 2 - 30t$$

At t = 2 sec, a = 2 - 30(2) m/s² = -58 m/s²

Problem 6.2

A particle's velocity is represented by $v(t)=22+2t$ m/s, t is in second. The particle's travel distance (m) after 10 sec is most nearly_____.

(write the answer in full number without unit such as 7 or 123 or 799 etc.)

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Solution: 320

$$v = \frac{ds}{dt}$$

$$s = \int v dt = \int_0^{10} (22 + 2t) dt = \left[22t + \frac{2}{2} t^2 \right]_0^{10} = 320 \text{ m}$$

Problem 6.3

A vehicle traveling at 50 miles per hour saw the yellow signal when it just approached the junction. He knew that the yellow signal at that junction lasts for 4 seconds and the junction is 150 feet long. The vehicle's acceleration (ft/sec^2) to cross the junction in time was most nearly:

A. 14.3
B. 16.8
C. 17.92
D. No acceleration is required

Solution: D

$$50 \text{ mph} = 50 \frac{\text{mile}}{\text{hr}} = 50 \frac{\text{mile}}{\text{hr}} \left(5280 \frac{\text{ft}}{\text{mile}} \right) \left(\frac{1}{3600} \frac{\text{hr}}{\text{sec}} \right) = 73.33 \text{ ft/s}$$

$$S = S_o + u_o t + \frac{1}{2} a t^2$$

$$150 = 73.33(4) + \frac{1}{2} a (4)^2 = 293.33 + 8a$$

a = - 17.92 ft/s^2 [-ve sign means, even with this deceleration, the vehicle can cross it. So, no acceleration is required]

Problem 6.4

A train starts from rest with an acceleration of 10 m/s^2. Parallel to this train, a car starts at the same time with uniform speed of 100 m/s. The train will overtake the car most nearly after:

A. 10 s B. 15 s C. 20 s D. 25 s

Solution: C

For train, $s = v_o t + \frac{1}{2} a t^2 = 0 + \frac{1}{2} (10 \, m/s^2)(t^2)$

$s = 5t^2$

For car, $s = vt = 100\ t$

Equating: $100\ t = 5\ t^2$

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$t(5t\text{-}100) = 0$

$t = 0$ or 20 sec

Therefore, $t = 20$ sec

Problem 6.5

Two engine-driven boats are racing with velocities of 10 m/s and 5 m/s. Their accelerations are 2 m/s^2 and 3 m/s^2, respectively. If the two boats reach the end at the same time, how long (sec) did those boats participate in the race?

A. 10 B. 15 C. 20 D. 5

Solution: A

For the 1st boat: $s = v_{o1}t + \frac{1}{2}a_1t^2 = 10t + \frac{1}{2}(2\ \text{m/s}^2)(t^2) = 10t + t^2$

For the 2nd boat: $s = v_{o2}t + \frac{1}{2}a_2t^2 = 5t + \frac{1}{2}(3\ \text{m/s}^2)(t^2) = 5t + 1.5t^2$

Equating: $10t + t^2 = 5t + 1.5t^2$

⇨ $0.5t^2 - 5t = 0$

⇨ $t(0.5t - 5) = 0$

⇨ $t = 0$ or 10 sec

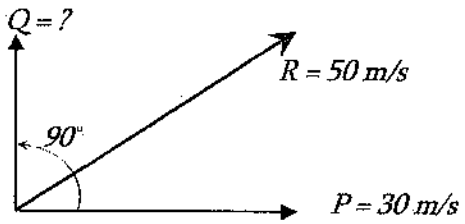
Therefore, $t = 10$ sec.

Problem 6.6

A baseball moving horizontally with a velocity of 30 m/s is hit by a bat at a right angle with the ball velocity. As a result, the ball acquires the velocity of 50 m/s. The batting velocity is most nearly:

A. 10 m/s B. 20 m/s C. 30 m/s D. 40 m/s

Solution: D



Using the Parallelograms rule, $R^2 = P^2 + Q^2 + 2PQ\cos 90$

⇨ $50^2 = 30^2 + Q^2 + 2(30)Q\cos 90$

⇨ $Q^2 = 50^2 - 30^2$

⇨ $Q = 40$ m/s

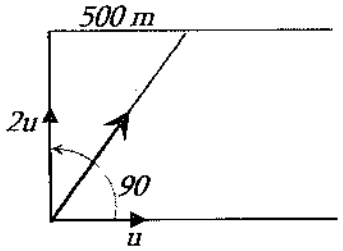
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Problem 6.7

A man swims at a right angle to the current of a river and meets the opposite bank 500 m from the starting point. If he swims at twice the speed of the current's speed, the width of the river (m) is most nearly:

A. 500 B. 750 C. 1,000 D. 1,250

Solution: C



The velocity of river and the velocity of swimming are constant. So, $s = vt$ equation is valid along the river and across the river.
Distance traveled along the river: $s = vt$, $t = 500/u$
Distance traveled across the river: $s = vt = 2u\,(500/u) = 1{,}000$ m

Problem 6.8

A man can swim directly across a river of 100-m width in 4 minutes if there is no current. He takes 5 minutes to cross directly when there is current. The velocity (m/min) of the current is most nearly:

A. 10
B. 15
C. 20
D. 25

Solution: B

Let,

u = river velocity, and v = swimmer velocity

If there is no current, $s = vt$

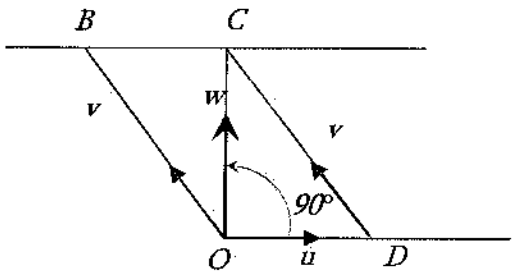
⇨ $v = s/t = 100$ m / 4 min = 25 m/min

If there is current, the following condition will occur. OB = swimmer velocity, $OC = w$ = resultant velocity, OD = river velocity. Draw a parallel line CD with OB; $OB = DC = v$ = swimmer velocity.

Again, $s = wt$

$w = 100$ m / 5 min = 20 min

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The resultant velocity will be perpendicular to the river velocity.

Therefore, $v^2 = u^2 + w^2$

$\Rightarrow \quad u = \sqrt{v^2 - w^2} = \sqrt{25^2 - 20^2} = 15 \text{ m/min}$

Problem 6.9

An airplane moving with a velocity of 50 km/h touches a straight highway and comes to rest in 300 m. If the retardation is uniform, the required time (sec) to come to a stop is most nearly:

A. 23 B. 33 C. 43 D. 49

Solution: C

$$u = 50\frac{\text{km}}{\text{h}} = \frac{50(1{,}000)}{3{,}600}\frac{\text{m}}{\text{s}} = 13.88 \text{ m/s}$$

$v^2 = v_o^2 - 2as$

$\Rightarrow \quad 0 = 13.88^2 - 2a(300)$

$\Rightarrow \quad a = 0.321 \text{ m/s}^2$

Again, $v = v_o - at$

$\Rightarrow \quad 0 = 13.88 - 0.321\, t$

$\Rightarrow \quad t = 42.9 \text{ s}$

Problem 6.10

A train traveling at 60 km/h is stopped at the next station in 10 sec by applying brakes. Most nearly, at what distance from the next station, were the brakes applied?

A. 63 m B. 73 m C. 83 m D. 93 m

Solution: C

Initial velocity,

$$u = 60\frac{\text{km}}{\text{h}} = \frac{60(1{,}000)}{3{,}600}\frac{m}{s} = 16.67 \text{ m/s}$$

Now, $v = v_o - at$

$\Rightarrow \quad 0 = 16.67 - a\,(10 \text{ s})$

$\Rightarrow \quad a = 1.67 \text{ m/s}^2$

$v^2 = v_o^2 - 2as$

$\Rightarrow \quad 0 = 16.67^2 - 2(1.67)(s)$

$\Rightarrow \quad s = 83.3 \text{ m}$

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Problem 6.11

A particle moving along a straight line with a uniform acceleration covers a distance of 72 cm at the 12th second, and a distance of 120 cm at the 20th second. If the acceleration of the particle is 6 cm/s², the initial velocity (cm/sec) of the particle is most nearly:

A. 1 B. 2 C. 2.5 D. 3

Solution: D

Traveled in 12 s, $S_{12} = S_o + u_o t + \frac{1}{2}at^2 = 0 + u_o(12) + \frac{1}{2}(6)(12)^2 = 12u_o + 432$

Traveled in 11 s, $S_{11} = S_o + u_o t + \frac{1}{2}at^2 = 0 + u_o(11) + \frac{1}{2}(6)(11)^2 = 11u_o + 363$

Traveled at the 12th sec = $S_{12} - S_{11}$

⇨ 72 cm = $(12u_o + 432) - (11u_o + 363)$

⇨ u_o=3 cm/s

Problem 6.12

A person goes to Walmart at a speed of 4 mph. He returns at a speed of 5 mph. His average speed (mph) is most nearly:

A. 4.444 B. 4.500 C. 4.540 D. 5.000

Solution: A

Let the one-way distance is 20 miles. Then, time to go = 20/4 = 5 hr and time to return = 20/5 = 4 hr. Then, average speed = total distance/total time= (20+20)/(5+4) = 4.444 mph

Particle's Rectilinear Motion under Gravity

All the equations listed in the previous section apply to particles in rectilinear motion under gravity except that the acceleration (a) is to be replaced by gravitational acceleration (g). It is always constant at a certain place in the universe and acts downward. The distance traveled, s is commonly replaced by the vertical distance, y or h. Therefore, the equations of motion can be written as –

$$v(t) = g(t - t_o) + v_o$$

$$y(t) = \frac{1}{2}g(t - t_o)^2 + v_o(t - t_o)$$

$$v^2 = v_o^2 + 2gh$$

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Problem 6.13

A ball projected vertically upward from the top of a tower with velocity of 14.5 m/s reaches the bottom after 5 sec. The height (m) of the tower is most nearly:

A. 30 B. 40 C. 50 D. 60

Solution: C

After being thrown upward from the top of tower, the ball will reach the peak, come back to the level of the top of tower, and then go to the ground. Therefore, the effective travel path is from the top of tower to the ground.

$$h = -u_o t + \frac{1}{2} g t^2 = -14.5(5s) + \frac{1}{2}(9.81)(5s)^2 = 50 \text{ m} \qquad [u_o \text{ upward}, g \text{ downward}]$$

Problem 6.14

A balloon is ascending vertically with a velocity of 9 m/s when a stone is let fall from it. If the stone reaches the ground in 10 s, the height (m) of the balloon when the stone is let fall is most nearly:

A. 300 B. 400 C. 500 D. 600

Solution: B

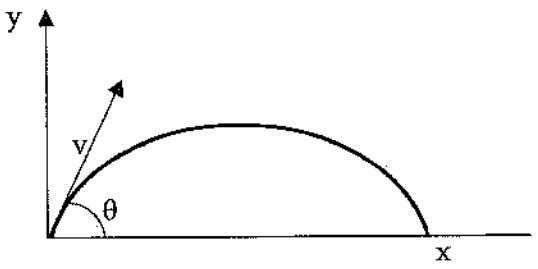
After letting the stone fall, it will travel upward a little bit then come back to its original releasing point, and then go to ground. Therefore, the effective travel path is from the releasing point to the ground.

$$h = -u_o t + \frac{1}{2} g t^2 = -9(10s) + \frac{1}{2}(9.81)(10s)^2 = 400 \text{ m} \qquad [u_o \text{ upward}, g \text{ downward}]$$

Projectile Motion

Projectile motion is a form of motion in which an object or particle is thrown with an angle to the Earth's surface and moves along a curved path under the action of gravity only. The velocity has both horizontal and vertical components. However, there is only vertical acceleration; that is the gravitational acceleration which acts downward. The equations for common projectile motion may be obtained from the constant acceleration equation as:

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$$a_x = 0$$
$$v_x = v_o \cos\theta$$
$$x = (v_o \cos\theta)t + x_o$$
$$a_y = -g$$
$$v_y = -gt + v_o \sin\theta$$
$$y = -\frac{1}{2}gt^2 + (v_o \sin\theta)t + y_o$$

Travel path of a projectile if thrown at 'v' velocity at an angle, θ with respect to horizon (x-axis)

Problem 6.15

A plane 2-miles above the ground is flying horizontally at 500 m/s. If it releases a bomb, the bomb will reach the ground in sec is most nearly:

A. 20 B. 26 C. 32 D. 52

Solution: B

$$2 \text{ miles} \times \left(5{,}280 \frac{\text{ft}}{\text{mile}}\right) \times \left(0.3048 \frac{\text{m}}{\text{ft}}\right) = 3{,}219 \text{ m}$$

$$\Rightarrow \quad h = h_o + u_o t + \frac{1}{2}gt^2$$

$$\Rightarrow \quad 3{,}219 \text{ m} = 0 + (0)(t) + \frac{1}{2}\left(9.8 \text{ m/s}^2\right)(t)^2$$

$$\Rightarrow \quad t = \sqrt{656.875} = 25.63 \text{ s}$$

Problem 6.16

A cannon-shell is fired with a velocity of 40 m/s toward an enemy plane at an angle of 30° with respect to the horizontal. Most nearly, at what height (m) will the shell strike a wall at 30 m away?

A. 10 B. 12 C. 14 D. 16

Solution: C

$v_{xo} = v_o \cos 30 = 40 \cos 30 = 34.64$ m/s

Horizontal Travel: $s = v_{xo} t$

$\Rightarrow$ 30 m = 34.64 m/s (t)

$\Rightarrow$ t = 0.866 sec

$v_{yo} = v_o \sin 30 = 40 \sin 30 = 20$ m/s

Vertical displacement after time, t:

$$y = v_{yo} t - \frac{1}{2}gt^2 = 20(0.866 \text{ sec}) - \frac{1}{2}(9.81)(0.866)^2$$

$$= 13.7 \text{ m}$$

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Problem 6.17

A fighter plane, while moving horizontally at a height of 490 m with a speed of 147 m/s dropped a bomb. Neglecting the air resistance, most nearly where will the bomb fall on the ground from the dropping point?

A. 1,070 m B. 1,270 m C. 1,470 m D. Cannot be determined

Solution: C

$$y = v_{yo}t + \frac{1}{2}gt^2; \qquad 490\text{ m} = 0 + \frac{1}{2}(9.81)(t)^2$$

$t = 10$ sec

Horizontal displacement, $x = v_{xo}\,t = 147$ m/s (10 sec) = 1,470 m

Problem 6.18

A fighter plane, while moving horizontally at a height of 480 m with a speed of 147 m/s dropped a bomb. Neglecting the air resistance, the velocity (m/s) of the bomb after 5 sec of dropping is most nearly:

A. 135 B. 145 C. 155 D. Cannot be determined

Solution: C

Horizontal velocity after 5 sec: $v_x = v_{xo} + a_x t = 147\ \text{m/s} + 0 = 147\ \frac{\text{m}}{\text{s}}$

Vertical velocity after 5 sec: $v_y = v_{yo} + a_y t = 0 + gt = 9.81(5) = 49.05\ \frac{\text{m}}{\text{s}}$

Velocity, $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(147)^2 + (49.05)^2} = 155$ m/s

Problem 6.19

A projectile with an initial velocity of 50 m/s is launched an angle of 45° with the horizon. The horizontal distance (m) traveled by the projectile is most nearly:

A. 552 B. 255 C. 127 D. None of the above

Solution: B

For projectile, the vertical velocity at the peak point is zero.

$$v_y = -gt + v_o \sin\theta$$

⇨ $0 = -9.81\ t + 50 \sin 45$

⇨ $t = 3.6$ sec (this time is needed to reach to the peak)

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Total travel time $T = 2\ t = 2(3.6) = 7.2$ sec

Horizontal distance traveled, $x = (v_o \cos\theta)t + x_o$

$x = 50 \cos 45\ (7.2 \text{ sec}) + 0 = 255$ m

Particle Curvilinear Motion

Curvilinear motion occurs when the particle moves in a curved path in plane or in 3D space. This section deals with the planar curvilinear motion. This type of motion can be explained using different co-ordinates systems. This is discussed in this section.

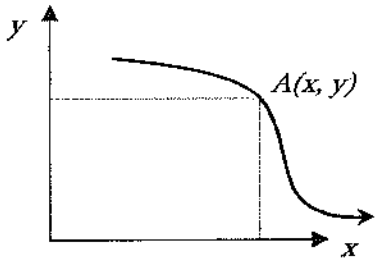
Rectangular Components of Curvilinear Motion

Consider a particle is moving along a non-straight path (curvilinear path) which is shown below using the rectangular coordinates system. At certain time, the position of the particle is A(x, y). If the unit vectors along x-, and y-axis are $\vec{i}$ and $\vec{j}$ respectively, then the position of the particle can be defined by –

$$\vec{r} = x\vec{i} + y\vec{j}$$

The velocity can be expressed as –

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(x\vec{i}) + \frac{d}{dt}(y\vec{j}) = v_x\vec{i} + v_y\vec{j}$$



The magnitude of the velocity can be determined as –

$$v = \sqrt{v_x^2 + v_y^2}$$

$$v_x = \frac{dx}{dt} = \dot{x}$$

$$v_y = \frac{dy}{dt} = \dot{y}$$

Similarly, the accelaration can be written as: $\vec{a} = \frac{d\vec{v}}{dt} = a_x\vec{i} + a_y\vec{j}$

The magnitude of the acceleration can be determined as: $a = \sqrt{a_x^2 + a_y^2}$

$$a_x = \frac{d^2x}{dt^2} = \ddot{x}; \qquad a_y = \frac{d^2y}{dt^2} = \ddot{y}$$

Problem 6.20

The position of a jet can be represented as $(x, y) = (0.3t^3,\ 0.2t^3)$ where x and y are in meter and t is in second. The acceleration (m/s²) of the jet after traveling 5 sec is most nearly:

A. 9

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B. 6
C. 10.8
D. 12.8

Solution: C

Velocity

$$v_x = \frac{dx}{dt} = \frac{d}{dt}\left(0.3t^3\right) = 0.9t^2$$

$$v_y = \frac{dy}{dt} = \frac{d}{dt}\left(0.2t^3\right) = 0.6t^2$$

Acceleration

$$a_x = \frac{d^2x}{dt^2} = \frac{d}{dt}\left(0.9t^2\right) = 1.8t$$

$$a_y = \frac{d^2y}{dt^2} = \frac{d}{dt}\left(0.6t^2\right) = 1.2t$$

At $t = 5$ sec

$$a_x = 1.8t = 1.8(5) = 9 \ \text{m/sec}^2$$

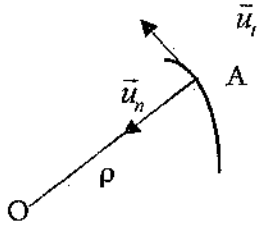
$$a_y = 1.2t = 1.2(5) = 6 \ \text{m/sec}^2$$

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(9)^2 + (6)^2} = 10.8 \ \text{m/sec}^2$$

Tangential and Normal Components of Curvilinear Motion

Consider a particle be moving in a curvilinear path as shown below, and its position at a certain time be A. The position of the particle at certain time is A. If $\vec{u}_t$ and $\vec{u}_n$ are the unit vectors along the tangential and normal directions respectively, then the velocity can be represented as –

$$\vec{v} = v_t\vec{u}_t = \frac{ds}{dt}\vec{u}_t = \dot{s}\vec{u}_t$$



Note that the velocity is always tangent to the path. It has no component along the normal direction. The acceleration can be written as –

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(v_t\vec{u}_t\right) = \vec{u}_t\frac{d}{dt}\left(v_t\right) + v_t\frac{d}{dt}\left(\vec{u}_t\right) = a_t\vec{u}_t + v_t\left(\frac{v_t}{\rho}\vec{u}_n\right)$$

Note, using the calculus, it can be shown that $\frac{d}{dt}\left(\vec{u}_t\right) = \frac{v_t}{\rho}\vec{u}_n$

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$$\vec{a} = a_t \vec{u}_t + \frac{v_t^2}{\rho}\vec{u}_n$$

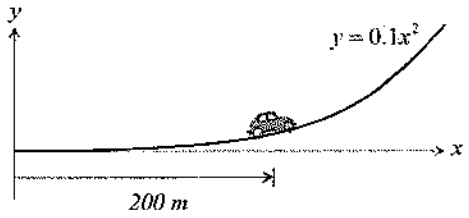
ρ = instantaneous radius of curvature

If the equation of the curve is known, the instantaneous radius of curvature can be determined as:

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$$

Problem 6.21

A car, shown below, is traveling along a path which can be expressed as $y = 0.1x^2$. After traveling 200 m, it attains a speed of 20 m/s and increase in speed of 2 m/s² along the tangential direction. The magnitude of the acceleration (m/s²) of the car at this instant is most nearly:



A. 1.0
B. 0.58
C. 2.0
D. 3.0

Solution: C

$$a = \sqrt{a_t^2 + a_n^2}$$

Given the increase in speed is 2 m/s² which is the tangential acceleration, $a_t = 2\frac{\text{m}}{\text{s}^2}$

The normal acceleration can be written as $a_n = \frac{v_t^2}{\rho}$ where, ρ (instantaneous radius of curvature) is unknown.

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$$

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$$y = 0.1x^2$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(0.1x^2\right) = 0.2x$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(0.2x) = 0.2$$

$$\frac{dy}{dx}\Big|_{x=200} = 0.2x = 0.2(200) = 40$$

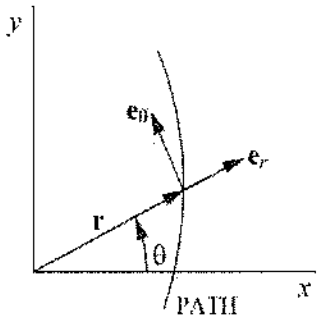
$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1+(40)^2\right]^{3/2}}{|0.2|} = 320{,}300 \text{ m}$$

$$a_n = \frac{v_t^2}{\rho} = \frac{(20)^2}{320{,}300 \text{ m}} = 0.0012 \frac{\text{m}}{\text{s}^2}$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(2)^2 + (0.0012)^2} = 2.0 \frac{\text{m}}{\text{s}^2}$$

Radial and Transverse Components of Curvilinear Motion

Consider a particle is moving in a curvilinear path shown below where r= radial position coordinate, θ= angle from the x-axis to $\vec{r}$. If $\vec{e}_r$ and $\vec{e}_\theta$ are the unit vectors with and normal to the position vector, $\vec{r}$ respectively, then the position vector can be represented as –



$$\vec{r} = r\vec{e}_r$$

The velocity can be represented as : $\vec{v} = \dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta$

The acceleration can be written as: $\vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\vec{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\vec{e}_\theta$

r= radial position coordinate
θ= angle from the x-axis to $\vec{r}$

$$\dot{r} = \frac{dr}{dt}; \qquad \ddot{r} = \frac{d^2r}{dt^2}$$

$$a_r = \ddot{r} - r\dot{\theta}^2; \quad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

Problem 6.22

The car travels along the circular curve of radius of 50 ft with a constant speed of 10 ft/s. The magnitude of the acceleration (ft/s²) of the car at this instant is most nearly:

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A. 0.5
B. 1.0
C. 1.5
D. 2.0

Solution: D

$$r = 50 \text{ ft}$$

$$\dot{r} = \frac{dr}{dt} = 0$$

$$\ddot{r} = \frac{d^2r}{dt^2} = 0$$

$$v_r = \dot{r} = 0$$

$$v_\theta = r\dot{\theta} = 50(\dot{\theta})$$

$$v = \sqrt{(v_r)^2 + (v_\theta)^2} = \sqrt{(0)^2 + (50\dot{\theta})^2} = 10 \frac{\text{ft}}{\text{sec}}$$

Solving $\dot{\theta} = 0.2 \frac{\text{rad}}{\text{sec}}$

$$\ddot{\theta} = \frac{d}{dt}\dot{\theta} = 0$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 50(0.2)^2 = -2 \frac{\text{ft}}{\text{sec}^2}$$

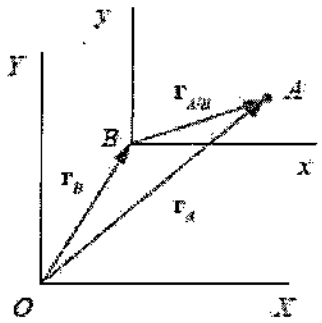
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 50(0) + 2(0)(0.2) = 0$$

$$a = \sqrt{(a_r)^2 + (a_\theta)^2} = \sqrt{(-2)^2 + (0)^2} = 2 \frac{\text{ft}}{\text{sec}^2}$$

Relative Motion

Relative motion is the calculation of the motion of an object with respect to some other moving object. Thus, the motion is not calculated with reference to the earth, but is the velocity of the object with regard to the other moving object as if it were static. For example, a person sitting in a bus is at zero velocity relative to the bus, but is moving at the same velocity as the airplane with respect to the ground. Relative motion is a concept, and its calculation occurs with relative velocity, relative speed, or relative acceleration which is the change in velocity divided by the change in time. Consider particles A and B are moving independently along two paths. The position of each particle is measured from the common origin O of the fixed XY reference axis. The origin of the second frame of reference xy is attached to particle B and moves with it. The axes of this frame are only permitted to translate relative to the fixed frame, XY. The relative position of "A with respect to B" is designated by a relative vector, $\vec{r}_{A/B}$.

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Then, the position vectors can be related as follows:

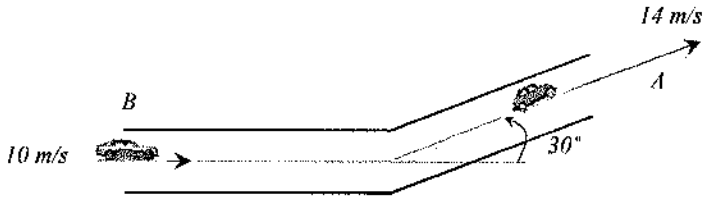
$$\vec{r}_A = \vec{r}_B + \vec{r}_{A/B}$$

The velocities and accelerations of the particles can be presented as follows:

$$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$$
$$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$$

Problem 6.23

At the instant shown, the car at A is traveling at 14 m/s with acceleration of 2 m/s² and the car at B is traveling at 10 m/s with acceleration of 3 m/s². The relative velocity (m/s) of A with respect to B at this instant is most nearly:



A. 3.31 B. 5.31 C. 7.31 D. 9.31

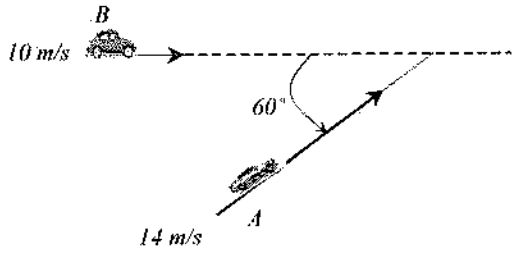
Solution: C

$$\vec{v}_A = 14\cos 30\vec{i} + 14\sin 30\vec{j} = 12.12\vec{i} + 7\vec{j}$$
$$\vec{v}_B = 10\vec{i}\ \frac{\text{m}}{\text{s}}$$
$$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$$
$$\vec{v}_{A/B} = \vec{v}_A - \vec{v}_B = 12.12\vec{i} + 7\vec{j} - 10\vec{i} = 2.12\vec{i} + 7\vec{j}$$
$$v_{A/B} = \sqrt{(2.12)^2 + (7)^2} = 7.31\frac{\text{m}}{\text{s}}$$

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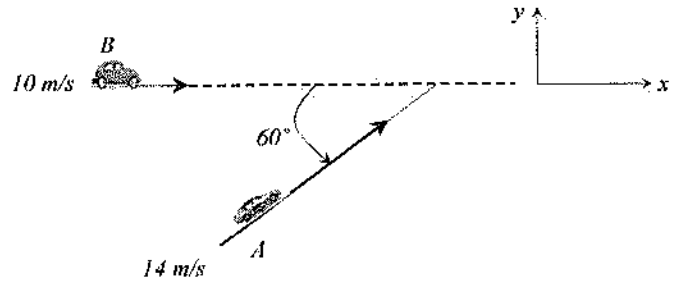
Problem 6.24

Two cars, A and B are running in two different roadways with velocity of 14 m/s and 10 m/s respectively as shown below. The roads are aligned at an angle of 60°. The relative velocity (m/s) of B with respect to A at this instant is most nearly:



A. 9.5
B. 12.5
C. 15.5
D. 17.5

Solution: B



Let, $\vec{i}$ and $\vec{j}$ be the unit vectors along x- and y-axis, respectively.

$$\vec{v}_A = 14\cos 60\,\vec{i} + 14\sin 60\,\vec{j}\ \frac{\text{m}}{\text{s}}$$

$$\vec{v}_B = 10\,\vec{i}\ \frac{\text{m}}{\text{s}}$$

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

$$\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A = 10\vec{i} - 14\cos 60\,\vec{i} - 14\sin 60\,\vec{j} = 3\vec{i} - 12.12\,\vec{j}$$

Magnitude of $v_{B/A} = \sqrt{(3)^2 + (-12.12)^2} = 12.5\ \frac{\text{m}}{\text{s}}$

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Section 2: Kinematics of Rigid Body

A rigid body may undergo two major types of motions:

a) Translation
b) Rotation about a fixed axis
c) General plane motion

General plane motion is commonly not covered. Let us discuss the other two types. Translation of rigid body occurs when all line segment of a body remains parallel to its initial position during the motion. Thus, the velocity can be presented as –

$$\vec{v}_A = \vec{v}_B$$

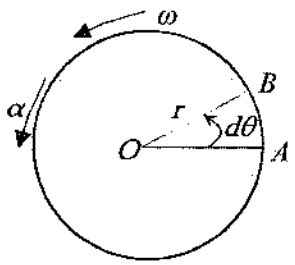
where $\vec{v}_A$ and $\vec{v}_B$ are the velocities measured at two locations A, and B. Like the velocity, the acceleration can be presented as –

$$\vec{a}_A = \vec{a}_B$$

This equation indicates that all points in a rigid body during rectilinear or curvilinear translation move at the same acceleration.

Rotation about a Fixed Axis

Let, a rigid body is rotating about a fixed axis which passage through O and is perpendicular to the OAB plane. The body was originally along OA position and reaches at B rotating $d\theta$ at time t.



Therefore, the rate change of instantaneous angular displacement is called the instantaneous angular velocity, ω. This means –

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Usually, the angular velocity means the instantaneous velocity. If the angular velocity is constant, then that circular motion is called the uniform circular motion. In case of uniform circular motion, the angular velocity can be written as –

$$\omega = \frac{\theta}{t}$$

$$\theta = \omega t$$

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This equation is similar to the equation of uniform linear motion, $s = vt$. The angular velocity is expressed in radian/sec. it is also called rotation or revolution per minute or rpm.

Time taken for a complete rotation is called the time period; one complete rotation means 2π radian angular displacement. So, if the time period is T, then –

$$\omega = \frac{2\pi}{T} = 2\pi n$$

where, $n = \frac{1}{T}$ is the number of rotation in unit time or frequency. Its unit is s^{-1} or Hz. If $d\theta$ is small, the length AB can be determined as $ds = rd\theta$. Then the linear velocity can be determined as $v = \frac{ds}{dt} = \frac{rd\theta}{dt} = r\left(\frac{d\theta}{dt}\right) = r\omega = \omega r$. Therefore,

$$v = \omega r$$

The angular acceleration is defined as –

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

The linear acceleration (a) can be determined as follows –

$$\frac{dv}{dt} = \frac{d}{dt}(\omega r)$$

$$a = \omega \frac{d}{dt}(r) + r\frac{d}{dt}(\omega)$$

$$a = \omega(0) + r\alpha$$

$$a = \alpha r$$

From the relationship of $\omega = \frac{d\theta}{dt}$ and $\alpha = \frac{d\omega}{dt}$ the following relationship can be derived –

$$\alpha d\theta = \omega d\omega$$

This linear acceleration is along the tangential direction and very often called tangential acceleration (a_t). Therefore,

$$a_t = \alpha r$$

The linear acceleration along the normal direction can be determined as follows –

$$a_n = \frac{v^2}{r} = \frac{(\omega r)^2}{r} = \omega^2 r$$

The magnitude of the linear acceleration can be determined as follows –

$$a = \sqrt{(a_t)^2 + (a_n)^2}$$

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a) For constant Acceleration

Applying Newton's law of motion, for constant angular acceleration, the equations for angular velocity and displacement are –

$$\alpha(t) = \alpha_o$$
$$\omega(t) = \alpha_o(t - t_o) + \omega_o$$
$$\theta(t) = \frac{1}{2}\alpha_o(t - t_o)^2 + \omega_o(t - t_o) + \theta_o$$

where

θ = angular displacement
θ_o = angular displacement at time t_o
ω = angular velocity
ω_o = angular velocity at time t_o
α_o = constant angular acceleration
t = time
t_o = some initial time

An additional equation for angular velocity as a function of angular position may be written as –

$$\omega^2 = \omega_o^2 + 2\alpha(\theta - \theta_o)$$

b) Non-constant Acceleration

When non-constant acceleration, $\alpha(t)$ is considered, the equations for the velocity and displacement may be obtained from –

$$v(t) = \int_{t_o}^{t} \alpha(\tau)d\tau + v_{t_o}$$
$$s(t) = \int_{t_o}^{t} v(\tau)d\tau + s_{t_o}$$

For variable angular acceleration –

$$\omega(t) = \int_{t_o}^{t} \alpha(\tau)d\tau + \omega_{t_o}$$
$$\theta(t) = \int_{t_o}^{t} \omega(\tau)d\tau + \theta_{t_o}$$

where τ is the variable of integration.

Rigid Body Motion About a Fixed Axis - Summary

Variable, α	Constant, $\alpha = \alpha_c$
$\alpha = \frac{d\omega}{dt}$	$\omega = \omega_o + \alpha_c t$
$\omega = \frac{d\theta}{dt}$	$\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha_c t^2$
$\alpha d\theta = \omega d\omega$	$\omega^2 = \omega_o^2 + 2\alpha_c(\theta - \theta_o)$

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Problem 6.25

An electric motor is rotating at 6,000 rpm when the switch is suddenly turned off. The fan decelerates at a constant rate and rests after 3 minutes. The angular deceleration of the electric fan is most nearly:

A. $\alpha_o = 4{,}000 \dfrac{rad}{\min^2}$

B. $\alpha_o = 400\pi \dfrac{rad}{\min^2}$

C. $\alpha_o = 4{,}000\pi \dfrac{rad}{\min^2}$

D. $\alpha_o = 4{,}000\pi \dfrac{rad}{\sec^2}$

Solution: C

$$\omega_o = 6{,}000\,\text{rpm} = 6{,}000\frac{\text{rev}}{\min}\left(2\pi\frac{\text{rad}}{\text{rev}}\right) = 12{,}000\pi\ \frac{\text{rad}}{\min}$$

$$\omega(t) = \omega_o + \alpha_o(t - t_o)$$

$$0 = 12{,}000\pi + \alpha_o(3\,\text{min})$$

$$\alpha_o = 4{,}000\pi \frac{rad}{\min^2}$$

Problem 6.26

The angular position of a car traveling around a curve is described by the $\theta(t) = 15t + 2t^2 - 6t^3 + 5$, where t is in seconds. The angular acceleration (rad/s^2) of the car at the beginning is most nearly:

A. 15
B. 36
C. 2
D. 4

Solution: D

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = \frac{d}{dt}\left(15 + 4t - 18t^2\right) = 4 - 36t$$

At, $t = 0$ sec, $\alpha = 4$ rad/sec^2

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Problem 6.27

An electric motor is rotating at 6,000 rpm when the switch is suddenly turned off. The fan decelerates at an angular deceleration of $\alpha_o = 4,000\pi \frac{rad}{\text{min}^2}$. The number of revolutions the fan will undergo before stopping is most nearly:

A. 9 B. 90 C. 900 D. 9,000

Solution: D

$$\omega_o = 6,000\,\text{rpm} = 6,000\frac{\text{rev}}{\text{min}}\left(2\pi\frac{\text{rad}}{\text{rev}}\right) = 12,000\pi\ \frac{\text{rad}}{\text{min}}$$

$$\alpha_o = 4,000\pi\ \frac{rad}{\text{min}^2}$$

$$\omega^2 = \omega_o^2 + 2(-a)(\theta - \theta_o)$$

$$0^2 = (12,000\pi)^2 - 2(4,000\pi)(\theta - \theta_o)$$

$\theta - \theta_o = 18,000\pi$ rad = $18,000\pi/2\pi = 9,000$ revolutions

Problem 6.28

The angular acceleration of a 0.4 ft diameter circular disk can be represented as $\alpha = 5t^2 + 6t + 2$ rad/sec^2, where t is in second. The original angular velocity of the disk was 6 rad/sec. The linear velocity (ft/s) after 5 seconds of rotation is most nearly:

A. 300 B. 60 C. 30 D. 10

Solution: B

Both the linear velocity and the linear acceleration require the angular velocity. So, let us determine angular velocity first.

$$\alpha = \frac{d\omega}{dt} = 5t^2 + 6t + 2\,;\ \text{On integrating}\ \int d\omega = \int (5t^2 + 6t + 2)dt$$

$$\int_6^{\omega} d\omega = \int_0^5 (5t^2 + 6t + 2)dt$$

$$[\omega]_6^{\omega} = \left[\frac{5t^3}{3} + 3t^2 + 2t\right]_0^5$$

$$[\omega - 6] = \left[\frac{5(5)^3}{3} + 3(5)^2 + 2(5)\right] - \left[\frac{5(0)^3}{3} + 3(0)^2 + 2(0)\right]$$

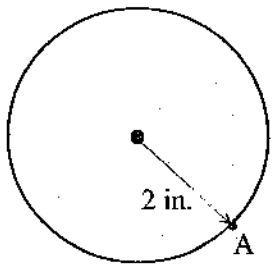
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$$\omega = 299.3\ \frac{\text{rad}}{\text{sec}}$$

Linear velocity, $v = \omega r = 299.3(0.2) = 59.9 \approx 60\ \frac{\text{ft}}{\text{sec}}$

Problem 6.29

The disk starts with an angular velocity of 1 rad/s, and is given an angular acceleration of $\alpha = 0.025\theta^{1.25}$ rad/s², where θ is in radians. The magnitudes of the normal components of acceleration (in/sec²) of a point A on the rim after 5 revolutions is most nearly:



A. 7.20
B. 104
C. 172
D. 3.98

Solution: B

$$\omega d\omega = \alpha d\theta$$

$$\int_{1\ \frac{rad}{sec}}^{\omega_A} \omega d\omega = \int_{0}^{\theta=10\pi} \left(0.025\theta^{1.25}\right) d\theta$$

$$\left|\frac{\omega^2}{2}\right|_1^{\omega_A} = \left[\frac{0.025}{2.25}\theta^{2.25}\right]_0^{10\pi}$$

$$\frac{\omega_A^2}{2} - \frac{(1)^2}{2} = \left[0.0111(10\pi)^{2.25}\right] - [0]$$

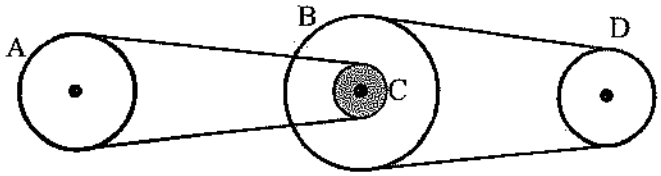
$$\omega = 7.20\ \frac{\text{rad}}{\text{sec}}$$

Normal acceleration, $a_n = \omega^2 r = (7.20)^2(2) = 103.7\ \frac{\text{in}}{\text{sec}^2}$

Problem 6.30

The power of a car engine is transmitted using the belt-and pulley arrangement shown below. The hub at C is rigidly connected to B and turns with it. The diameters of pulley A, B, C, and D are 0.5 ft, 0.7 ft, 0.2 ft and 0.4 ft respectively. If the engine turns pulley B at 50 rad/s, the angular velocity of the pulley A (rad/sec) is most nearly:

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A. 16
B. 18
C. 20
D. 22

Solution: C

$\omega_C = \omega_B = 50\dfrac{\text{rad}}{\text{sec}}$ as the hub at C is rigidly connected to B.

Linear speed at the periphery of Pulley C, $v_C = \omega_C r_C = 50\dfrac{\text{rad}}{\text{sec}}(0.1\,\text{ft}) = 5\dfrac{\text{ft}}{\text{sec}}$

This linear speed of C is transferred to the periphery of Pulley A.

$v_A = v_C$

$v_A = \omega_C r_C$

$$\omega_A = \frac{5\dfrac{\text{ft}}{\text{sec}}}{r_A} = \frac{5\dfrac{\text{ft}}{\text{sec}}}{0.25\,\text{ft}} = 20\frac{\text{rad}}{\text{sec}}$$

Section 3: Kinetics of Particle

Weight

The weight is the gravitational force executed on a body. It always acts downward toward the center of the earth. It can be calculated as follows:

$$W = mg$$

W = weight of a body (N or lbf)
m = mass of a body (kg, lbm)
g = local gravitational constant (typically, 9.81 m/s² or 32.2 ft/s²)

Newton's Second Law

Newton's Second law of motion states "rate of change of momentum of a body is proportional to applied force on the body and the direction along which the force acts, the change of momentum also occurs along that direction." Mathematically, Newton's Second law can be expressed as:

$$\sum F = \frac{d(mv)}{dt}$$

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$\sum F$ = the sum of the applied forces acting on the particle
m = the mass of the particle
v = the velocity of the particle

For constant mass, $\sum F = \dfrac{d(mv)}{dt} = ma$

Problem 6.31

Two forces, $\vec{F}_1 = 5\vec{i} + 2\vec{j} - 2\vec{k}$ N and $\vec{F}_2 = -3\vec{i} + \vec{j} - 7\vec{k}$ N are applied on a 100-g particle. The acceleration of the particle (m/s²) is most nearly:

A. 0.97 B. 9.7 C. 97 D. 970

Solution: C

Resultant Force, $\vec{F} = \vec{F}_1 + \vec{F}_2 = \left(5\vec{i} + 2\vec{j} - 2\vec{k}\right) + \left(-3\vec{i} + \vec{j} - 7\vec{k}\right) = 2\vec{i} + 3\vec{j} - 9\vec{k}$

$$|F| = \sqrt{F_x^2 + F_y^2 + F_z^2} = \sqrt{(2)^2 + (3)^2 + (-9)^2} = 9.7\,\text{N}$$

$$a = \frac{F}{m} = \frac{9.7\,\text{N}}{0.1\,\text{kg}} = 97\,\text{ms}^{-2}$$

Problem 6.32

A truck weighing 5 tons is moving with a speed of 36 km/hr. To stop the truck in 4 m, the amount of force in Newton to be applied is most nearly:

A. 65,200 B. 62,500 C. 52,500 D. 72,500

Solution: B

$$v_o = 36\frac{\text{km}}{\text{hr}} = 36\frac{\text{km}}{\text{hr}}\left(1{,}000\frac{\text{m}}{\text{km}}\right)\left(\frac{1}{3{,}600}\frac{\text{hr}}{\text{sec}}\right) = 10\,\text{ms}^{-1}$$

$$v^2 = v_o^2 - 2as$$

$$0 = 10^2 - 2a(4\,\text{m})$$

$$a = 12.5\ \text{m/s}^2$$

$F = ma = 5{,}000$ kg (12.5 m/s²) = 62,500 N

Problem 6.33

A body was at rest. A force of 15 N acted on it for 4 sec and then that force stopped acting. The body then moved 54 m in the next 9 sec. The mass of the body is most nearly:

A. 5 kg B. 8 kg C. 10 kg D. 12 kg

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Solution: C

Given:	*9 sec:*	*4 sec:*
F= 15 N	The force does not act after 4 sec. So, it moved next 9-sec at uniform velocity.	$v = v_o + at$
t_1 = 4 sec		⇨ 6 = 0 + a (4 sec)
t_2 = 9 sec		a = 1.5 m/s²
s = 54 m		
v_o = 0	$v = s / t_2$ = 54 m / 9 s = 6 m/s	Now, $F = ma$
m = ?		$m = F/a$ = 15 N / 1.5 m/s²
		= 10 kg

Problem 6.34

A 10-kg block begins sliding down in a 25° inclined plane. Starting from the rest, the block attains a velocity of 5 m/s after 5 seconds. Most nearly, how far will the block travel in another 5 seconds?

A. 12.5 m B. 37.5 m C. 75 m D. 100 m

Solution: B

Acceleration, $a = \dfrac{v - v_o}{t} = \dfrac{5\,m/s}{5\,s} = 1\,\dfrac{m}{s^2}$

Travel in the second 5 sec, $s = s_o + v_o t + \dfrac{1}{2}at^2$

⇨ $s = 0 + 5\,m/s\,(5\,s) + \dfrac{1}{2}(1\,m/s^2)(5^2) = 37.5\,m$

[Note: the travel amount in the first 5 sec is not calculated here as v_o is taken 5 m/s. The travel in the second 5 sec is determined here]

Potential Energy

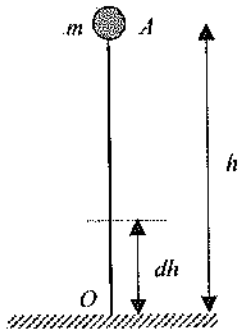
The energy that is acquired due to the position of a body or energy acquired by the body due to the change of position of the particles within the body is called the potential energy of the body. For example, a piece of brick is kept above the roof or pulling the spring or pushing the spring. Potential Energy (PE) is measured by the work that is available when the body returns to its initial normal position or configuration from the present position or configuration. There may be three types of PE:

i) Gravitational PE
ii) Elastic PE
iii) Electric PE (not included in the NCEES FE Exam)

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Gravitational Potential Energy

Let an object of mass, m be raised above the surface of the earth to a height, dh against the gravity.



Work done due to this movement –

$$d\vec{W} = \vec{F}.d\vec{h}$$
$$dW = F.dh$$

The total work done in raising the body to a height, h at position, A is the summation of the small work done as –

$$V = \int dW = \int_0^h F\,dh = \int_0^h mg\,dh = mg\int_0^h dh = mg\left[h\right]_0^h = mgh$$

Elastic Potential Energy

For a linear elastic spring with modulus, stiffness, or spring constant (k), the force in the spring is $F_s = ks$, where, s = the change in length of the spring from the unreformed length of the spring. In changing the deformation in the spring from position s_1 to s_2, the change in the potential energy stored in the spring is determined as:

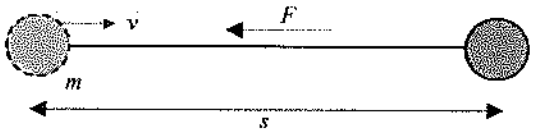
$$PE_2 - PE_1 = \frac{1}{2}k\left(s_2^2 - s_1^1\right)$$

Kinetic Energy

When a nail is struck by a hammer strongly on a wall the nail enters inside the wall overcoming the resistance of the wall. The hammer can do this work because of its motion, i.e., due to the kinetic energy of the hammer, the nail can overcome the resistance of the wall. You might have seen sailing boats in the river. Kinetic energy of the current in the river drives away the boat. When wind flows strongly, the boat moves faster if sail is hoisted. Again, while competing for high jump, and long jump do not jump from rest, but do jump after running a certain distance. As a result, they can go longer distance by using the kinetic energy. From this discussion, it is observed that while stopping a moving body by applying external force, the total work done by the body before to rest

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is the measure of kinetic energy of the body. In short, energy possessed by a body by virtue of its motion is called kinetic energy. Consider a body of mass, m is moving along AB with a velocity of v. A constant force F is applied opposite to the motion along BA. Due to this, uniform retardation will be produced. Let the uniform retardation = a, and the body came to rest at point B after covering a distance, s from the point, A. Hence the final velocity is $v = 0$.



Kinetic energy = work done before coming to rest
= Force x Distance Traveled before coming to rest
= Fs

From the Newton's Second Law of Motion, $F = ma$

Again, $v^2 = v_o^2 + 2as$

$$0 = v^2 - 2as$$

$$s = \frac{v^2}{2a}$$

Kinetic energy = $T = Fs = (ma)\left(\frac{v^2}{2a}\right) = \frac{1}{2}mv^2$

Work

If force is applied on a body and if there is a displacement of the body, then the product of the force and the component of displacement along the direction of force is called work (U). It is defined as: $U = \int F.\, dr$

Variable force:	$U_F = \int F\cos\theta\, ds$
Constant force:	$U_F = (F_c \cos\theta)\, \Delta s$
Weight:	$U_W = -W\Delta y$
Spring:	$U_s = -\left(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2\right)$

where: s_1 and s_2 are two different positions of the applied force end of the spring with $|s_2| > |s_1|$; θ is the direction of displacement with respect to the direction of the force applied.

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Work-Energy Theorem

If T_i and V_i are, respectively, the kinetic and potential energy of a particle at state i, then for conservative systems (no energy dissipation or gain), the law of energy is-

$$T_2 + V_2 = T_1 + V_1$$

If non-conservative forces are present, and the work done by these forces is present, then the work done by these forces must be accounted for. Hence –

$$T_2 + V_2 = T_1 + V_1 + U_{1\to 2}$$

$U_{1\to 2}$ = the work done by non-conservative forces is moving between state 1 and state 2. Care must be exercised during computations to correctly compute the algebraic sign of the work term. If the forces serve to increase the energy of the system, $U_{1\to 2}$ is positive. If the forces, such as friction, serve to dissipate energy, $U_{1\to 2}$ is negative. If multiple types of work are done such as work done by engine, brake, etc. then –

$$T_2 + V_2 = T_1 + V_1 + \sum U_{1\to 2}$$

Problem 6.35

If a spring is elongated by 15 cm its potential energy is 60 J. The amount of force (N) required to elongate this spring by 30 cm is most nearly:

A. 5,333
B. 1,600
C. 800
D. 400

Solution: B

$$W = \frac{1}{2}kx^2$$

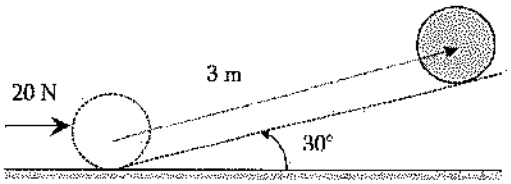
$$k = \frac{2W}{x^2} = \frac{2(60\,\text{J})}{(0.15\,\text{m})^2} = 5{,}333\ \text{Nm}^{-1}$$

$$\text{Now, } F = kx = \left(5{,}333\,\frac{\text{N}}{\text{m}}\right)(0.30\,\text{m}) = 1{,}600\,\text{N}$$

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Problem 6.36

If a horizontal force of 20 N is applied to a body of 1 kg weight, it moves 3 m in an upward inclined plane as shown below.



The amount of work (J) done is most nearly:

A. 67 B. 52 C. 46 D. 40

Solution: A

$$T_2 + V_2 = T_1 + V_1 + U_{1\to2}$$

$T_2 = 0$
$T_1 = 0$
$V_1 = mgh = mg(0) = 0$
$V_2 = mgh = (1\text{ kg})(9.81)(3 \sin 30) = 14.715\text{ J}$

$$T_2 + V_2 = T_1 + V_1 + U_{1\to2}$$
$$0 + 14.715 = 0 + 0 + U_{1\to2}$$
$$U_{1\to2} = 14.715\text{ J}$$

Total work done, $W = F.s + U_{1\to2} = (20\text{ N})(3 \cos 30) + (1\text{ kg})(9.81)(3 \sin 30) = 66.7\text{ J}$

Problem 6.37

A body of 10 kg is dropped from a height of 100 m. Its kinetic energy in Joule when it is 60 m above the ground is most nearly:

A. 5,886 B. 3,924 C. 4,623 D. 3,323

Solution: B

Velocity after falling 40 m (60 m above the ground)

$$v^2 = v_0^2 + 2as$$
$$v = \sqrt{o + 2(9.8\text{ m/s}^2)(40\text{ m})}$$
$$v = 28\text{ m/s}$$

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$$T = \frac{1}{2}mv^2 = 3{,}924 \text{ J}$$

Its kinetic energy when it is 60 m above the ground is 3,924 J.

Problem 6.38

A body is allowed to fall down freely from a height of 30 m. Where will its kinetic energy be twice the potential energy?

A. 20 m from ground
B. 15 m from ground
C. 10 m from ground
D. 5 m from ground

Solution: C

Total energy = $mgh = 30mg$
Let, at x m from ground, $T = 2\ V$.
V at x m height = mgx
T at x m height = $2V = 2mgx$
Then, $V + T = 30mg$
$mgx + 2\ mgx = 30\ mg$
$x = 10$ m

Problem 6.39

A 4 kg mass is allowed to fall down freely from a height of 25 m by the gravitational attraction. After 2 sec, its kinetic energy in Joule is most nearly:

A. 679 B. 770 C. 967 D. 212

Solution: B

KE after 2 sec = ½ mv^2
Velocity after 2 sec, $v = v_o + gt$
⇨ $v = 0 + 2g$
⇨ $v = 2g$
KE after 2 sec = ½ mv^2 = ½ $m\ (2g)^2 = 2mg^2 = 2\ (4\text{ kg})\ (9.81\text{ m/s}^2)^2 = 770$ J

Problem 6.40

A 4-kg mass is allowed to fall down freely from a height of 25 m by the gravitational attraction. After 2 sec, the potential energy (J) is most nearly:

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A. 679 B. 769 C. 6131 D. 212

Solution: D

PE after 2 sec = mgh

Travel after 2 sec, $h = v_o t + \frac{1}{2} g t^2 = 0 + \frac{1}{2}(9.81)(2)^2 = 19.6 m$

Height, h from the ground = 25 m – 19.6 m = 5.4 m

PE after 2 sec = mgh = 4 kg (9.81 m/s²) (5.4 m) = 212 J

Problem 6.41

An object of mass of 5 kg falls from a height of 5 m on a pin and the pin enters 10 cm inside the ground. The average resisting force (N) of the ground is most nearly:

A. 2,500 B. 4,299 C. 2,230 D. 2,930

Solution: A

Work done by the resisting force, $W = F.s = F(0.1 \text{ m})$

Total fall = 5 m + 0.1 m = 5.1 m

Work done by the object = mgh = 5 (9.81) (5.1 m) = 250 J

Now, $0.1F = 250$ => $F = 2{,}500$ N

Problem 6.42

A car of mass of 2,000 kg while descending with a velocity of 16 m/s along a road inclined to 30° with the horizontal, the driver brought it to rest within 40 m by applying brake. The opposing force (kN) applied on the car is most nearly:

A. 16.2 B. 12.6 C. 6.0 D. 22.0

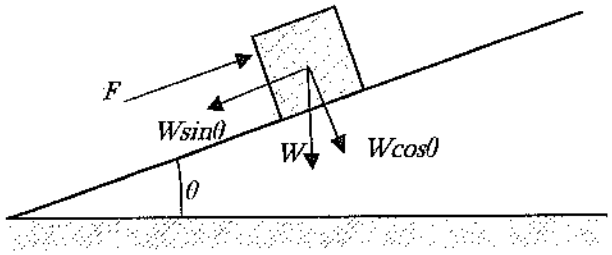
Solution: A

Here, the kinetic energy of the car was resisted by the applied force.

KE of the car = $\frac{1}{2} mv^2$ = ½ (2,000 kg) (16 m/s)² = 256,000 J

The problem did not mention the coefficient of friction. This means friction is to be neglected.

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Resistive work, $W = \Sigma F.s = (F - mg \sin 30)\ (40\ \text{m})$

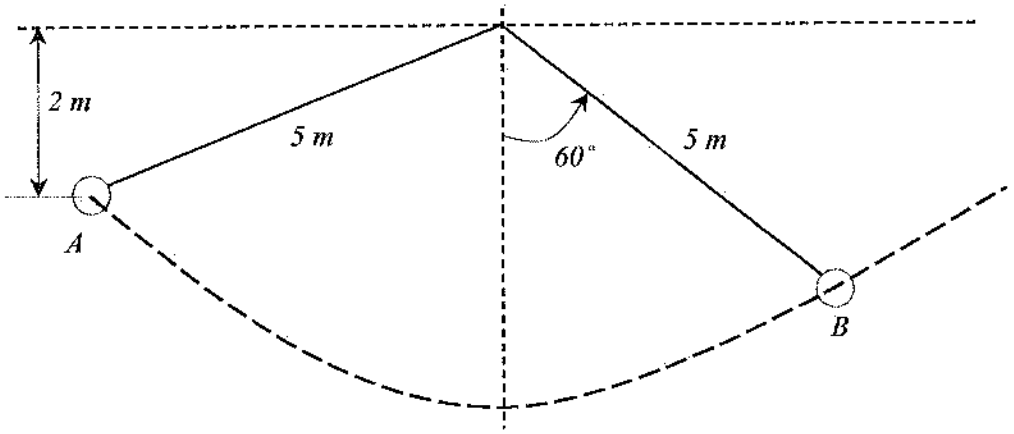
Equating KE = W

⇨ $(F - \text{mg} \cos 60)\ (40\ \text{m}) = 256{,}000$

⇨ $F = 16{,}210\ \text{N} = 16.2\ \text{kN}$

Problem 6.43

If the pendulum shown below is released from A, what will be velocity (m/s) at position B? Consider no friction in the operating system.



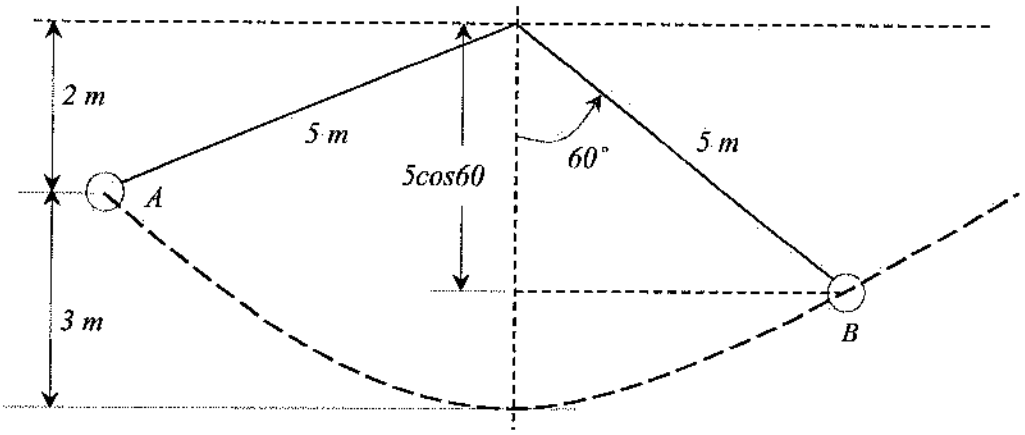
A. 0 B. 0.97 C. 3.13 D. 4.23

Solution: C

Total energy at A plus any work done on it equals the total energy at B, i.e.,

$$T_A + V_A + U_{A \to B} = T_B + V_B$$

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$$T_A = \frac{1}{2}mv_A^2 = \frac{1}{2}m(0)^2 = 0$$

$$V_A = mgh_A = \text{m}(9.81)(3.0) = 29.43\text{m Joule}$$

$$V_B = mgh_B = \text{m}(9.81)(5\text{-}5\cos 60) = 24.53\text{m Joule}$$

$U_{A\to B} = 0$ [no work is done on it]

$$T_B = \frac{1}{2}mv_B^2$$

Now, $T_A + V_A + U_{A\to B} = T_B + V_B$

$$0 + 29.43m + 0 = \frac{1}{2}mv_B^2 + 24.53m$$

$$v_B = 3.13\frac{\text{m}}{\text{sec}}$$

Problem 6.44

The displacement of a particle is $\bar{s} = 3i - 2j + k$ m when $\bar{F} = 5i + 3j - 2k$ N is applied on it. The work (J) done by the force is most nearly:

A. 5 B. 6 C. 7 D. 8

Solution: C

$$W = \bar{F}\bar{s} = (5i + 3j - 2k).\ (3i - 2j + k) = 15 - 6 - 2 = 7\text{ J}$$

Power and Efficiency

Sometimes, it is important to know how quickly a work can be done rather than the amount of work. For example, a pump capable of drawing one million gallons of water in its life may be

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worthier than a pump of capable of drawing two million gallons. This is possible if the first pump can do it in 5 years and the second one takes 25 years. Therefore, rate of work is sometimes important considering the busy life this day. Power is the time rate of doing work of an agent, and it is measured by work done in unit time. It can be shown as –

$$\text{Power} = \frac{\text{Work}}{\text{Time}}$$

Its unit is Joule/sec or Watt or in US unit ft.lb/sec. Another larger unit is also used called kilowatt (kW). Another unit, Horse Power (hp) is very often used. 1 hp = 746 kW or 1 hp= 550 ft.lb/sec.

Sometimes, power is also related to velocity as follows:

$$\text{Power} = \frac{\text{Work}}{\text{Time}} = \frac{W}{t} = \frac{Fs}{t} = F\left(\frac{s}{t}\right) = Fv$$

If the displacement of a body, instead of being along the force, is along a direction making an angle, θ with the applied force, then –

$$\text{Power} = Fv\cos\theta = \vec{F}.\vec{v}$$

In case of rotational motion, we know,

$$\text{Work} = \text{Torque } (M) \times \text{Angular Displacement } (\theta)$$

$$\text{Power} = \frac{\text{Work}}{\text{Time}} = \frac{M\theta}{t}$$

When we get work from a machine or a body that work is smaller than the energy supplied to that machine or body to do work. It is not only true for machine, but also true in our practical life where at a part of work is utilized than the supplied energy. The rest of the energy is lost. In case of engine, this dissipation of energy is used as friction, to warm up the engines etc. This dissipation of energy cannot be stopped completely, but by applying different technologies this dissipation can be reduced. In this case, equation of energy is equal to supplied energy = effective energy + dissipated energy. The efficiency of a machine is defined as the ratio of the output energy to the input or supplied energy. It is commonly denoted as η.

$$\eta = \frac{\text{Output Energy}}{\text{Input Energy}}$$

Let, E_1 be the energy supplied and E_2 be the dissipation of energy. Then, the output is $E_1 - E_2$.

$$\eta = \frac{E_1 - E_2}{E_1} = \left(1 - \frac{E_2}{E_1}\right) \times 100\%$$

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Problem 6.45

The energy per second (J/s) to be used to lift a stone of mass of 300 kg on top of the roof at speed of 0.1 m/s by a crane is most nearly:

A. 29.4 B. 294 C. 199 D. 251

Solution: B

Power = $Fv = mgv$ = 300 kg x 9.81 m/s^2 x 0.1 m/s = 294 J/s

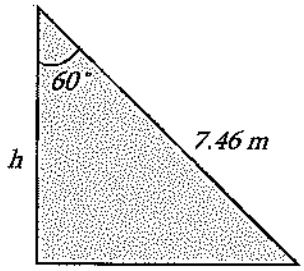
Problem 6.46

A ladder of 7.46-m length is in inclined position at angle of 30° with the roof of the building. A person weighing 60 kg is climbing to the roof in 30 sec using the ladder taking a load of 15 kg on his head. The applied power (watt) is most nearly:

A. 183.7
B. 127.2
C. 91.4
D. 55.1

Solution: C

$$\text{Power} = \frac{\text{Work}}{\text{Time}} = \frac{(60+15)\,\text{kg}(9.81)(7.46\cos 60)}{30\ \text{sec}} = 91.4\,\text{W}$$



Problem 6.47

The depth and the diameter of a well full of water are 12 m and 1.8 m respectively. A pump can make the well empty in 24 minutes. The power of the pump (hp) is most nearly:

A. 3.33 B. 1.67 C. 0.84 D. 2.56

Solution: B

$$m = \text{volume x density} = (\pi r^2 h)\rho = (\pi)(0.9\,\text{m})^2(12\,\text{m})\left(1{,}000\,\frac{\text{kg}}{\text{m}^3}\right) = 30{,}536\,\text{kg}$$

$$\text{Power} = \frac{mgh}{t} = \frac{(30{,}536\,\text{kg})(9.81)\left(\frac{12}{2}\,\text{m}\right)}{24(60)\,\text{sec}} = 1{,}246\,\text{W} = \frac{1{,}246\,\text{W}}{746} = 1.67\,\text{hp}$$

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Problem 6.48

A motor of power 3,430 W pumps water to a height of 7.20 m from a well. If the efficiency of the motor is 90%, the amount of water (kg) can be pumped in 1 minute is most nearly:

A. 3,237 B. 2,913 C. 2,622 D. 43.7

Solution: C

Let the mass be m kg. Efficiency of the motor is 90% means the effective efficiency is = 0.9(3,430 W) = 3,087 W.

$$\text{Power} = \frac{W}{t} = \frac{mgh}{t}$$

$$3{,}087 = \frac{m(9.81)(7.2)}{60\,\text{sec}}$$

$$m = 2{,}622 \text{ kg}$$

Impulse and Momentum

Assuming constant mass, the equation of motion of a particle may be written as –

$$F = \frac{d(mv)}{dt}$$

$$mdv = Fdt$$

For a system of particles, by integrating and summing over the number of particles, this may be expanded to –

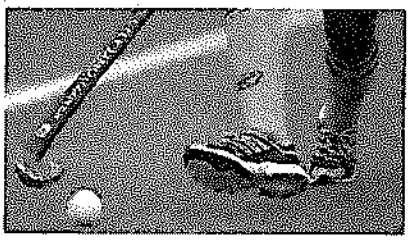
$$\sum m_i (v_i)_{t_2} = \sum m_i (v_i)_{t_1} + \sum \int_{t_1}^{t_2} F_i dt$$

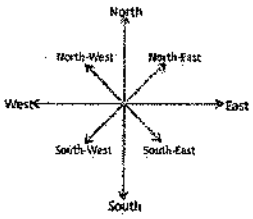
The term on the left side of the equation is the linear momentum of a system of particles at t_2. The first term on the right side of the equation is the linear momentum of a system of particles at time, t_1. The second term on the right side of the equation is the impulse of the force F from time t_1 to t_2. It should be noted that the above equation is a vector equation. Component scalar equations may be obtained by considering the momentum and force in a set of orthogonal directions.

Problem 4.49

A 0.2-kg hockey puck is traveling to the north with a velocity of 10 m/s when it is struck by a hockey stick and given a velocity of 20 m/s along north-east. The magnitude of the net impulse (kg.m/sec) exerted by the hockey stick on the puck is most nearly:

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A. 2.95 B. 7.95 C. 11.95 D. 1.95

Solution: A

$$\vec{v}_o = 10\vec{j}$$
$$\vec{v} = 20\cos 45\ \vec{i} + 20\sin 45\ \vec{j}$$
$$\vec{J} = m(\vec{v} - \vec{v}_o) = 0.2\left[\left(20\cos 45\vec{i} + 20\sin 45\vec{j}\right) - 10\vec{j}\right] = 2.83\vec{i} + 0.83\vec{j}$$
$$J = \sqrt{(2.83)^2 + (0.83)^2} = 2.95\ \frac{\text{kg.m}}{\text{sec}}$$

Impact

During an impact, momentum is conserved while energy may or may not be conserved. For direct central impact with no external forces –

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

m_1, m_2 = the masses of the two bodies
v_1, v_2 = the velocities of the bodies just before impact
v_1', v_2' = the velocities of the bodies just after impact

For impacts, the relative velocity expression is –

$$e = \frac{(v_2')_n - (v_1')_n}{(v_1)_n - (v_2)_n}$$

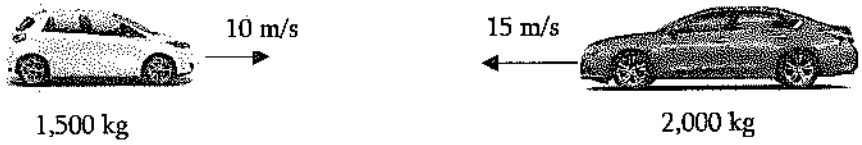
e = coefficient of restitution
$(v_i)_n$ = the velocity normal to the plane of impact just before impact
$(v_i')_n$ = the velocity normal to the plane of impact just after impact

The value of e is such that:
$0 \le e \le 1$, with limiting values
$e = 1$, perfectly elastic (energy conserved)
$e = 0$, perfectly plastic (no rebound)

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Problem 6.50

A 1,500 kg car is traveling at 10 m/s. Another 2,000 kg car is traveling at 15 m/s towards the first car. After the collision, the combined velocity (m/s) is most nearly:



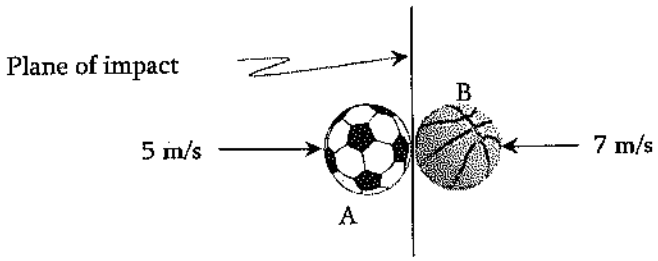
A. 12.86 m/s in the travel direction of the first car
B. 12.86 m/s in the travel direction of the second car
C. 4.3 m/s in the travel direction of the first car
D. 4.3 m/s in the travel direction of the second car

Solution: D

$$m_1v_1 + m_2v_2 = (m_1 + m_2)v'$$
$$\Rightarrow 1,500(10) + 2,000(-15) = (1,500 + 2,000)v'$$
$$\Rightarrow 15,000 - 30,000 = 3,500v'$$
$$\Rightarrow v' = -4.3 \text{ m/s}$$ [velocity after collision is 4.3 m/s in the travel direction of the second car]

Problem 6.51

A 1-kg ball, A and a 2-kg ball, B collide as shown below. If the coefficient of restitution of the balls is 0.75, for each ball just after collision, the final velocity (m/s) of the 1-kg ball, A is most nearly:



A. 0 B. 3 C. 6 D. 9

Solution: D

Applying conservation of momentum principle along the motion

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$$m_A v_{A1} + m_B v_{B1} = m_A v_{A2} + m_B v_{B2}$$
$$(1\,\text{kg})\left(5\frac{\text{m}}{\text{s}}\right) + (2\,\text{kg})\left(-7\frac{\text{m}}{\text{s}}\right) = (1\,\text{kg})v_{A2} + (2\,\text{kg})v_{B2}$$
$$v_{A2} + 2v_{B2} = -9 \ldots\ldots\ldots\ldots\ldots(1)$$

Now,

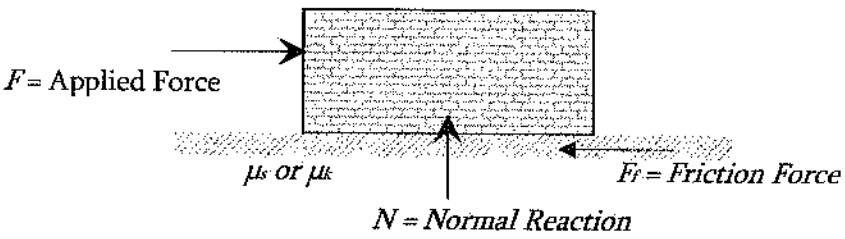
$$e = \frac{v_{B2} - v_{A2}}{v_{A1} - v_{B1}}$$
$$0.75 = \frac{v_{B2} - v_{A2}}{5\frac{\text{m}}{\text{s}} - \left(-7\frac{\text{m}}{\text{s}}\right)}$$
$$v_{B2} - v_{A2} = 9 \ldots\ldots\ldots\ldots\ldots(2)$$

Solving Equations (1) and (2), $v_{A2} = -9\frac{\text{m}}{\text{s}} = 9\frac{\text{m}}{\text{s}}(\leftarrow)$

Friction

When one surface tries to move over another surface, the roughness between these two surfaces tries to resist the movement. This impediment to move one surface over another one is known as friction. The impediment force is known as frictional force. It always acts to the opposite direction of the impending movement. The Laws of Friction are:

1) Friction force is independent of the area of contact.
2) Friction force is proportional to the normal force N.
3) Experiments show that the force necessary to initiate slip is greater than that necessary to maintain the motion.



The formula expressing the Laws of Friction is:

$F \le \mu N$, where: μ = the coefficient of friction.

In general,

$F < \mu_s N$, no slip occurring

$F = \mu_s N$, at the point of impending slip

$F > \mu_k N$, when slip is occurring

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Here,

μ_s= the coefficient of static friction

μ_k=the coefficient of kinetic friction

The third law of friction says, $\mu_s > \mu_k$

Problem 6.52

A 100-kg block resting on a horizontal surface is being pulled horizontally by 2,000 N force. The coefficient of kinetic friction between the block and the plane is 0.2. The velocity (m/s) of the block after traveling 10 m is most nearly:

A. 7 B. 11 C. 15 D. 19

Solution: D

$$T_1 + U_{1\to 2} = T_2$$
$$T_2 = T_1 + U_{1\to 2}$$
$$\frac{1}{2}mv_2^2 = \frac{1}{2}mv_1^2 + U_{1\to 2}$$
$$F_{friction} = \mu N = \mu(mg) = 0.2(100)(9.81) = 196.2\ \text{N}$$
$$U_{1\to 2} = \sum F.s = (2,000 - 196.2)10 = 18,038\ \text{J}$$
$$\frac{1}{2}mv_2^2 = \frac{1}{2}m(0)^2 + 18038$$
$$v_2 = 18.99 \approx 19\frac{\text{m}}{\text{sec}}$$

Section 4: Kinetics of Rigid Body

Mass Moment of Inertia

Mass moment of inertia is important in rigid body kinetics as it measures the resistance of a body to angular acceleration. Larger the mass moment of inertia, larger the resistance of the body to angular acceleration. If a rigid body rotates around an axis, then moment of inertia of that body with respect to that axis means the summation of the product of square of distance from the axis and mass of each of the particles of that body. Mathematically, the moment of inertia can be expressed in the following way:

$$I = \int r^2 dm$$

where dm is the mass of infinitesimally small element of the body at distance, r from the axis. Moment of inertia does not depend on angular velocity of particles but depends on the distribution of particles about the axis of rotation.

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The definitions for the mass moments of inertia are:

$$I_x = \int (y^2 + z^2) dm; \quad I_y = \int (x^2 + z^2) dm; \quad I_z = \int (x^2 + y^2) dm; \quad I = \int r^2 dm$$

The mass of inertia may be a calculated about any axis through the application of the above definitions. However, once the moments of inertia have been determined about an axis passing through a body's mass center, it may be transformed to another parallel axis. The transformation equation is mentioned below: $I_{new} = I_c + md^2$

I_{new} = the mass moment of inertia about any specified axis
I_c = the mass moment of inertia about an axis that is parallel to the above specified axis but passes through the body's mass center
m = the mass of the body
d = the normal distance from the body's mass center to the above-specified axis

The mass radius of gyration is defined as $r_m = \sqrt{\frac{I}{m}}$

Problem 6.53

A wheel has a mass of 5 kg and a radius of gyration of 25 cm. The mass moment of inertia (kg.m^2) is most nearly:

A. 0.3125
B. 0.03125
C. 3.125
D. None of the above

Solution: A

$$I = mr_m^2 = 5 \text{ kg } (0.25 \text{ m})^2 = 0.3125 \text{ kg.m}^2$$

Problem 6.54

A uniform rod is 2.0 ft long and has a mass of 15 kg. The rod's mass moment of inertia (kg.ft^2) about the axis perpendicular to the rod passing through the centroid is most nearly:

A. 20
B. 10
C. 5.0
D. 30

Solution: C

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$$I_{yc} = I_{zc} = \frac{ML^2}{12} = \frac{(15\text{kg})(2.0\text{ft})^2}{12} = 5.0\ \text{kg}-\text{ft}^2$$

Angular Momentum

The vector product of radius vector of a rotating particle and the linear momentum is called the angular momentum. Suppose, r= radius of a particle with respect to the center of rotation, and P= linear momentum of the body, then the angular momentum (H) can be expressed as –

$$\vec{H} = \vec{r} \times \vec{P}$$

It is vector quantity and the magnitude of angular momentum can be determined as $H = rP\sin\theta$, where θ is the angle between $\vec{r}$ and $\vec{P}$. The direction of H is to be determined by the cross-product rule. If the angle between $\vec{r}$ and $\vec{P}$ is 90°, then –

$$H = rP\sin\theta = rP = r(mv) = rm(\omega r) = \omega(mr^2) = \omega I = I\omega$$

where, ω = angular momentum

Problem 6.55

A body of mass of 500 g is revolving in a circular path of radius of 2 m. If the time period of revolution is 10 sec, the angular momentum (kg.m²/s) of the body is most nearly:

A. 0.756 B. 1.256 C. 1.756 D. 1.956

Solution: B

$$H = I\omega$$

$$\omega = \frac{2\pi}{T}$$

$$H = mr^2\frac{2\pi}{T} = 0.5\,\text{kg}(2\,\text{m})^2\left(\frac{2\pi}{10}\ \frac{\text{rad}}{\text{sec}}\right) = 1.256\ \text{kg.m}^2\text{s}^{-1}$$

Torque or Moment of a Force

Moment of a force means the action of force which creates rotation of a body. Torque means the action of force which causes twisting of a body. However, both moment of a force and torque are mathematically same. That is torque or moment of force about a chosen axis is the product of the force and its moment arm. Let us consider a thin sheet AB, fixed in horizontal position in such a way that it can rotate about the point, A and about the perpendicular axis xy. If the sheet is rotated by applying a force on any point, C then, the produced torque (M) is –

$$\vec{M} = \vec{dx}\vec{F} = rF\sin\theta$$

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Again, Torque, $M = Fr = mr^2 \frac{d\omega}{dt} = I\frac{d\omega}{dt} = I\alpha$

Problem 6.56

A wheel has mass of 5 kg and radius of gyration of 25 cm. In order to produce angular acceleration of 4 rad/s² in the wheel the magnitude of torque (N.m) is to be applied is most nearly:

A. 1.25 B. 2.5 C. 0.97 D. 1.93

Solution: A

$$\text{Mass moment of inertia, } I = mr_m^2 = 5\,\text{kg}\left(0.25\,\text{m}\right)^2 = 0.3125\ \text{kg.m}^2$$

$$\text{Torque, } M = I\alpha = 0.3125\ \text{kg.m}^2\left(4\frac{\text{rad}}{\text{sec}^2}\right) = 1.25\,\text{N.m}$$

Kinetic Energy

Kinetic energy is the ability of a rigid body to do work while in motion. Depending on the type of motion, kinetic energy can be expressed as –

For Translation. When a rigid body of mass m is subjected to either rectilinear or curvilinear translation (i.e. no rotation), the kinetic energy (T) of the body is –

$$T = \frac{1}{2}mv^2$$

where v is the translation velocity at the instant considered.

For Rotation about a Fixed Axis. When a rigid body of mass m is subjected to rotation about a fixed axis, the kinetic energy (T) of the body is –

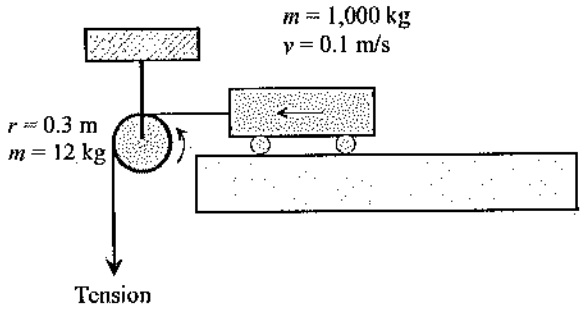
$$T = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\omega r\right)^2 = \frac{1}{2}\left(mr^2\right)\left(\omega\right)^2 = \frac{1}{2}I_o\omega^2$$

where I_o is the mass moment of inertia of the body about the axis perpendicular to the plane of motion and passing through the fixed point of rotation.

Problem 6.57

A 1,000-kg block is to be rolled over a frictionless surface at a velocity of 0.1 m/s over a 0.6-m diameter frictionless pulley applying a tension. The kinetic energy (J) of the 12-kg pulley is most nearly:

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A. 0.06 B. 0.03 C. 0.12 D. 0.18

Solution: B

It is subjected to rotation about a fixed axis. The kinetic energy (T) of the pulley is $T = \frac{1}{2} I_o \omega^2$

Now, $I_o = \frac{1}{2} mr^2 = \frac{1}{2}(12\text{ kg})(0.3\text{ m})^2 = 0.54\text{ kg.m}^2$

$$\omega = \frac{v}{r} = \frac{0.1 \frac{\text{m}}{\text{s}}}{0.3\text{ m}} = 0.33 \frac{\text{rad}}{\text{sec}}$$

Therefore, $T = \frac{1}{2} I_o \omega^2 = \frac{1}{2}(0.54)(0.33)^2 = 0.03\text{ J}$

Conservation of Impulse and Momentum

Conservation of linear momentum states that the sum of all the impulses by the external forces in a period equals the change of the linear momentum of the body during that time frame. It can be written as –

$$mv_1 + \sum \int_{t_1}^{t_2} F dt = mv_2$$

where

mv_1 = Initial linear momentum

mv_2 = Final linear momentum

$\sum \int_{t_1}^{t_2} F dt$ = sum of linear impulses imposed on the body

The conservation principal of angular impulse and momentum say, the initial angular momentum of a particle plus any angular impulse done on the particle equals the final angular momentum of the particle. In vector form, it can be written as –

$$(\bar{H}_1)_o + \sum \int_{t_1}^{t_2} \bar{M}_o dt = (\bar{H}_2)_o$$

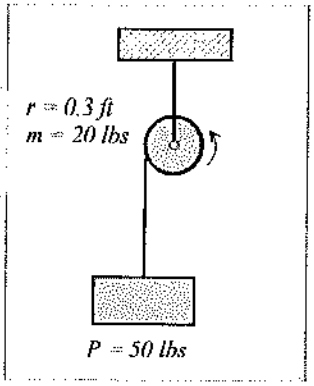
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In scalar form, it can be written as –

$$(H_1)_{x,y,or\,z} + \sum \int_{t_1}^{t_2} M_{x,y,or\,z}\,dt = (H_2)_{x,y,or\,z}$$

Problem 6.58

The disk shown, has a weight of 20 lbs and is pinned at its center. Neglect the mass of the cord. If a vertical force of 50 lbs is applied to the cord wrapped around the disk's outer rim, the angular velocity (rad/sec) of the disk after 5 seconds starting from rest is_____.



(write the answer in nearest hundreds such as 8700 or 1100 or 400 without comma for thousands)

Solution: 2700

$$(H_1)_o + \sum \int_{t_1}^{t_2} M_o\,dt = (H_2)_o$$

$$I_o\omega_1 + \sum \int_{t_1}^{t_2} M_o\,dt = I_o\omega_2$$

$$I_o\omega_1 + \sum \int_{t_1}^{t_2} (rF)\,dt = I_o\omega_2$$

$$I_o(0) + 50\,\text{lbs}(0.3\,\text{ft})(5\,\text{sec}) = \left[\frac{1}{2}\left(\frac{20\,\text{lbs}}{32.2}\right)(0.3\,\text{ft})^2\right]\omega_2$$

$$\omega_2 = 2,683\ \frac{\text{rad}}{\text{sec}} \approx 2700\ \text{rad/sec}$$

"Your exam results are based on the total number of correct answers that you selected. There are no deductions for wrong answers. The score is then converted to a scaled score, which adjusts for any minor differences in difficulty across the different exam forms. This scaled score represents an examinee's ability level and is compared to the minimum ability level for that exam, which has been determined by subject-matter experts through psychometric statistical methods. NCEES does not publish the passing score. NCEES scores each exam with no predetermined percentage of examinees that should pass or fail. All exams are scored the same way. First-time takers and repeat takers are graded to the same standard." NCEES (2016) at http://ncees.org/engineering/engineering-scoring/

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7 MECHANICS OF MATERIALS

Fundamentals of Engineering (FE) CIVIL Exam Specification mentions, 7-11 questions may appear in the exam based on Mechanics of Materials as follows:

A. Shear and moment diagrams
B. Stresses and strains (e.g., axial, torsion, bending, shear, thermal)
C. Deformations (e.g., axial, torsion, bending, thermal)
D. Combined stresses
E. Principal stresses
F. Mohr's circle
G. Column analysis (e.g., buckling, boundary conditions)
H. Composite sections
I. Elastic and plastic deformations
J. Stress-strain diagrams

Stress-Strain Curve for Mild Steel

Members in civil engineering structures undergo different types of loading such as tension, compression, moment, torsion, etc. Tension is very common in truss, hanger, cables, reinforcement, etc. While designing the tension members, it is imperative to know the mechanical properties of materials in tension. Some mechanical properties of a tension member are modulus of elasticity, yield stress, ultimate stress, failure stress, and so on. Stress (σ) is defined as the reaction force over area. It can be expressed as follows:

$$\sigma = \frac{P}{A}$$

P = reaction force, A = Cross-sectional area.

Strain (ε) can be defined as follows:

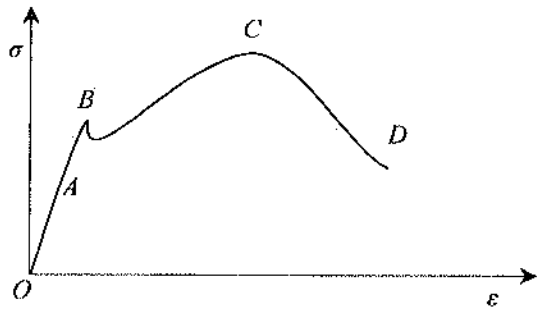
$$\varepsilon = \frac{\Delta L}{L}$$

where ΔL = change in length, L = original length. Finally, the modulus of elasticity (E) is expressed as follows:

$$E = \frac{\sigma}{\varepsilon}$$

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The schematic stress-strain curve of mild steel is shown below.

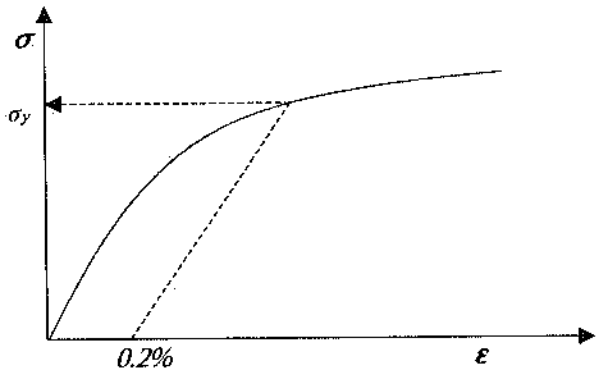


A few definitions from this curve, and some other properties are listed below.

A) Modulus of elasticity: The slope of axial stress and axial strain diagram up to proportional limit (*A*).
B) Proportional limit: The linear portion of the stress-strain diagram (*OA* region).
C) Elastic limit: The stress level up to which no plastic strain occurs (*B*).
D) Yield point: The stress level above which plastic strain occurs (*B*).
E) Ultimate stress: The maximum stress a ductile material sustains before failing (*C*)
F) Fracture stress: The stress at which a material fails (*D*).
G) Modulus of Resilience: Area under stress-strain curve up to proportional limit.
H) Modulus of Toughness: Area under stress-strain curve up to failure.
I) Strain hardening: For ductile material, after yielding, an increase in load can be supported until reaches a maximum stress (or ultimate stress) (region *C* to *D*). This rise in stress after yield point is called strain hardening.
J) Necking: Upon applying tensile stress, the cross-sectional area of a ductile material decreases, forming a neck before failure. This contraction of area before failure is called necking.
K) Strain Softening: After the ultimate stress and before the failure point, the stress capacity keeps on decreasing with continuous development of plastic strain due to significant molecular displacement in the materials. This region is called strain softening.
L) Elastic or recovery strain: The strain that is recovered fully upon removal of loads.
M) Permanent or plastic strain: The strain that cannot be recovered any amount upon removal of loads.
N) Shear modulus or modulus of rigidity: The slope of shear stress and shear strain diagram up to the proportional limit.

If the yield point on the σ-ε curve is not distinct, then a straight line parallel to the initial line is drawn at 0.2% strain point, as shown below. The intersection of the straight line and the original σ-ε curve is considered the yield point (σ_y).

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Determining yield stress if yield point is not distinct

Engineering Strain

Engineering strain, commonly known as strain, is the change of length over the initial length. It can be expressed as follows:

$$\varepsilon = \frac{\Delta L}{L_o}$$

where:

ε = engineering strain (or just strain)
ΔL = change in length of member
L_o = original length of member; very often L_o is written as L

Percent Elongation

$$\%\ \text{elongation} = \left(\frac{\Delta L}{L_o}\right) x100$$

Percent Reduction in Area (RA)

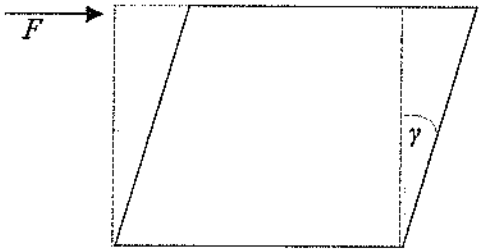
The % reduction in area from initial area, A_i, to final area, A_f, is: $\%\ RA = \left(\frac{A_i - A_f}{A_i}\right) x100$

True Stress and True Strain

True stress is defined as the force over the final area of the samples; and true strain is the differential change in length over the initial length of the sample [True strain = ln $(1 + \varepsilon)$]. Please see the Material Science/Structure of Matter Section also.

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Shear Stress-Strain



Shear stress, $\tau = F/A$
Shear strain, γ = Change in angle which was initially $\pi/2$.
$G = \tau/\gamma$

Shear modulus or modulus of rigidity (G) is defined as the ratio of shear and shear strain. If Poisson's Ratio (ν) of the material is known, the shear modulus and modulus of elasticity can be correlated as follows:

$$G = \frac{E}{2(1+\nu)}$$

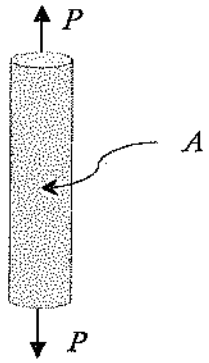
Poisson's Ratio can be defined as the ratio of lateral strain (ε_t) and longitudinal strain (ε_a). If tension force is applied axially on a body, its length increases and the lateral dimension decreases. If a compressive force is applied the scenario is the opposite. Consequently, if lateral strain is positive, longitudinal strain is negative and vice-versa. Therefore, Poisson's Ratio is always negative. Its value is between 0 and 0.5. Mathematically, Poisson's ratio can be expressed as:

$$\nu = -\frac{\frac{\Delta r}{r_o}}{\frac{\Delta l}{l_o}} = -\frac{\varepsilon_t}{\varepsilon_a}$$

where l_o and r_o are the original length and lateral dimension, respectively; Δl and Δr are the change in length and lateral dimension, respectively.

Uniaxial Loading and Deformation

Let us assume a load of P is applied on the body of cross-sectional area, A. Stress-strain of the tension can be defined as shown.



$$\sigma = \frac{P}{A}$$

$$\varepsilon = \frac{\Delta L}{L}$$

$$E = \frac{\sigma}{\varepsilon} = \frac{P/A}{\Delta L/L} = \frac{PL}{A\Delta L}$$

The deformation or change in length can be calculated as: $\Delta L = \frac{PL}{AE}$

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Problem 7.1

A tensile force of 120 kN is applied on a 0.02-m diameter and 2-m long rod. After applying the load, the diameter of the rod decreases to 0.01998 m and the length increases to 2.01 m. The engineering stress on that rod is most nearly:

A. 318 MPa B. 382 MPa C. 382 kPa D. 382 Pa

Solution: B

$$\sigma_E = \frac{F}{A_o} = \frac{120 \text{ kN}}{\frac{\pi}{4}(0.02 \text{ m})^2} = 381{,}970 \text{ kPa} = 382 \text{ MPa}$$

Note: By stress, we mean engineering axial stress. The original diameter is to be used.

Problem 7.2

A tensile force of 120 kN is applied on a 0.02-m diameter and 2-m long rod. After applying the load, the diameter decreases to 0.01998 m and the length increases to 2.01 m. The true stress on that rod is most nearly:

A. 393 MPa B. 742 MPa C. 383 MPa D. 472 kPa

Solution: C

$$\sigma_T = \frac{F}{A_f} = \frac{120 \text{ kN}}{\frac{\pi}{4}(0.01998 \text{m})^2} = 382{,}736 \text{ kPa} = 383 \text{ MPa}$$

Note: We do not use true stress in engineering works. In true stress, the final diameter is to be used.

Problem 7.3

A tensile force of 100 kN is applied on a 0.02-m diameter and 2-m long rod. After applying the load, the diameter of the rod decreases to 0.01998 m and the length increases to 2.01 m. The engineering axial strain is most nearly:

A. 0.2% B. 0.5% C. 0.8% D. 1.0%

Solution: B

$$\text{Engineering axial strain } \varepsilon_a = \frac{\Delta L}{L_o} = \frac{L - L_o}{L_o} = \frac{2.01 - 2.0}{2.0} = 0.005 = 0.5\%$$

Note: By strain, we mean engineering axial strain.

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Problem 7.4

A tensile force of 100 kN is applied on a 0.02-m diameter and 2-m long rod. After applying the load, the diameter of the rod decreases to 0.01998 m and the length increases to 2.01 m. The true axial strain is most nearly:

A. 0.2% B. 0.5% C. 0.8% D. 1.0%

Solution: B

$$\varepsilon_T = \text{true strain} = \ln(1+\varepsilon) = = \ln\left(1+\frac{\Delta L}{L}\right) = \ln\left(1+\frac{2.01-2.00}{2.00}\right) = 0.00498 = 0.498\%$$

Problem 7.5

A tensile force of 100 kN is applied on a 0.02-m diameter and 2-m long rod. After applying the load, the diameter of the rod decreases to 0.01998 m and the length increases to 2.01 m. The lateral (transverse) strain is most nearly:

A. 0.1% B. 0.2% C. 0.3% D. 0.4%

Solution: A

$$\varepsilon_t = \frac{\Delta d}{d_o} = \frac{d-d_o}{d_o} = \frac{0.01998-0.02}{0.02} = -0.001 = -0.1\%$$

Note: If 'true' or 'engineering' is not mentioned, it means 'engineering'.

Problem 7.6

A tensile force of 100 kN is applied on a 0.02-m diameter and 2-m long rod. After applying the load, the diameter of the rod decreases to 0.01998 m and the length increases to 2.01 m. The Poisson's Ratio of this material is most nearly:

A. 0.1 B. 0.15 C. 0.20 D. 0.25

Solution: C

$$\text{Lateral strain, } \varepsilon_t = \frac{\Delta d}{d_o} = \frac{d-d_o}{d_o} = \frac{0.01998-0.02}{0.02} = -0.001 = -0.1\%$$

$$\text{Axial strain, } \varepsilon_a = \frac{\Delta L}{L_o} = \frac{L-L_o}{L_o} = \frac{2.01-2.0}{2.0} = 0.005 = 0.5\%$$

$$\text{Poisson's Ratio, } \nu = \frac{\varepsilon_t}{\varepsilon_a} = \frac{-0.001}{0.005} = -0.2$$

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Problem 7.7

A tensile force of 200 kN is applied on a 0.02-m diameter and 2-m long rod. After applying the load, the diameter of the rod decreases to 0.01998 m and the length increases to 2.01 m. Assuming no permanent deformation occurs in that material, the modulus of elasticity (GPa) is most nearly:

A. 64 B. 127 C. 155 D. 210

Solution: B

$$\text{Axial stress, } \sigma_E = \frac{F}{A_o} = \frac{200 \text{ kN}}{\frac{\pi}{4}(0.02 \text{ m})^2} = 636{,}618 \text{ kPa} = 637 \text{ MPa}$$

$$\text{Axial strain, } \varepsilon_a = \frac{\Delta L}{L_o} = \frac{L - L_o}{L_o} = \frac{2.01 - 2.0}{2.0} = 0.005 = 0.5\%$$

$$\text{Elasticity, } E = \frac{\sigma_E}{\varepsilon_a} = \frac{637 \text{ MPa}}{0.005} = 127{,}400 \text{ MPa} = 127 \text{ GPa}$$

Problem 7.8

A steel rod is 0.02-m in diameter and 2.5-m in length. The modulus of elasticity of this material is 210 GPa. The change in length after applying a tensile force of 150 kN is most nearly:

A. 5.7 mm B. 5.7 nm C. 5.7 μm D. 5.7 cm

Solution: A

$$\Delta L = \frac{FL}{AE} = \frac{150{,}000 \text{ N}(2.5 \text{ m})}{\frac{\pi}{4}(0.02 \text{ m})^2 (210 \text{x} 10^9)} = 0.00568 \text{ m} \approx 5.7 \text{ mm}$$

Problem 7.9

A steel rod is 0.02 m in diameter, and 2.5 m in length. The modulus of elasticity of this material is 210 GPa and a Poisson's Ratio of 0.2. The change in diameter after applying a tensile force of 150 kN is most nearly:

A. 0.0009 m B. 0.009 mm C. 0.09 mm D. 0.9 mm

Solution: B

$$\text{Axial stress, } \sigma = \frac{F}{A} = \frac{150 \text{ kN}}{\frac{\pi}{4}(0.02 \text{ m})^2} = 477{,}463 \text{ kPa} = 0.477 \text{ GPa}$$

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Axial strain, $\varepsilon = \dfrac{\sigma}{E} = \dfrac{0.477}{210} = 0.00227$

Lateral strain, $\varepsilon_l = -\nu\sigma = -0.2(0.00227) = -0.000454$

Change in diameter, $\Delta D = \varepsilon_l D = -0.000454\ (0.02\ \text{m}) = -0.000009\ \text{m} = -0.009\ \text{mm}$

Problem 7.10

A steel bar has a length of 2.5 m and cross-section of 200-mm x 100-mm. The modulus of elasticity of this material is 210 GPa and Poisson's Ratio is 0.3. The change in width after applying a tensile force of 150 kN is most nearly:

A. 0.21 m B. 0.21 mm C. 2.1 mm D. 0.0021 mm

Solution: D

Axial stress, $\sigma = \dfrac{F}{A} = \dfrac{150\ \text{kN}}{(0.2)(0.1)} = 7500\ \text{kPa}$

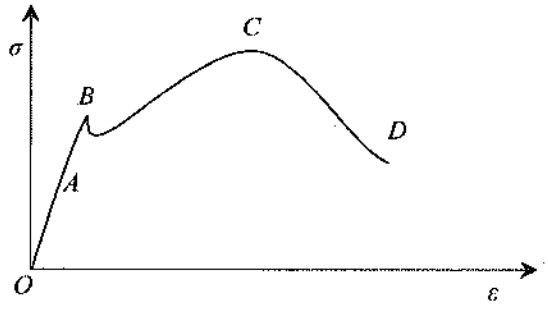
Axial strain, $\varepsilon = \dfrac{\sigma}{E} = \dfrac{7500{,}000\ \text{Pa}}{210{,}000{,}000{,}000\ \text{Pa}} = 0.0000357$

Lateral strain, $\varepsilon_l = -\nu\varepsilon = -0.3(0.0000357) = -0.0000107$

Change in width, $\Delta w = -\varepsilon_l w = -0.0000107\ (200\ \text{mm}) = -0.0021\ \text{mm}$

Problem 7.11

Which of the following statements is true for the following figure?



A. A = Proportion limit, D = Failure stress
B. A = Yield point, D = Ultimate stress
C. B = Proportion limit, O = No loading Point
D. A = Linear elastic limit, C = Failure stress

Solution: A

A = proportion limit; B = Yield Point; C = Ultimate stress; D = Failure stress
[For shading type question, this topic is very important]

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Problem 7.12

In a tensile test, if the yield point is not distinct, the yield point is determined by plotting an offset line parallel to the initial stress-strain curve. The parallel line is plotted at the stain of -

A. 0.0002% B. 0.2% C. 0.0035 D. 0.35%

Solution: B

From the Mechanics of Materials, 0.002 or 0.2%.

Problem 7.13

The correct relationship among modulus of elasticity (E), Poisson's Ratio (ν), and Shear Rigidity (G) is most nearly:

A. $G = \frac{2}{E(1-\nu)}$ B. $G = \frac{E}{2(1+\nu)}$ C. $G = \frac{E}{2(1-\nu)}$ D. $G = \frac{2}{E(1+\nu)}$

Solution: B

Thermal Deformations

It is well known that the length of a material increases or decreases with increase or decrease in temperature, respectively. The change in length due to change in temperature is called thermal deformation. The change in length per unit length (thermal strain) due to temperature change can be calculated as:

$$\varepsilon_{temp} = \frac{\delta_t}{L} = \alpha \Delta T$$

$$\text{Or, } \delta_t = \alpha L \Delta T$$

where:

α = linear coefficient of thermal expansion and contraction
ΔT = algebraic change in temperature of the member
L = original length of the member
δ_t = algebraic change in length of the member

From Hooke's law, the thermal stress can be calculated as: $\sigma = E\varepsilon_{temp} = E\alpha\Delta T$

Problem 7.14

A 5-m steel rod has a coefficient of thermal expansion and contraction of 6×10^{-6} per °F. If the temperature of the body is decreased by 50 °F, the thermal strain is most nearly:

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A. 1.5×10^{-3} B. 3×10^{-3} C. 3×10^{-4} D. None of the above

Solution: C

Thermal strain = $\alpha(\Delta T) = 6 \times 10^{-6}\,(50\ ^\circ\text{F}) = 3 \times 10^{-4}$

Problem 7.15

A 10-m steel rod has a coefficient of thermal expansion and contraction of 6×10^{-6} per °F. If the temperature of the body is decreased by 50 °F, the thermal contraction (m) is most nearly:

A. 1.5×10^{-3} B. 3×10^{-3} C. 3×10^{-4} D. None of the above

Solution: B

Thermal contraction = $\alpha L (\Delta T) = 6 \times 10^{-6}\,(10\text{ m})\,(50\ ^\circ\text{F}) = 3 \times 10^{-3}$ m.

Problem 7.16

A 10-m steel rod is snugly supported by two rigid supports at the end. The coefficient of thermal expansion and contraction of the rod is 6×10^{-6} per °F and the elastic modulus is 210 GPa. If the temperature of the body is increased by 50 °F, the thermal stress developed at the supports is most nearly:

A. 36 MPa B. 63 kPa C. 13.6 kPa D. 63 MPa

Solution: D

Thermal strain, $\varepsilon = \alpha(\Delta T) = 6 \times 10^{-6}\,(50\ ^\circ\text{F}) = 3 \times 10^{-4}$.

$$\sigma = E\varepsilon = 210\text{ GPa}\left(3 \times 10^{-4}\right) = 630 \times 10^{-4}\text{ GPa} = 63\text{ MPa}$$

Cylindrical Pressure Vessel

For internal pressure, the stress at the inside wall is: $\sigma_t = P_i\left(\dfrac{r_o^2 + r_i^2}{r_o^2 - r_i^2}\right)$

For external pressure, the stress at the outside wall is: $\sigma_t = -P_o\left(\dfrac{r_o^2 + r_i^2}{r_o^2 - r_i^2}\right)$

σ_t = tangential (hoop) stress

P_i = internal pressure

P_o = external pressure

r_i = inside radius

r_o = outside radius

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For vessels with end caps, the axial stress (σ_a) is: $\sigma_a = P_i\left(\frac{r_i^2}{r_o^2 - r_i^2}\right)$

When the thickness of cylinder wall is about one-tenth or less of inside radius, the cylinder can be considered as a thin-walled. In which case, the internal pressure is resisted by the hoop stress and the axial stress.

$$\sigma_t = \frac{p_i r}{t} \text{ and } \sigma_a = \frac{p_i r}{2t}$$

where:

t = wall thickness

$r = \frac{r_i + r_o}{2}$

Problem 7.17

A cylindrical vessel has an inner diameter of 2 ft and a wall thickness of 1 inch. After applying a pressure of 100 psi, a crack was developed longitudinally at the vessel wall. The failure/cracking stress in psi of the thin walled pressure vessel is most nearly:

A. 625 B. 650 C. 1,250 D. 2,100

Solution: C

Longitudinal crack developed means it was failed in hoop stress.

Inner radius, $r_i = 12$ in

Outer radius, $r_o = 12 \text{ in} + 1 \text{ in} = 13$ in

Average radius, $r = \frac{r_i + r_o}{2} = \frac{12 + 13}{2} = 12.5$ in

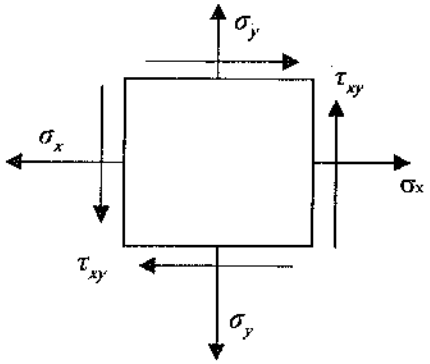
$\frac{t}{r_i} = \frac{1 \text{ in}}{12 \text{ in}} < \frac{1}{10}$; so it is a thin-walled pressure vessel.

$\sigma_t = \frac{p_i r}{t} = \frac{100 \text{ psi}(12.5 \text{ in})}{1 \text{ in}} = 1{,}250$ psi [Some books use m instead of r]

Principal Stresses

Principal stresses are used to define different failure criteria of material. They occur at the point and orientation of a member where there is no shear stress. If axial and shear stresses in two orthogonal directions of a point are known, the principal stresses can be determined using the following equation:

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$$\sigma_{1or3} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

where:

Algebraically the largest principal stress = σ_1

Algebraically the smallest principal stress = σ_3

Mohr's Circle-Stress, 2D

Mohr's Circle can be used more easily to determine the principal stresses at a point in a structural member. To construct a Mohr's circle, the following sign conventions are used.

1) Tensile normal stress components are plotted on the horizontal axis and are considered positive. Compressive normal stress components are negative.
2) For constructing Mohr's circle only, shearing stresses are plotted above the normal stress axis when the pair of shearing stresses, acting on opposite and parallel faces of an element, form a clockwise couple. Shearing stresses are plotted below the normal axis when the shear stresses form a counterclockwise couple.

The circle drawn with the center on the normal stress (horizontal) axis with center, *C*, and radius, *R*, where:

$$C = \frac{\sigma_x + \sigma_y}{2}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Then, the two non-zero principal stresses can be found as:

$\sigma_1 = C + R$

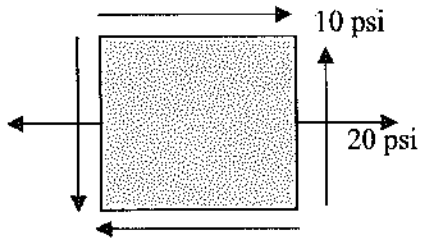
$\sigma_3 = C - R$

The maximum in-plane shear stress can be calculated as, $\tau_{max} = R$

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Problem 7.18

An element is subjected to the following plane stress condition. The maximum principal stress in psi is most nearly:



A. 30 B. 7.5 C. 200 D. 24.14

Solution: D

$$\text{Radius, } R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{20-0}{2}\right)^2 + 10^2} = 14.14 \text{ psi}$$

$$C = \frac{\sigma_x + \sigma_y}{2} = \frac{20+0}{2} = 10 \text{ psi}$$

$$\sigma_1 = C + R = 10 \text{ psi} + 14.14 \text{ psi} = 24.14 \text{ psi}$$

Problem 7.19

What is the value of in-plane shear stress when principal stresses occur?

A. $\frac{\sigma_x + \sigma_y}{2}$ B. $\frac{\sigma_x - \sigma_y}{2}$ C. 0 D. can have any value

Solution: C

Principal stresses lie at the x-axis of Mohr Circle. At x-axis, $y = 0$ (in-plane shear stress = 0)

Problem 7.20

The value of axial stress when the maximum in-plane shear stresses occur is most nearly:

A. $\frac{\sigma_x + \sigma_y}{2}$ B. $\frac{\sigma_x - \sigma_y}{2}$ C. 0 D. can have any value

Solution: A

The axial stress is $\frac{\sigma_x + \sigma_y}{2}$ when the maximum in-plane shear stress occurs. Or, simply the center of the Mohr Circle.

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Problem 7.21

What are principal stresses?

A. The average of axial stresses
B. The maximum in-plane shear stress
C. The maximum and the minimum normal stresses where shear stress are zero
D. All of the above

Solution: C

The maximum and the minimum normal stresses where shear stress are zero.

Problem 7.22

What is the angle between the planes of the two principal stresses?

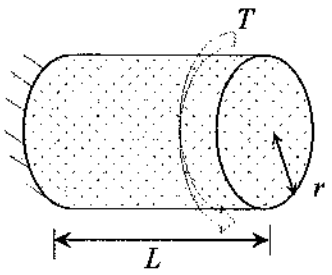
A. 180°
B. 90°
C. 270°
D. 360°

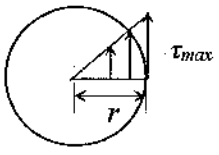
Solution: B

The angle in the Mohr Circle is halved for real element. Angle between σ_1 and σ_3 is 180° in the Mohr Circle. Thus, in the element it is 180/2 = 90°.

Torsion

If an object is subjected to a torque, T, then shear stress is developed in the section with no stress at the center and the maximum shear stress at the outmost fiber as follows.





The maximum shear stress due to torsion stress in circular solid shafts is presented by:

$$\tau_{max} = \frac{Tr}{J}$$

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where: J = polar moment of inertia

$J = \frac{\pi}{2} r^4$ for solid shaft

$J = \frac{\pi}{2}\left(r_0^4 - r_i^4\right)$ for hollow shaft with r_o and r_i as the outer and the inner radii, respectively.

τ is proportional to the distance from center. This means the τ is the maximum at the out-most fibers. The angle of twist (ϕ) can be calculate as using the shear modulus, G: $\phi = \int_0^L \frac{T}{GJ} dz = \frac{TL}{GJ}$

For Hollow, Thin-Walled Shafts, the shear stress can be determined as: $\tau = \frac{T}{2A_m t}$

where:

t = thickness of shaft wall

A_m = the area of a solid shaft of radius/equal to the mean radius of the hollow shaft

Problem 7.23

If a torque is applied to a circular shaft, the maximum shear stress is developed at-

A. the center of the shaft
B. the inner fiber of the shaft
C. the outer fiber of the shaft
D. Cannot be determined unless the body is known

Solution: C

Problem 7.24

A circular shaft of inner diameter 12 inch and outer diameter of 14 inch is subjected to a torque of 100 kip.ft. The maximum shear stress developed (ksi) at the shaft is most nearly:

A. 0.4 B. 2.22 C. 4.84 D. 6.23

Solution: C

$$J = \frac{\pi}{2}\left(r_0^4 - r_i^4\right) = \frac{\pi}{2}\left(7^4 - 6^4\right) = 1{,}736 \text{ in}^4$$

$c = r_o = 7$ inch

Maximum shear stress, $\tau_{max} = \frac{Tc}{J} = \frac{100(12) \text{ kip.in}(7 \text{ in})}{1{,}736 \text{ in}^4} = 4.84 \text{ ksi}$

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Problem 7.25

A 2-ft circular shaft of inner diameter of 12 inch and outer diameter of 14 inch is subjected to a torque of 10,000 kip.ft. The shear modulus of the shaft material is 10,000 ksi. The angle of twist in radian of the shaft is most nearly:

A. 0.00166 B. 0.0166 C. 0.166 D. 1.66

Solution: C

$$J = \frac{\pi}{2}\left(r_0^4 - r_i^4\right) = \frac{\pi}{2}\left(7^4 - 6^4\right) = 1{,}736 \text{ in}^4$$

$T = 120{,}000$ kip.in

$L = 2$ ft $= 24$ in

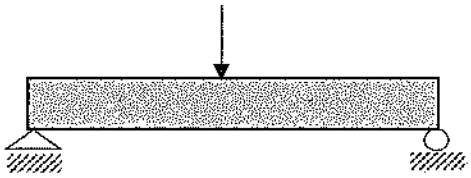
$G = 10{,}000$ ksi

Angle of twist, $\varphi = \frac{TL}{JG} = \frac{120{,}000 \text{ kip.in}(24 \text{ in})}{1{,}736 \text{ in}^4(10{,}000 \text{ ksi})} = 0.166 \text{ rad}$

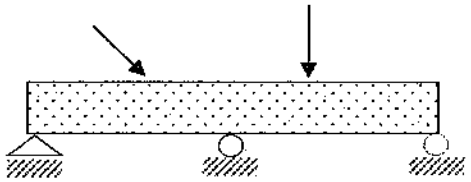
Beams

Beam is a major structural member in civil engineering. Beam is defined as the essential moment carrying member which can support axial force as well. As loads act perpendicularly to its longitudinal axis, shear force also occurs in beams. Deflection is also a major design consideration. Steel beams are relatively long and slender. Therefore, lateral torsional buckling, web yielding, web crippling, etc. are critical for steel beam. Depending on the support condition, it can be broadly divided into four types as follows:

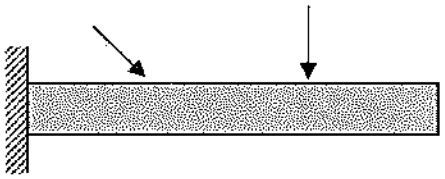
Type 1 Simply supported beam:
One pin, one roller support



Type 2 Continuous beam:
One beam with three or more supports

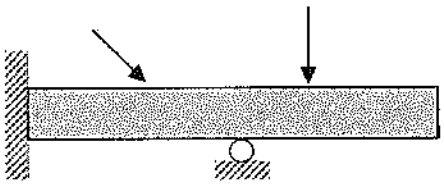


Type 3 Cantilever:
One end rigidly supported



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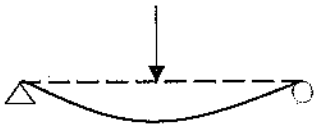
Type 4 Overhanging:
One or both ends are free, it may be continuous



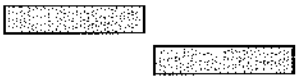
If a beam has rectangular or triangular loading, it can be converted into a concentrated loading to determine the support reactions. Note that while determining shear and moment the loading cannot be converted into concentrated load.

Shearing Force and Bending Moment Sign Conventions

1) The bending moment is positive if it produces bending when the beam concave upward (compression in top fibers and tension in bottom fibers).
2) The shearing force is positive if the right portion when the beam is downward with respect to the left.



Positive Moment
Compressive stress at the top fiber
Tensile stress at the bottom fiber



Positive Shear
Right portion of the beam moves downward with respect to the left

For Shear-Force diagram:

Step 1: Determine all the support reactions using free body diagram of the whole beam and then apply equilibrium equations.
Step 2: Follow the loads. For example, at the left support, if the left support reaction is upward then go upward, and then proceed accordingly.

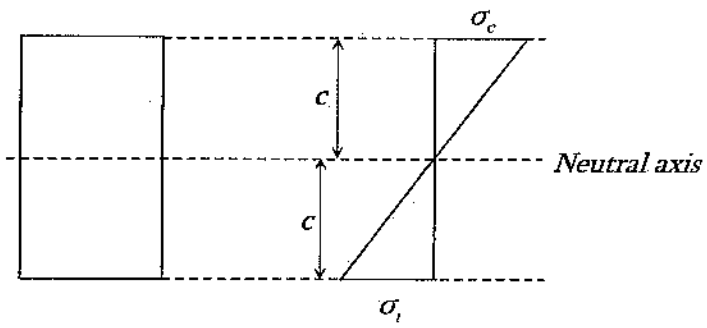
For moment diagram:

Step 1: Determine the area of the shear force; that is the moment.
Step 2: Proceed accordingly.

Stresses in Beams

When positive bending occurs in beam, it bends downward. The bottom fiber experiences tensile stress and the top fiber experiences compressive stress.

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The normal stress in a beam due to bending:

$$\sigma = \frac{Mc}{I} = \frac{M}{S}$$

where:

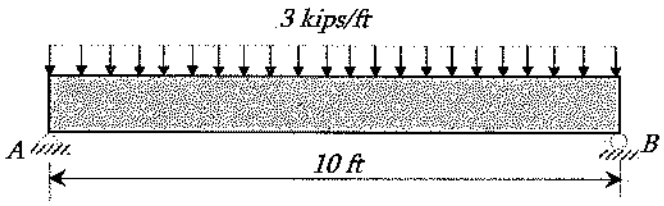
M = the moment at the section
I = the moment of inertia of the cross section
y = the distance from the neutral axis to the fiber above or below the neutral axis
$S = \frac{I}{c}$ = section modulus. If the allowable bending stress, σ_{all} is known, then, the required section modulus (S_{req}) can be calculated as:

$$S_{req} = \frac{M}{\sigma_{all}}$$

Now, from the section catalog, a section can be chosen which has S_{req} equal to or more than required. This is the design philosophy for bending. Sometimes, 'f' is used instead of 'σ' to express axial stress.

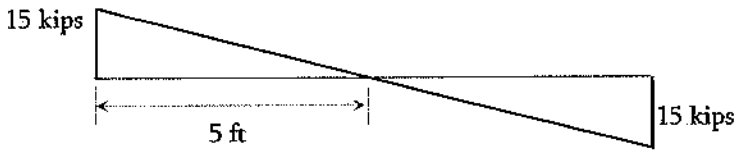
Problem 7.26

For the following beam, which of the following is a correct shear force diagram?

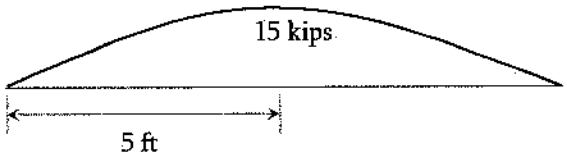


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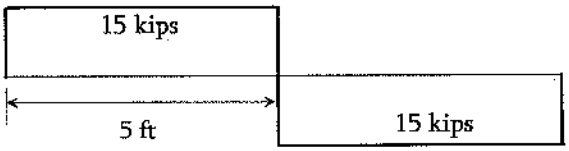
A.



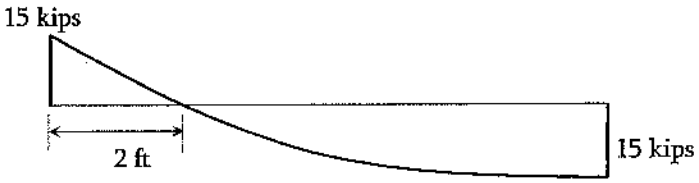
B.



C.



D.



Solution: A

$$\sum M_B = 0$$

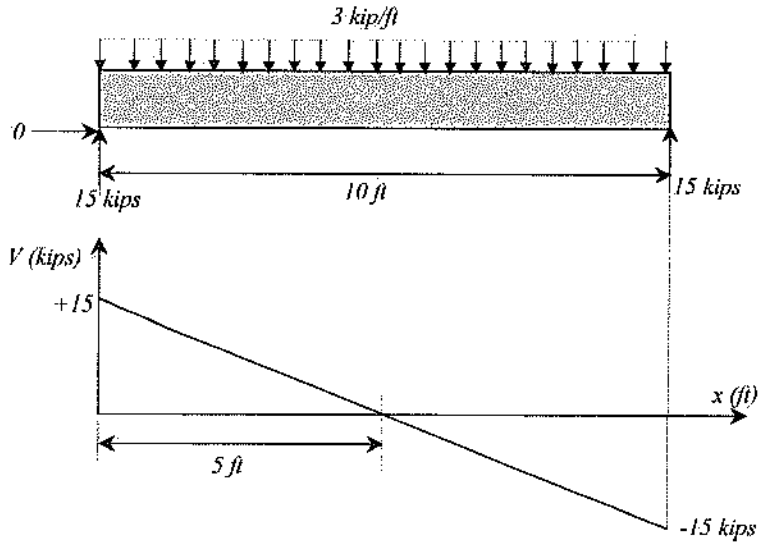
Reaction at the left support, $A_y = \dfrac{3(10)\text{kips}\ (5\,\text{ft})}{10\,\text{ft}} = 15\,\text{kips}$

$$\sum F_y = 0$$

$\Rightarrow\ A_y + B_y = 30$

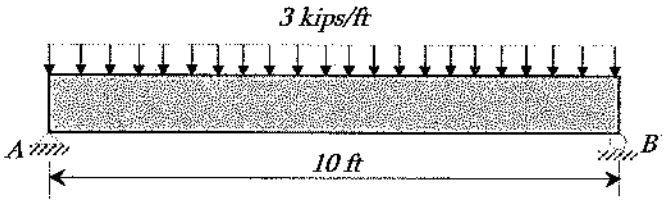
$\Rightarrow\ B_y = 15$ kips

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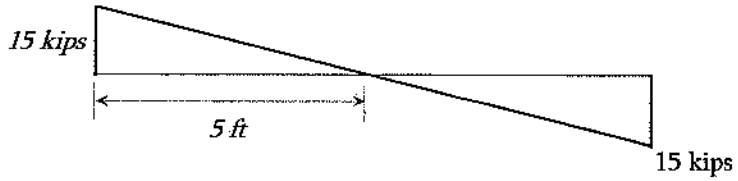


Problem 7.27

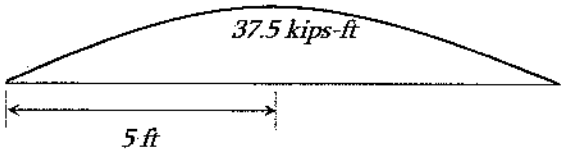
For the following beam, which of the following is the correct Moment diagram?



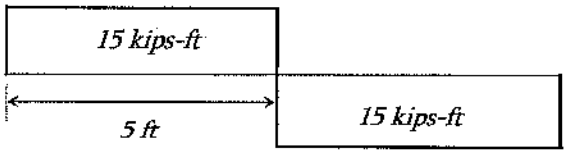
A.



B.

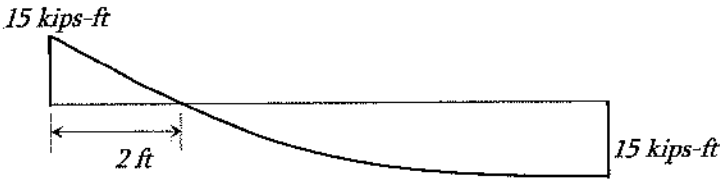


C.



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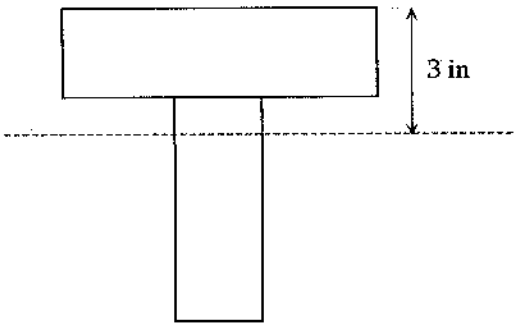
D.



Solution: B

Problem 7.28

For the following beam section, what is the bending stress at the top fiber? The maximum moment is 100 kips-inch, and the moment of inertia is 136 in^4.



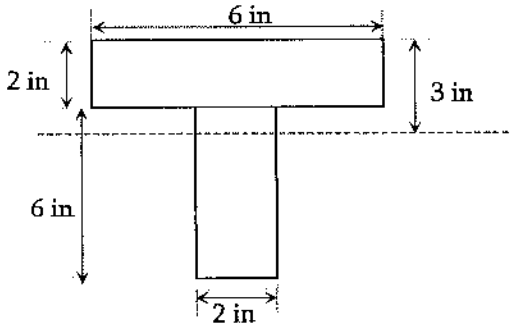
A. 2.2 psi
B. 2.2 kPa
C. 2.2 ksi
D. More data is required to calculate

Solution: C

The bending stress at the top fiber, $\sigma = \frac{Mc}{I} = \frac{100 \text{ kips.in}\,(3 \text{ in})}{136 \text{ in}^4} = 2.2 \text{ ksi}$

Problem 7.29

For the following beam section, which of the following is true?



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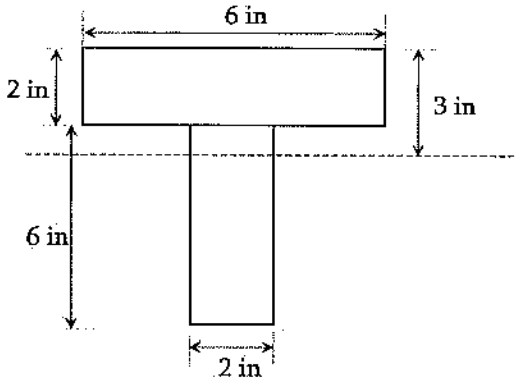
A. Shear stress variation is linear
B. Bending stress variation is linear
C. Shear stress variation is oval shaped
D. None of the above

Solution: B

Bending stress variation is linear through the cross-section; it does not matter if the cross-section is not uniform.

Problem 7.30

A beam section shown below, has a maximum positive moment of 100 kip.ft. What is most nearly the developed maximum compressive bending stress at that section? The neutral axis is 3-inch down the top fiber.



A. 20.5 ksi
B. 26.5 ksi
C. 36.1 ksi
D. 44.1 ksi

Solution: B

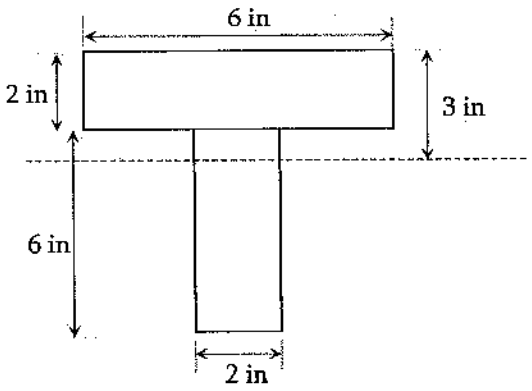
Moment of inertia, $I = \frac{2(6)^3}{12} + (2)(6)(5-3)^2 + \frac{6(2)^3}{12} + (2)(6)(7-5)^2 = 136 \text{ in}^4$

Positive moment causes compressive stress at the top fiber. So, the maximum compressive bending stress, $\sigma_c = \frac{Mc}{I} = \frac{100\text{x}12\text{kip.in}\ (3\text{in})}{136 \text{ in}^4} = 26.5 \text{ ksi}$

Problem 7.31

A beam section shown below, has a maximum positive moment of 100 kip.ft. What is most nearly the developed maximum tensile bending stress at that section? The neutral axis is 3-inch down the top fiber.

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A. 20.5 ksi B. 26.5 ksi C. 36.1 ksi D. 44.1 ksi

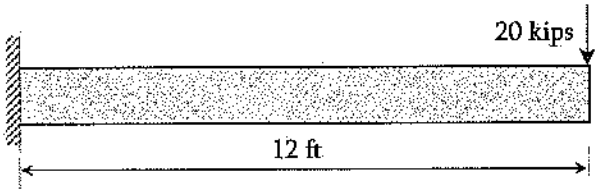
Solution: D

Moment of inertia, $I = \frac{2(6)^3}{12} + (2)(6)(5-3)^2 + \frac{6(2)^3}{12} + (2)(6)(7-5)^2 = 136 \text{ in}^4$

Positive moment causes tensile stress at the bottom fiber. So, the maximum tensile bending stress, $\sigma_t = \frac{Mc}{I} = \frac{100\text{x}12 \text{ kip.in } (5 \text{ in})}{136 \text{ in}^4} = 44.1 \text{ ksi}$

Problem 7.32

A beam shown below, has a square cross section of 8 in. on each side. The absolute maximum bending stress in ksi in the beam is most nearly:



A. 13.8
B. 23.8
C. 29.8
D. 33.8

Solution: D

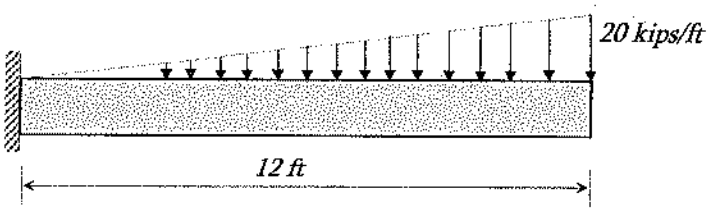
M_{max} = 20 kips (12 ft) = 240 kips.ft
I = 8(8)³/12 = 341.33 in⁴
c = 4 in

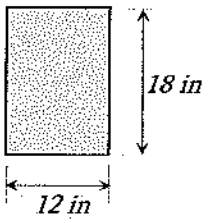
$$\sigma = \frac{Mc}{I} = \frac{240\text{x}12 \text{ kip.in } (4 \text{ in})}{341.33 \text{ in}^4} = 33.8 \text{ ksi}$$

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Problem 7.33

The maximum bending stress in ksi in the following beam is most nearly:





A. 13.8 B. 17.8 C. 22.8 D. 39.8

Solution: B

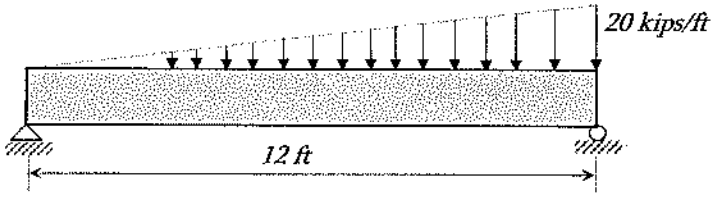
M_{max} = Force (distance) = [½(20 kips)(12 ft)] [8 ft] = 960 kips.ft

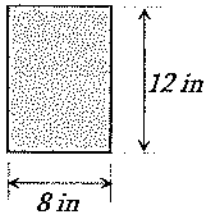
$I = 12(18)^3/12 = 5{,}832$ in^4 ; $c = 9$ in

$$\sigma = \frac{Mc}{I} = \frac{960\text{x}12\,\text{kip.in}\ (9\,\text{in})}{5{,}832\ \text{in}^4} = 17.8\ \text{ksi}$$

Problem 7.34

The maximum bending stress in ksi in the following beam is most nearly:





A. 11.6 B. 6.6 C. 22.8 D. 19.8

Solution: A

$$\Rightarrow \sum M_B = 0$$

$$\Rightarrow A_y = \frac{\frac{1}{2}(20)(12)(4\,\text{ft})}{12\,\text{ft}} = 40\ \text{kips}$$

$$\sum F_y = 0$$

$$\Rightarrow A_y + B_y = \frac{1}{2}(20)(12)$$

$$\Rightarrow B_y = 80\ \text{kips}$$

Let the point of $V = 0$ be 'x' ft from the left.

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$$A_y - \frac{1}{2}\left(\frac{20}{12}x\right)x = 0$$

$$\Rightarrow \quad 40 - 0.833x^2 = 0$$

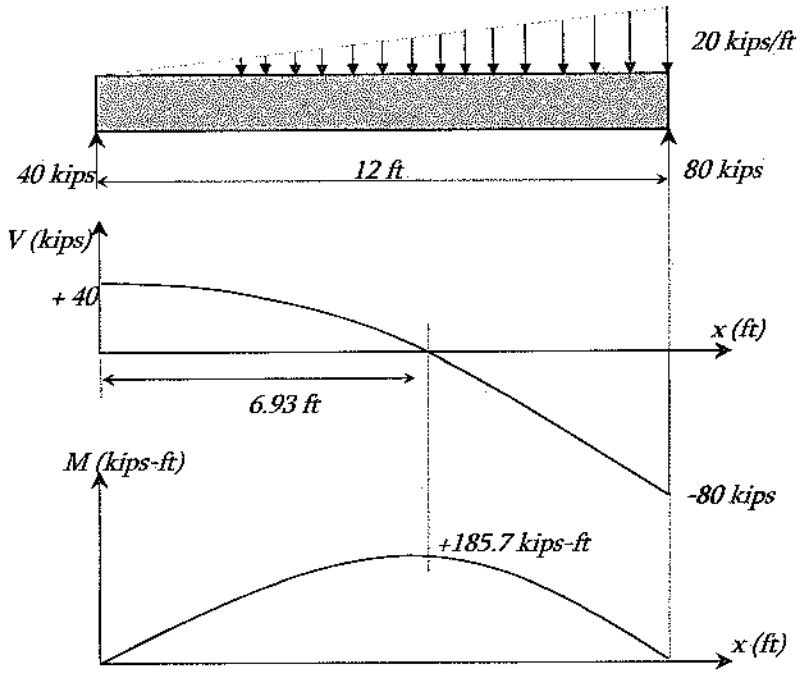
$$\Rightarrow \quad x = 6.93 \text{ ft}$$

M_{max} = Area under the V-Diagram = [2/3(40 kips)(6.93 ft)] = 185.7 kip.ft

$I = 8(12)^3/12 = 1{,}152 \text{ in}^4$

$c = 6$ in

$$\sigma = \frac{Mc}{I} = \frac{185.7 \text{x} 12 \text{kip.in } (6\text{in})}{1{,}152 \text{ in}^4} = 11.6 \text{ ksi}$$



Problem 7.35

The allowable bending stress of steel W-beam is 30 ksi. The maximum moment calculated from the analysis is 500 kips-ft. Is W18 x 40 section (which has section modulus of 68.4 in^3) adequate for this bending stress?

A. Yes
B. No
C. Adequate but too much over designed

Solution: B

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Section Modulus required,
W18 x 40 section has $S_x = 68.4$ in^3 which is less than the required. Therefore, this section is not adequate for this beam.
Note: W 18 x 106 section has $S_x = 204$ in^3, which is just more than the required, and can be selected.

Transverse shear stress:

Shear stress occurs due to the shear force in the beam section. Shear stress (f_v or τ) can be expressed as follows:

$$\text{Shear stress, } \tau = \frac{VQ}{It}$$

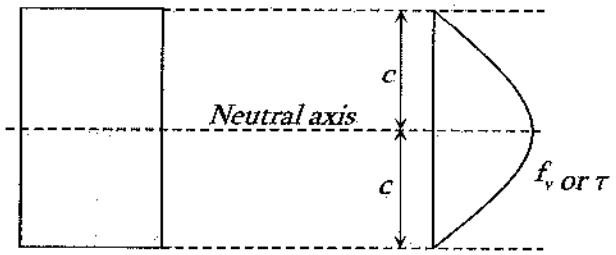
where:

V = Shear force at the concern point
I = Moment of inertia of the section
t = thickness of the section at the concern point
$Q = A\bar{y}$ (A = area of the section above or below the concern point, $\bar{y}$ = centroid of this area (A) with respect to the neutral axis.

Using the above equation two generalized cases can be developed:

1. For rectangular section, $\tau = 1.5\frac{V}{A}$
2. For wide flange section, $\tau = \frac{V}{dt_w}$

Shear stress profile in a rectangular beam section is given below. It shows that shear stress is zero at the upper and the lower ends, and its maximum at the neutral axis or at the centroid. This is true for any kind of section.



Transverse shear flow:

The shear force per unit length of the beam can be determined as follows:

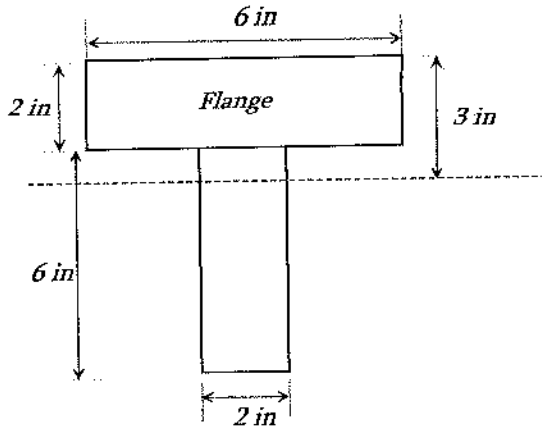
$$q = \frac{VQ}{I}$$

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This shear force value is required to determine the amount of force applied on nail or glue in a beam section.

Problem 7.36

For the following beam section, what is the shear stress at the bottom of the flange? The shear force is 100 kips, the moment of inertia is 136 in^4, and the centroid is located at 3-inch down from the top of the flange.



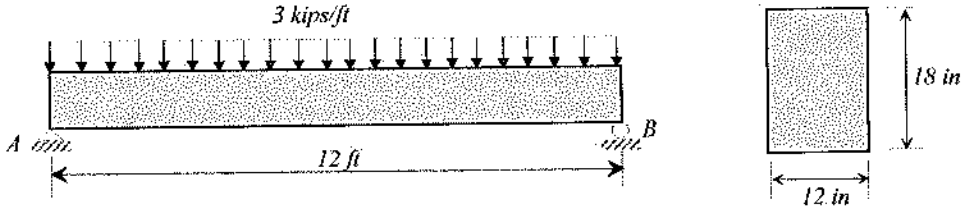
A. 2.49 ksi B. 2.94 ksi C. 4.92 ksi D. 8.82 ksi

Solution: B

$$\tau = \frac{VQ}{It} = \frac{100 \text{ kips}\,(12 \text{ in}^2)(1+1) \text{ in}}{136 \text{ in}^4 (6 \text{ in})} = 2.94 \text{ ksi}$$

Problem 7.37

A 12-ft simply supported beam is having a uniformly distributed load of 3 kip/ft as shown below. The cross-section of the beam is 12-in x 18-in. The maximum shear stress developed in the beam is most nearly:



A. 8 ksi B. 105 ksi C. 125 psi D. 140 psi

Solution: C

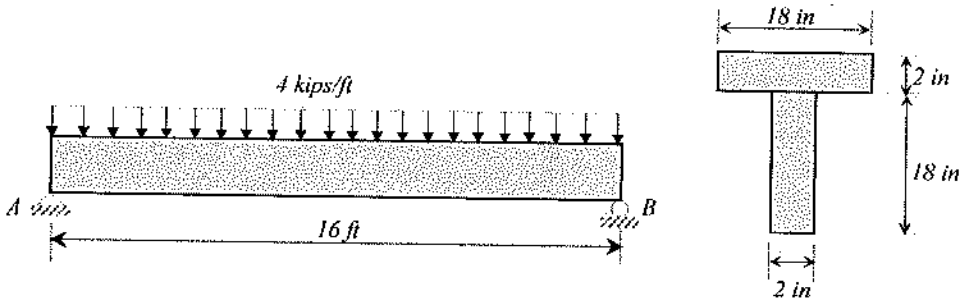
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Maximum shear force, $V_{max} = \frac{wL}{2} = \frac{3(12)}{2} = 18 \text{ kips}$

$$\tau_{max} = \frac{VQ}{It} = 1.5\frac{V}{A} = 1.5\frac{18 \text{ kips}}{(12 \text{ in})\ (18 \text{ in})} = 0.125 \text{ ksi} = 125 \text{psi}$$

Problem 7.38

A 16-ft simply supported beam is having a uniformly distributed load of 3 kip/ft as shown below. The cross-section of the beam is a T-section with 18-in x 2-in flange and 18-in x 2-in web. What is most nearly the maximum shear stress developed in the beam? Consider the flange a wide flange.



A. 100 ksi B. 800 psi C. 130 ksi D. 1,235 ksi

Solution: B

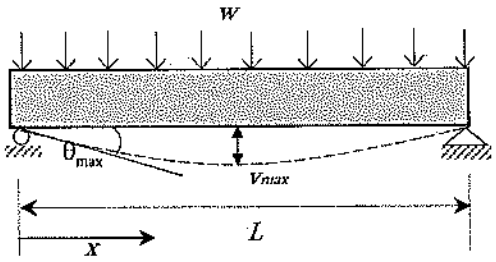
Maximum shear force, $V_{max} = \frac{wL}{2} = \frac{4(16)}{2} = 32 \text{ kips}$

Maximum shear stress, $\tau_{max} = \frac{V}{dt_w} = \frac{32 \text{ kips}}{(20 \text{in})(2 \text{ in})} = 0.8 \text{ksi} = 800 \text{ psi}$

Deflection of Beams

It is a must that any structural member will deflect more or less (visible or invisible) upon applying load. While designing any structure, it is a design requirement to let the deflection reaching a certain limit so that the users themselves feel safe and comfortable. The maximum deflection that is allowed is called the allowable deflection. The possible deflection for different structures under different loads types are given in the NCEES FE Ref. Handbook. For example, for a simply supported beam with uniformly distributed load:

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The maximum deflection, $v_{max} = -\dfrac{5wL^4}{384EI}$

The maximum deflection slope, $\theta_{max} = -\dfrac{wL^3}{24EI}$

The elastic curve, $v = -\dfrac{wx}{24EI}\left(x^3 - 2Lx^2 + L^3\right)$

Therefore, the design philosophy for deflection is that the maximum possible deflection, due to the applied loads on a member, must be equal or less than the allowable. If not, choose a larger section which satisfies the allowable deflection limit.

Composite Sections

The bending stresses in a beam composed of dissimilar materials (material 1 and material 2) where $E_1 > E_2$ are:

$$\sigma_1 = n\frac{My}{I_T}$$

$$\sigma_2 = \frac{My}{I_T}$$

where:

I_T = the moment of inertia of the transformed section

n = the modular ratio, $\dfrac{E_1}{E_2}$

E_1 = elastic modulus of material 1

E_2 = elastic modulus of material 2

The modulus section is transformed into a section composed of a single material. The centroid and then the moment of inertia are found on the transformed section for use in the bending stress equations.

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Problem 7.39

Which of the following statements is true for a beam?

A. Shear force causes twist in a beam
B. Bending causes compressive stress at upper fiber and tensile stress at the bottom fiber.
C. Shear force occurs perpendicular to the applied force.
D. Bending causes angular twist in the beam

Solution: B

Positive bending causes compressive stress at the upper fiber and tensile stress at the bottom fiber. The neutral axis remains unaffected. Torsion produces twist in the beam.

Problem 7.40

Which of the following relationships is the right definition of moment?

A. Moment = Force x distance
B. Moment = Torque x perpendicular distance
C. Moment = Force x perpendicular distance
D. None of the above

Solution: C

Moment = Force x perpendicular distance. Sometimes, the word 'perpendicular' is not written but it is the perpendicular distance.

Problem 7.41

The unit of bending stress or shear stress is most nearly:

A. Pound per square feet
B. Newton per feet
C. Pound per cubic feet
D. None of the above

Solution: A

The right unit of bending stress or shear stress is pound per square feet. In SI unit it is Newton per square meter.

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Problem 7.42

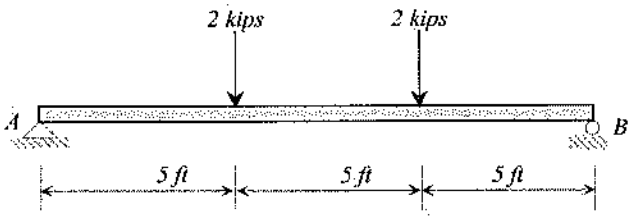
Which of the following statements is true?

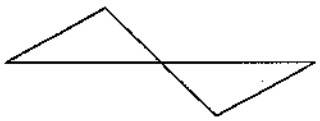
A. The failure stress is greater than the ultimate stress for tensile test in mild steel
B. Yield stress is the limiting stress of the linear portion of stress-strain curve
C. Poisson's Ratio of steel varies between 0.6 to 0.7
D. Allowable stress is less than the failure stress

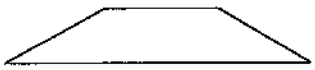
Solution: D

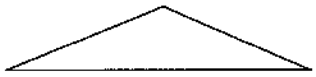
Problem 7.43

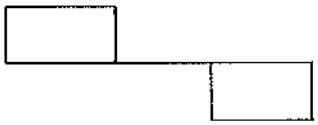
Which of the following diagrams is the correct qualitative bending moment diagram for the following beam?



A. 

B. 

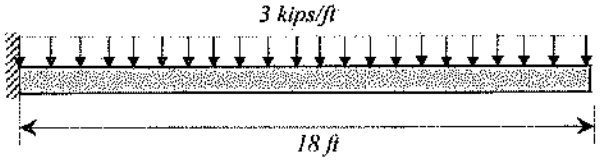
C. 

D. 

Solution: B

Problem 7.44

What is the maximum deflection (in.) of the following beam? $E = 29 \times 10^3$ ksi, and $I = 1{,}200$ in^4.



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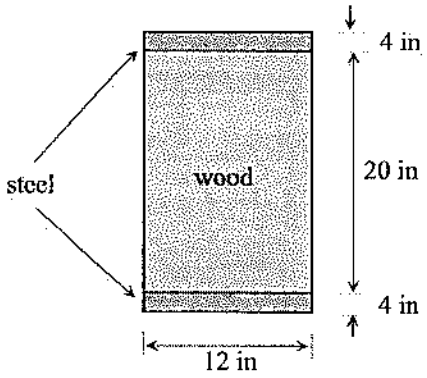
A. 0.90
B. 1.95
C. 2.35
D. 5.62

Solution: B

The maximum deflection, $\upsilon_{max} = \frac{wL^4}{8EI} = \frac{3\ \text{k/ft}(\frac{1}{12})(18\text{x}12\ \text{in})^4}{8(29\text{x}10^3\ \text{ksi})(1{,}200\ \text{in}^4)} = 1.95\ \text{in}$

Problem 7.45

The following beam is composed of wood and two steel plates at the top and at the bottom of wood as shown. The beam has a maximum moment of 100 kip-in. What is the maximum bending stress developed in the steel? $E_s = 210$ GPa, $E_w = 21$ GPa.



A. 95 psi
B. 95 ksi
C. 0.95 ksi
D. 0.95 psi

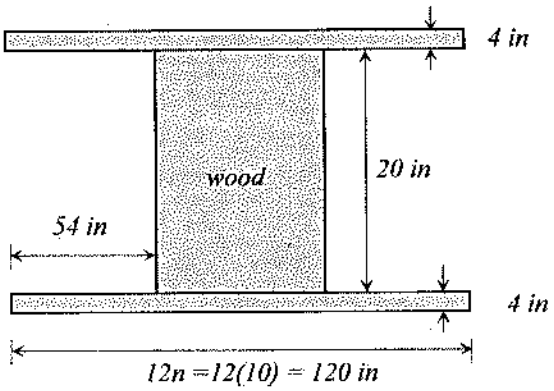
Solution: A

$$n = \frac{E_s}{E_w} = \frac{210}{21} = 10$$

Moment of Inertia, $I = \frac{120(28)^3}{12} - 2\frac{54(20)^3}{12} = 147{,}520\ \text{in}^4$

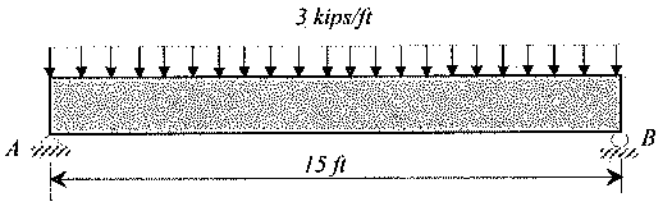
Maximum bending stress in steel, $\sigma = n\frac{Mc}{I} = 10\frac{100\ \text{kip.in}\,(14\,\text{in})}{147{,}520\ \text{in}^4} = 0.095\ \text{ksi} = 95\,\text{psi}$

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Problem 7.46

A simply supported 15 ft long beam has a load of 3 kip/ft. Is the section adequate for deflection criteria? E= 29x10³ ksi, I= 882 in⁴, and the allowable deflection limit $\Delta_{all} = \dfrac{L}{360}$.



A. Yes
B. No

Solution: A

The maximum deflection, $\Delta_{max} = \dfrac{5wL}{384\,EI} = \dfrac{5(3\text{ kips/ft }/(12\text{ in/ft}))(15x12\text{ in})^4}{384(29x10^3\text{ksi})(882\text{ in}^4)} = 0.13\text{ in.}$

The allowable deflection limit, $\Delta_{all} = \dfrac{L}{360} = \dfrac{15x12\text{ in}}{360} = 0.5\text{ in.}$

$\Delta_{all} > \Delta_{max}$; Therefore, the section is adequate considering the deflection.

Problem 7.47

A steel W-section beam has the maximum moment of 100 kip-ft and allowable bending stress of 20 ksi. Which of the following sections is adequate to support this bending?

A. W16x36 B. W12x40 C. W10x39 D. W10x60

Solution: D

The required section modulus, $S = \dfrac{M}{\sigma} = \dfrac{100(12)\text{ kip.in}}{20\text{ ksi}} = 60\text{ in}^3$

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W10 x 60 has $S > 60$ in^3. All other sections have S-values lower than 60 in^3.

Problem 7.48

Which of the following conditions is a correct design principle?

A. Actual deflection > allowable deflection
B. Actual axial stress > allowable axial stress
C. Actual shear stress < allowable shear stress
D. All of the above

Solution: C

Actual axial stress, shear stress, and deflection must be equal or less than the allowable ones. In practice, it would be expected that actual axial stress, shear stress, and deflection be less than the allowable ones.

Problem 7.49

Which of the following parameters is a factor for beam deflection?

A. Load magnitude
B. Beam span length
C. Moment of inertia and the beam's modulus of elasticity
D. All of the above

Solution: D

Load magnitude, beam span length, the moment of inertia of the beam cross-section, and the beam's modulus of elasticity are factors in the amount of deflection that results.

Columns

There is no sharp definition of short column and long column. The understanding is that short column tends to fail in compressive yielding. On the other hand, if a column is tall, it fails by lateral displacement at the middle (called buckling). In compressive yielding failure, material is utilized perfectly and shows visible deformation before failure. On the other hand, bucking failure is sudden and material properties are not perfectly utilized.

Depending on the shape and length of the column, Leonhard Euler developed an equation to determine the critical/maximum load below which no buckling is to be occurred. That load is called Euler Buckling load. It is determined as:

$$P_{critical} = \frac{\pi^2 E I_{min}}{(KL)^2}$$

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$P_{critical}$ = Euler Buckling load
E = Modulus of elasticity or Young's Modulus
I_{min} = Minimum moment of inertia
K = Effective length factor
L = length of the column

Then, critical stress ($f_{critical}$) can be calculated as- $f_{critical} = \frac{P_{critical}}{A}$

where: A = cross-sectional area of the column.

The value of K depends on the support conditions of the column. The NCEES FE Ref. Handbook lists K-values for different conditions in Civil Engineering Section. Theoretical effective-length factors (K_s) for columns include:

Pinned-pinned, $K = 1.0$
Fixed-fixed, $K = 0.5$
Fixed-pinned, $K = 0.7$
Fixed-free, $K = 2.0$

Critical buckling stress for long columns: $\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E}{\left(\frac{KL}{r_{min}}\right)^2}$

r_{min} = radius of gyration, $\sqrt{\left(\frac{I}{A}\right)}$

$\frac{KL}{r_{min}}$ = effective slenderness ratio for the column

Problem 7.50

KL/r is known as-

A. Buckling load
B. Buckling length to radius ratio
C. Slenderness Ratio
D. Compact Ratio

Solution: C

KL/r is known as Slenderness Ratio.

Problem 7.51

For calculating the buckling stress, which radius of gyration should be used?

A. Radius of gyration about the x-axis

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B. The smaller radius of gyration
C. Radius of gyration about the y-axis
D. None of the above

Solution: B

The smaller radius of gyration is used for calculating the buckling stress. It gives conservative results.

Problem 7.52

Lateral bucking occurs for-

A. Long-thick column
B. Short-thin column
C. Short-thick column
D. Long-thin column

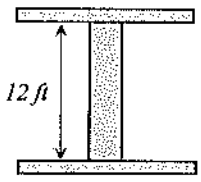
Solution: D

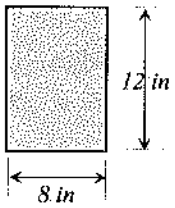
Long thin column fails by lateral bucking. Therefore, the design guidelines restrict the length and cross-sectional area.

Problem 7.53

A 12-ft long column is pin supported at both ends. The cross-section of the column is shown below. What is most nearly the critical buckling stress (ksi) of the column? $E = 2{,}000$ ksi.

A. 3.1 B. 4.1 C. 5.1 D. 6.1





Solution: C

Moment of inertia about horizontal axis, $I_x = \dfrac{8(12)^3}{12} = 1{,}152 \text{ in}^4$

Moment of inertia about vertical axis, $I_y = \dfrac{12(8)^3}{12} = 512 \text{ in}^4$ [this is the minimum]

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Minimum radius of gyration, $r_{min} = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{512\ \text{in}^4}{8(12)\ \text{in}^2}} = 2.31\ \text{in}$

Critical buckling stress, $\sigma_{cr} = \frac{\pi^2 E}{\left(\frac{KL}{r_{min}}\right)^2} = \frac{\pi^2 (2x10^3\ \text{ksi})}{\left(\frac{1.0(12x12)\text{in}}{2.31\ \text{in}}\right)^2} = 5.1\ \text{ksi}$

Problem 7.54

The Euler Buckling formula characterizes –

A. Bending stress
B. Lateral deflection
C. Elastic buckling
D. Comparatively short column

Solution: C

Elastic Strain Energy

If the strain remains within the elastic limit, the work done during deflection (extension) of a member will be transformed into potential energy and can be recovered. If the final load is P and the corresponding elongation of a tension member is δ (whose initial length is L), then the total energy (U) stored is equal to the work (W) done during loading.

$$U = W = \frac{1}{2} P\delta$$

The strain energy per unit volume (u) is given as: $u = \frac{U}{AL} = \frac{1}{2}\frac{\sigma^2}{E}$ (for tension)

Problem 7.55

A tensile force of 100 kN is applied on a 0.02 m diameter 2 m length rod. After applying the load, the diameter of the rod decreases to 0.01998 m and the length increases to 2.01 m. The work (J) done during the loading is most nearly:

A. 0.5 B. 500 C. 50 D. 5,000

Solution: B

Work done, $U = W = \frac{1}{2} P\delta = \frac{1}{2}(100\ \text{kN})(0.01\ \text{m}) = 0.5\ \text{kJ} = 500\ \text{J}$

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Problem 7.56

A tensile force of 100 kN is applied on a 0.02-m diameter 2-m length rod. After applying the load, the diameter of the rod decreases to 0.01998 m and the length increases to 2.01 m. The strain energy per unit volume is most nearly:

A. 0.795 MJ/m³ B. 0.975 MJ/m³ C. 0.795 J/m³ D. 0.795 kJ/m³

Solution: A

Axial stress, $\sigma = \frac{F}{A} = \frac{100 \text{ kN}}{\frac{\pi}{4}(0.02 \text{ m})^2} = 318{,}309 \text{ kPa} = 318 \text{ MPa}$

Axial strain, $\varepsilon_a = \frac{\Delta L}{L_o} = \frac{L - L_o}{L_o} = \frac{2.01 - 2.0}{2.0} = 0.005 = 0.5\%$

Elasticity, $E = \frac{\sigma_E}{\varepsilon_a} = \frac{318 \text{ MPa}}{0.005} = 63600 \text{ MPa} = 63.6 \text{ GPa}$

Strain energy per unit volume, $u = \frac{U}{AL} = \frac{\sigma^2}{2E} = \frac{(318 \text{ MPa})^2}{2(63.6x10^3 \text{ MPa})} = 0.795 \text{ MJ/m}^3$

Material Properties

Some common material properties are listed in the NCEES FE Ref. Handbook. Please skim through it so that you become familiar with these two tables.

Problem 7.57

The Modulus of Rigidity of typical brass material is most nearly:

A. $40x10^3$ ksi B. $5.8x10^3$ ksi C. $29x10^3$ ksi D. 210 GPa

Solution: B

The Modulus of Rigidity of Brass is $5.8x10^3$ ksi from Table 1.

Miscellaneous Problems

Problem 7.58

Modulus of Rigidity is defined as the –

A. Capacity of deflection resistance
B. Initial slope of shear stress versus shear strain curve
C. Slope of stress-strain curve

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D. Energy required before the failure

Solution: B

Modulus of Rigidity or Shear Modulus is defined as the Initial slope of shear stress versus shear strain curve

Problem 7.59

Modulus of Elasticity is defined as the –

A. Capacity of deflection resistance
B. Slope of stress versus strain curve until elastic region
C. Initial slope of stress-strain curve
D. Energy required before the failure

Solution: C

Modulus of elasticity is defined as the initial slope of stress versus strain curve or any point up to the proportional limit.

Problem 7.60

Hooke's law is valid up to -

A. Yield point
B. Elastic region
C. Linear stress-strain region
D. Any point in stress-strain curve

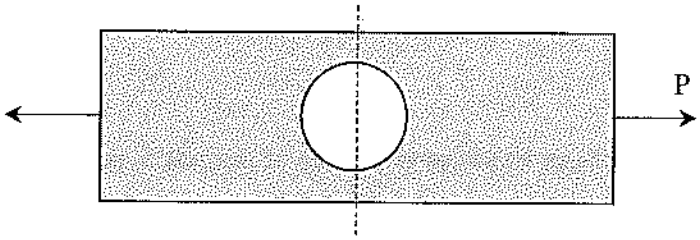
Solution: C

Hooke's law is valid up to the proportional limit or linear portion of stress-stress curve. It is valid up to yield point or up to elastic region if the whole elastic region is linear, which is very rare.

Problem 7.61

Which of the following statements is true for the following member? The cross-sectional area is the minimum along the dash line.

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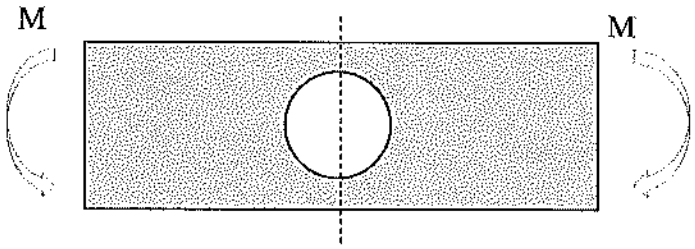
A. Axial stress is equal to P/A_{min}
B. Axial stress is greater than P/A_{min}
C. Axial stress is smaller than P/A_{min}
D. None of the above

Solution: B

For a sudden change in cross-section, the actual stress is greater than the calculated one using $\sigma = \frac{P}{A}$ or $\sigma = \frac{Mc}{I}$ or $\tau = \frac{VQ}{It}$. That increasing factor is called the stress concentration factor.

Problem 7.62

Which of the following statements is true for the following member? The cross-sectional area is the minimum along the dash line.



A. Bending stress is equal to $\sigma = \frac{Mc}{I}$
B. Bending stress is greater than $\sigma = \frac{Mc}{I}$
C. Bending stress is smaller than $\sigma = \frac{Mc}{I}$
D. None of the above

Solution: B

See the previous problem.

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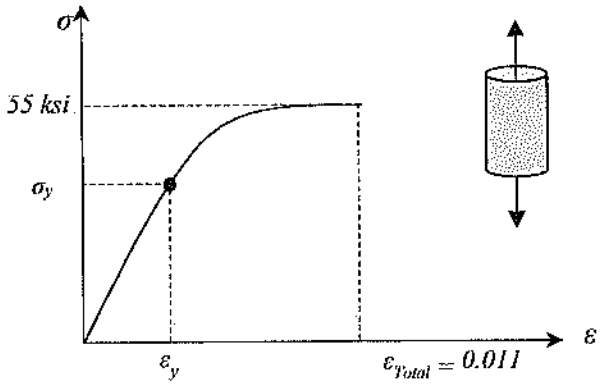
Problem 7.63

For an A36 steel, the total of 0.011 strain occurs if the stress of 55 ksi is applied. What will be most nearly its permanent strain for this stress level (55 ksi)?

A. 0.011 B. 0.00124 C. 0.00223 D. 0.0098

Solution: D

$E = 29\text{x}10^3$ ksi for any kind of steel; A36 steel means, $\sigma_y = 36$ ksi



Elastic/yield strain, $\varepsilon_y = \dfrac{\sigma_y}{E} = \dfrac{36\text{ ksi}}{29\text{x}10^3\text{ ksi}} = 0.00124$

$\varepsilon_{Total} = \varepsilon_y + \varepsilon_{Permanent}$

$\varepsilon_{Permanent} = \varepsilon_{Total} - \varepsilon_y = 0.011 - 0.00124 = 0.0098$

Problem 7.64

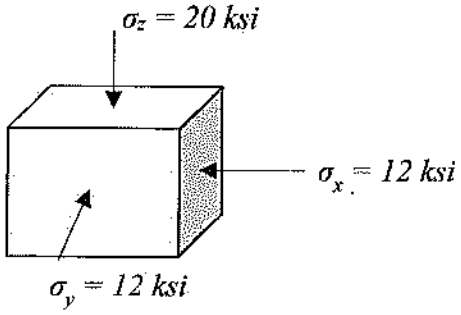
On a cube element, a vertical compressive stress of 20 ksi is applied. The resulting lateral compressive stress is 12 ksi in both orthogonal lateral directions. The modulus of elasticity of the material is 23,000 ksi and the Poisson's Ratio is 0.25. The vertical compressive strain of the cube element is most nearly:

A. 0.00061 B. 0.00087 C. 0.00097 D. 0.0097

Solution: A

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Let us assume the compressive stress is positive. The stress block is shown; the balancing stresses are not shown for simplicity.

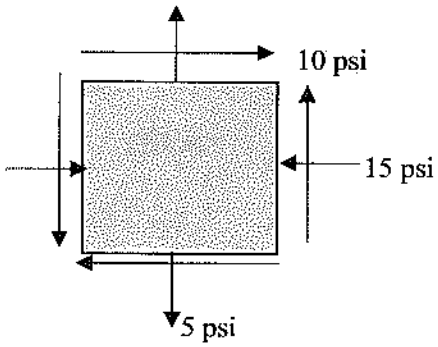


$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right] = \frac{1}{23{,}000\ \text{ksi}}\left[20\,\text{ksi} - 0.25(12\,\text{ksi} + 12\,\text{ksi})\right] = 0.00061$$

Remember: $\varepsilon = \dfrac{\sigma}{E}$ is true for 1D body only.

Problem 7.65

An element is subjected to the following plane stress conditions. The principal stresses (psi) are most nearly:



A. 9.14, - 19.14
B. 14.14, - 5.00
C. 14.14, 22.14
D. None of the above

Solution: A

From the NCEES FE Ref Handbook, the Mohr Circle Radius,

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{-15-5}{2}\right)^2 + 10^2} = 14.14\ \text{psi}$$

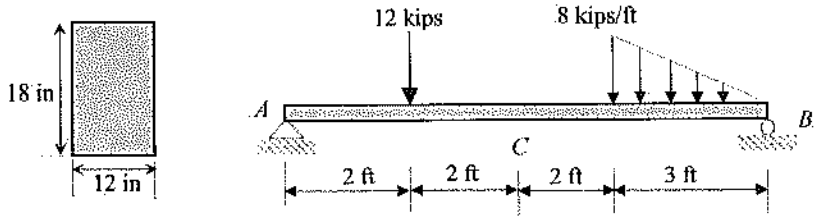
$$C = \frac{\sigma_x + \sigma_y}{2} = \frac{-15+5}{2} = -5\ \text{psi}$$

$\sigma_1 = C + R$ = -5 psi + 14.14 psi = 9. 14 psi
$\sigma_3 = C - R$ = -5 psi - 14.14 psi = -19. 14 psi

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Problem 7.66

For the following simply supported beam, the maximum bending stress at '*C* is most nearly:



A. 444 ksi B. 444 psi C. 328 psi D. 1,423 psi

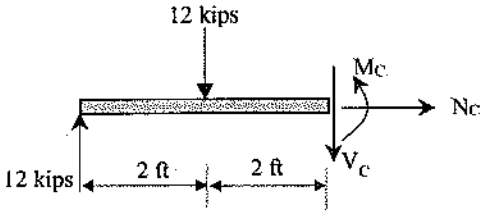
Solution: B

To determine the maximum bending stress at *C*, we need the internal moment at C first.

$\sum M_B = 0$

½ (8 kips/ft)(3 ft) (2 ft) + 12 kips (7 ft) – A_y (9 ft) = 0

A_y = 12 kips



$\sum M$ at $C = 0$

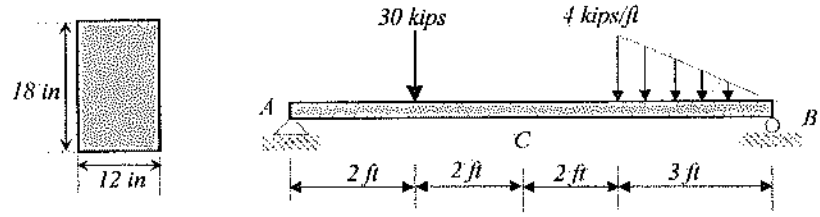
$-M_c - 12$ kips (2 ft) + 12 kips (4 ft) = 0

M_c = 24 kips-ft = 288 kips-in

$$\sigma = \frac{Mc}{I} = \frac{288 \text{ kip.in } (9 \text{ in})}{\frac{12(18)^3}{12} \text{ in}^4} = 0.44 \text{ ksi} = 444 \text{psi}$$

Problem 7.67

For the following simply supported beam, the maximum shear stress (psi) at '*C* is most nearly:



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A. 0.037 B. 37 C. 6.54 D. 5.33

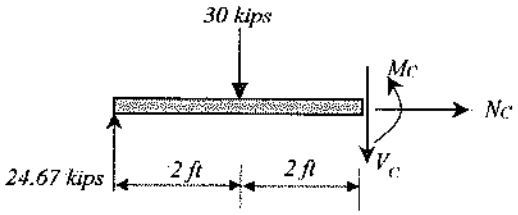
Solution: B

To determine the maximum shear stress at C, we need to find the internal shear at C first.

$\sum M_B = 0$

½ (4 kips/ft)(3 ft) (2 ft) + 30 kips (7 ft) – A_y (9 ft) = 0

A_y = 24.67 kips



$\sum F_y = 0$

- V_C - 30 kips + 24.67 kips = 0

V_C = 5.33 kips

For rectangular section, $\tau_{max} = \frac{VQ}{It} = 1.5\frac{V}{A} = 1.5\frac{5.33 \text{ kips}}{(12 \text{ in})\ (18 \text{ in})} = 0.037 \text{ ksi} = 37 \text{ psi}$

Problem 7.68

A cylindrical vessel has an outer diameter of 14 inch and a wall thickness of 0.5 inch. After applying a pressure of 100 psi, a transverse crack was developed at the vessel wall. The axial cracking stress (psi) of the pressure vessel is most nearly:

A. 500
B. 675
C. 900
D. 1,200

Solution: B

Transverse crack developed means it was failed in axial stress.

Outer radius, $r_o = \frac{14 \text{ in}}{2} = 7 \text{ in}$

Inner radius, $r_i = 7 - 0.5 = 6.5$ in

Average radius, $r = \frac{r_i + r_o}{2} = \frac{6.5 + 7}{2} = 6.75$ in

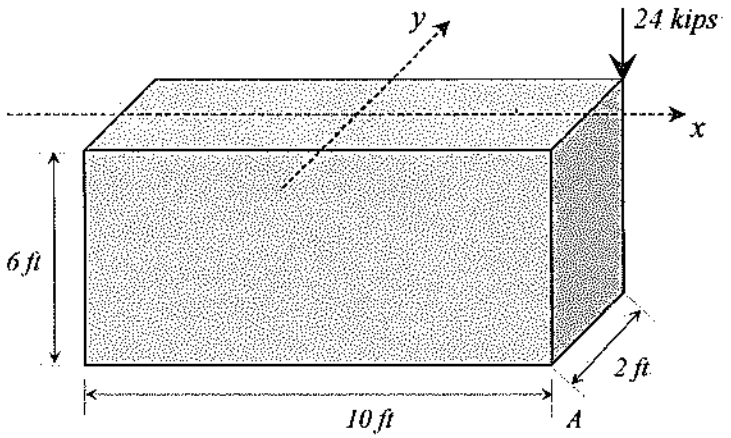
$\frac{t}{r_i} = \frac{0.5 \text{ in}}{6.5 \text{ in}} = \frac{1}{13} < \frac{1}{10}$; so it is a thin-walled pressure vessel.

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$$\sigma_a = \frac{p_i r}{2t} = \frac{100 \text{ psi}(6.75 \text{ in})}{2(0.5) \text{ in}} = 675 \text{ psi}$$

Problem 7.69

A compression block is supporting a corner load of 24 kips. The base dimension of the block is 10-ft x 2-ft, and the height is 6 ft. The total axial stress (ksf) at the other corner, *A*, due to this eccentric loading is most nearly:



A. 1.2
B. 1.3
C. 1.1
D. 6.1

Solution: B

Total axial stress = normal stress ± bending stress

$$\sigma_{Total} = \sigma_N \pm \frac{M_x c_x}{I_x} \pm \frac{M_y c_y}{I_y}$$

$$= -\frac{P}{A} + \frac{M_x c_x}{I_x} - \frac{M_y c_y}{I_y}$$

$$= -\frac{24 \text{ kips}}{(10)(2)} + \frac{24(1)}{6.67} - \frac{(124)(5)}{166.67}$$

$$= -1.2 + 3.6 - 3.7$$

$$= -1.3 \text{ ksf}$$

M_x = moment with respect to x-axis
= 24 k(1 ft) = 24 k-ft
c_x = point of concern from the x-axis = 1 ft
I_x = moment of inertia with respect to x-axis

$$= \frac{bh^3}{12} = \frac{10(2)^3}{12} = 6.67 \text{ ft}^4$$

M_y = 24 k(5 ft) = 120 k-ft
C_y = 5 ft
I_y = moment of inertia with respect to x-axis

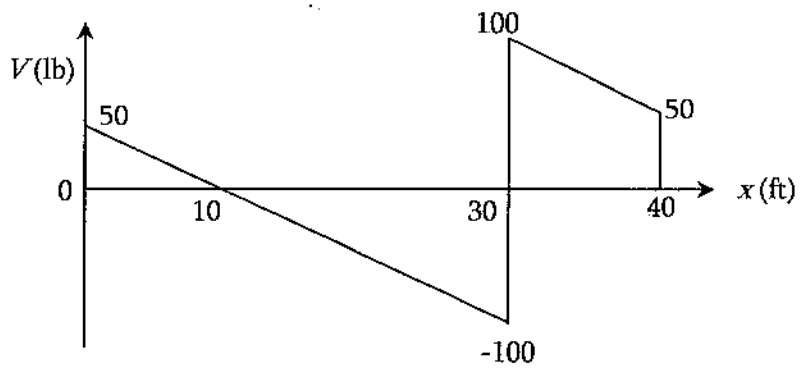
$$= \frac{bh^3}{12} = \frac{2(10)^3}{12} = 166.67 \text{ ft}^4$$

- M_x produces tensile stress at A
- M_y produces comp. stress at A

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Problem 7.70

The shear force diagram of a particular beam is shown below. All the lines shown are straight lines. The shear force is expressed in pound, the distance is expressed in feet. The maximum bending moment (lb.ft), regardless of positive or negative, in the beam is most nearly:

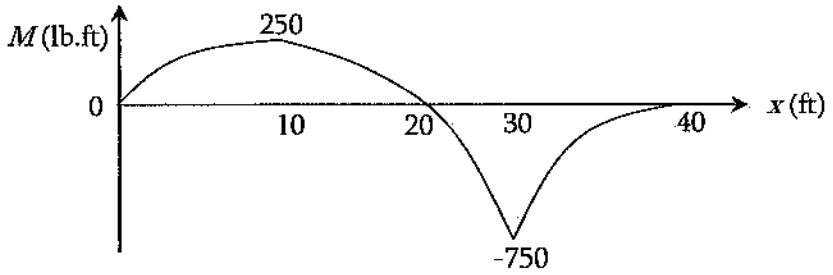


A. 570
B. 425
C. 750
D. 250

Solution: C

M is maximum where $V = 0$. Therefore, the potential points are $x = 10$, 30 ft from left.

Sum of area of V-diagram from left to 10 ft = ½ (50 lb) (10 ft) = 250 lb.ft
Sum of area of V-diagram from left to 30 ft = ½ (50 lb) (10 ft) + ½ (-100 lb) (30 – 10 ft) = -750 lb.ft



Beyond the upmost effort, this book might have few errors or inappropriate explanation. The author is apologizing for that. The author is requesting the readers to report any concern about the book at books.mrislam@gmail.com. Any suggestion to improve this book, or any issue reported will be fixed in the next edition/printing with proper acknowledgement.

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8 MATERIALS

Fundamentals of Engineering (FE) CIVIL Exam Specification mentions, 4-6 questions may appear in the exam based on Materials as follows:

A. Mix design (e.g., concrete and asphalt)
B. Test methods and specifications (e.g., steel, concrete, aggregates, asphalt, wood)
C. Physical and mechanical properties of concrete, ferrous and nonferrous metals, masonry, wood, engineered materials (e.g., FRP, laminated lumber, wood/plastic composites), and asphalt

The fact is that this topic is embedded in Mechanics of Materials, Structural Steel Design, Reinforced Concrete Design, Transportation Engineering, and so on. Go through those topics for your understanding. Few areas which are not included in those areas are discussed here.

Material Types

Ductile Material. The material which shows significant elongation/deformation before failing is called ductile material. Examples are mild steel, rubber band, etc.
Brittle Material. The material which does not show significant elongation/deformation before failing is called brittle material. Glass, concrete, etc. are examples of brittle material.

Material Behaviors

Elastic Material. Elasticity is the ability of a material to resist deformation or stress and to return to its original size and shape when the stress is removed. Materials deform when forces are applied on them. If the material is elastic, the object returns to its initial shape and size when these forces are removed. This behavior does not depend on the loading rate. For very small amount of stress, few materials can be considered as elastic. With increase in load/stress, material goes beyond the elasticity and permanent deformation may occur. Then, the material does not come back to its original position and shape upon releasing the load. Elasticity can be linear; stress is proportional to strain. This is called Hooke's Law. Elasticity can be non-linear; strain is not proportional to stress.
Plastic Material. Plasticity is the permanent deformation behavior of a material. Upon releasing load from a body, if it does not come back to its original position and shape, then its behavior is somewhat plastic. For perfectly plastic body, upon releasing load, no part of the deformation heals.

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Viscous Material. Viscous material shows time dependent deformation behavior. It has two important properties: creep and relaxation.

Creep. It is defined as the time rate of deformation. For viscous material, if a fixed amount of load or stress is sustained on a body, the deformation or strain increases with time.

Relaxation. For viscous material, if a fixed amount of deformation or stain is sustained on a body, the load or stress relaxes with time.

Fatigue Behavior. For a small amount of loading a body may not fail. However, if that small amount of load is repeated for many times, it may fail. This behavior is called fatigue. Fatigue occurs when a material undergoes repeated cycles of loading. For example, if you bend a hair clip a certain amount it may not fail. But, if keep on bending and releasing a hair clip for thousands of times, it may fail. There is a limit of applied stress or strain, for which a material does not fail or can take infinite amount of loading is called fatigue endurance limit.

Tensile Strength

For plain carbon steels, there is a general relationship between Brinell Hardness and tensile strength as follows:

TS (psi) ≈ 500 BHN
TS (MPa) ≈ 3.5 BHN

Fracture Toughness

The combination of applied stress and the crack length in a brittle material. It is the stress intensity when the material will fail.

$$K_{IC} = Y\sigma\sqrt{\pi a}$$

K_{IC} = fracture toughness
σ = applied engineering stress
a = crack length
Y = geometrical factor (Y = 1.1 for exterior crack; Y = 1 for interior crack)

The critical value of stress intensity at which catastrophic crack propagation occurs, K_{Ic}, is a material property.

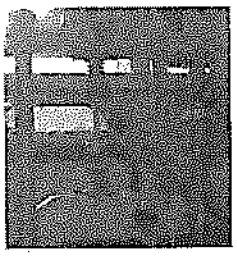
Composite Materials

$$\rho_c = \sum f_i \rho_i$$
$$C_c = \sum f_i C_i$$
$$\sigma_c = \sum f_i \sigma_i$$
$$E_c = \sum f_i E_i$$

Subscript, c = composite; i = any part
σ = strength parallel to fiber direction
ρ = density
C = heat capacity of composite per unit volume
E = Young's modulus

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Material Testing



Tensile Test

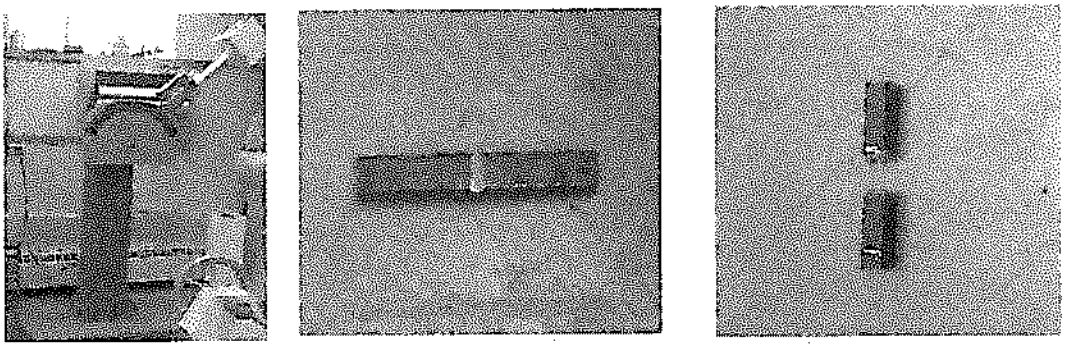
Tensile test in material is conducted to determine the modulus, yield point, ultimate stress, failure stress, etc. A sample is clamped and pulled axially. The force and deformation are recorded. The test can be force controlled or displacement controlled. The stress-strain diagram is plotted to determine the desired mechanical properties.

Compressive Test

Compressive test is conducted by applying axial compressive load in a sample to determine the confined and unconfined compressive strength of material. Examples of compressive strength tests are determining unconfined compressive strength of soil, compressive strength of concrete, etc.

Impact Test

The impact test determines the amount of energy absorbed by a material during fracture. It is also known as the Charpy V-notch test. This absorbed energy is a measure of a given material's notch toughness. The test apparatus is mainly a pendulum of known mass and length. It is dropped from a known height to impact a notched specimen. The energy transferred to the material can be inferred by comparing the difference in the height of the hammer before and after the fracture (energy absorbed by the fracture event).



a) Impact Test Equipment b) Notched speciment c) Failed speciment

Fig. Impact testing

Hardness Test

Hardness test measures the resistance to indentation/penetration of a material. For a given indenter with fixed load, the more the penetration the lower the hardness of the material surface.

Now practice the following problems related to mechanics of materials, and material testing. There are some topics which are not discussed above; however, those are discussed below using problems.

Problem 8.1

The tensile strength capacity of a material is 50 ksi. The allowable stress of that material if the factor of safety is 2.0 is most nearly:

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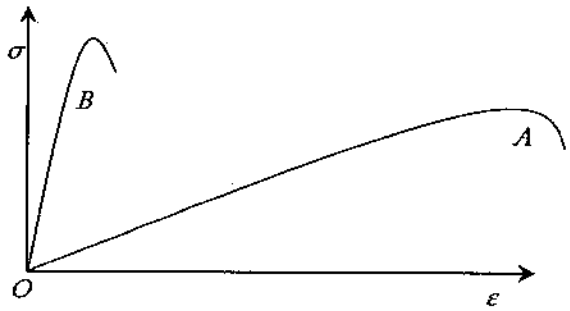
A. 50 ksi
B. 25 ksi
C. 100 ksi
D. Cannot be determined

Solution: B

Allowable stress=failure stress /factory of safety =50 ksi/2 = 25 ksi

Problem 8.2

Which of the following materials (*A* or *B*) is ductile?



Solution: A

Problem 8.3

What is the toughness?

A. Time dependent damage
B. Damage by repeated load
C. Energy absorbed up to elastic limit
D. Energy absorbed up to failure

Solution: D

Time dependent damage: Creep Behavior
Damage by repeated load: Fatigue
Energy absorbed up to elastic limit: Modulus of Resilience
Energy absorbed up to failure: Toughness

Problem 8.4

Which test measure the capacity of a surface to resist deformation?

A. Creep

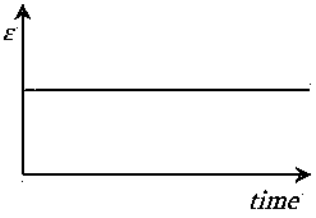
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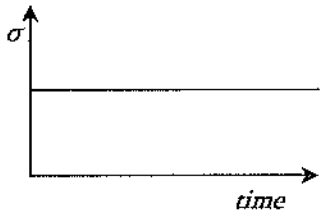
B. Fatigue
C. Hardness
D. Toughness

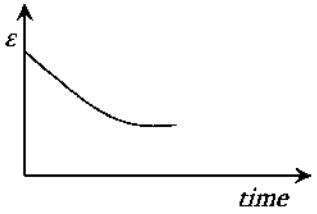
Solution: C

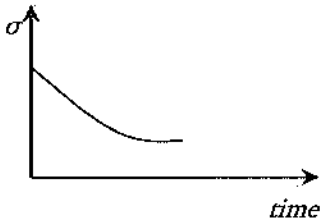
Problem 8.5

Which of the following curves is due to creep test?

A.


B.


C.


D.


Solution: B (In creep test, constant stress is applied on viscoelastic materials; then, strain increases with time)

Note: Viscoelastic materials have both elastic and viscous properties. Elastic means deformation occurs instantly after applying load and come back to original position upon releasing load. Viscous materials take time to deform upon loading and take time to restore upon releasing the load.

Problem 8.6

Which of the following statements is true?

A. Hardness test measures the toughness of a material
B. Stiffness and hardness are the same property
C. The region between proportional limit and the yield point is called plastic region
D. None of the above

Solution: D

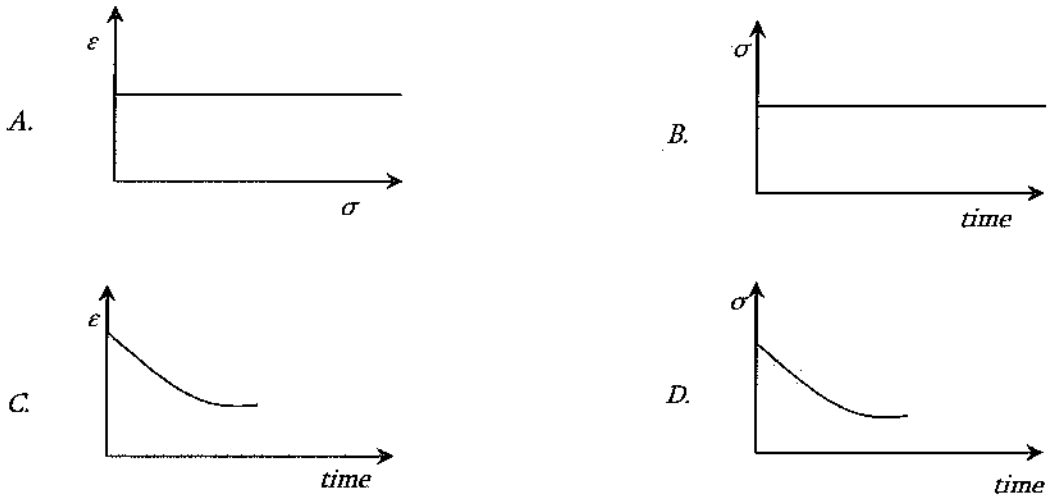
Hardness test measures the resistance of surface deformation of a material.
Stiffness and hardness are not the same property.

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The region between proportional limit and yield point is called non-linear elastic region.

Problem 8.7

Which of the following curves is due to relaxation testing?



Solution: D

In relaxation test, constant strain with time is applied on viscoelastic materials; then, stress decreases with time.

Problem 8.8

Impact test is not done –

A. to determine the amount of energy required to cause failure in standardized test samples
B. repeatedly over a range of temperatures
C. to determine the ductile to brittle transition temperature
D. to determine the surface resistance against deformation

Solution: D

Hardness test is done to determine the surface resistance against deformation. The first three options are true for impact test.

Problem 8.9

Thermal expansion coefficient is related to-

A. True strain
B. Engineering strain

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C. True stress
D. Engineering stress

Solution: B

The thermal expansion coefficient is the ratio of engineering strain to the change in temperature.

Problem 8.10

While mechanical processing of metal, raising temperature does not cause –

A. recovery (stress relief)
B. increase strength and lowers ductility
C. recrystallization
D. grain growth

Solution: B

Raising temperature causes (1) recovery (stress relief), (2) recrystallization, and (3) grain growth. On the other hand, Cold working (plastically deforming) a metal increases strength and lowers ductility. See the Materials Section of the NCEES Ref. Handbook.

Problem 8.11

Using the amount of load required to produce a small amount compressive deformation in a steel bar, which of the following parameters can be determined?

A. Hardness B. Ductility C. Modulus D. Compressive strength

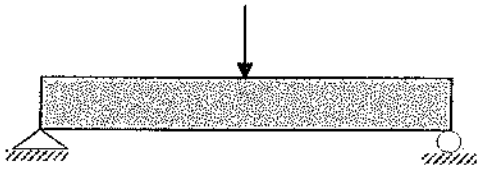
Solution: C

Modulus is defined as $E = \frac{\sigma}{\varepsilon} = \frac{\frac{P}{A}}{\frac{\Delta L}{L}} = \frac{PL}{A\Delta L}$. The area ($A$) and the initial length (L) are typically known. Therefore, using the amount of load required to produce a small amount compressive deformation in a steel bar, modulus of steel can be determined.

Problem 8.12

A 4-ft long simply supported beam is tested until failure by applying a concentrated load at the mid-span. The beam is 6-in wide and 10-in high. Crack initiates at the bottom fiber of the beam, roughly at the mid-span at 3,000 lb of loading. The tensile strength of the beam is most nearly:

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A. 240 psi B. 300 psi C. 360 psi D. None of the above

Solution: C

Positive bending moment causes tensile stress at the bottom fiber of beam and compressive stress at the top fiber of the beam.

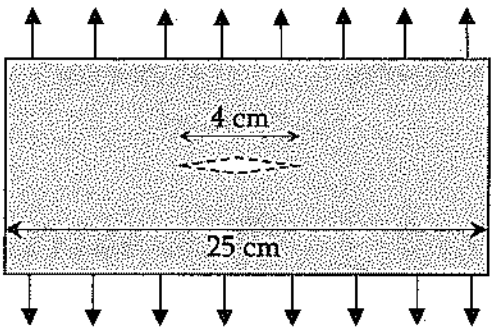
Maximum moment, $M = PL/4 = 3{,}000$ lb (4 x 12) in / 4 = 36,000 lb-in

$$I = \frac{bh^3}{12} = 6(10)^3/12 = 500 \text{ in}^4$$

$$\sigma = \frac{Mc}{I} = 36{,}000 \text{ lb-in (5 in) / 500 in}^4 = 360 \text{ psi}$$

Problem 8.13

A 1.0-cm-thick brass plate is 25 cm wide. It has an interior crack of 4 cm length. If a tensile force of 5 kN is applied as shown below, the stress intensity factor at the end of the crack tip is most nearly:



A. 2.0 MPa.$\sqrt{\text{m}}$ B. 0.71 MPa.$\sqrt{\text{m}}$ C. 0.78 MPa.$\sqrt{\text{m}}$ D. 0.5 MPa.$\sqrt{\text{m}}$

Solution: D

From the NECCES FE Ref. Handbook, Materials Science/Structure of Matter,

$Y = 1.0$ for interior crack

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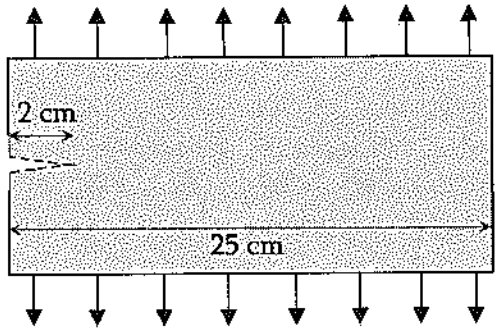
$a = 4\text{ cm}/2 = 2\text{ cm} = 0.02\text{ m}$

$$\sigma = \frac{F}{A} = \frac{5{,}000\ \text{N}}{\left(\frac{25}{100}\text{m}\right)\left(\frac{1}{100}\text{m}\right)} = 2\text{x}10^{6}\ \frac{\text{N}}{\text{m}^2}$$

$$K_{IC} = Y\sigma\sqrt{\pi a} = (1.0)(2\,\text{MPa})\sqrt{(\pi)(0.02\,\text{m})} = 0.5\ \text{MPa}.\sqrt{\text{m}}$$

Problem 8.14

A 1.0-cm-thick brass plate is 25 cm wide. It has an exterior crack of 2 cm length. If a tensile force of 5 kN is applied as shown below, the stress intensity factor at the end of the crack tip is most nearly:



A. 2.0 MPa.$\sqrt{\text{m}}$ B. 0.71 MPa.$\sqrt{\text{m}}$ C. 0.78 MPa.$\sqrt{\text{m}}$ D. 0.55 MPa.$\sqrt{\text{m}}$

Solution: D

$Y = 1.1$ for exterior crack

$a = 2\text{ cm} = 0.02\text{ m}$

$$\sigma = \frac{F}{A} = \frac{5{,}000\ \text{N}}{\left(\frac{25}{100}\text{m}\right)\left(\frac{1}{100}\text{m}\right)} = 2\text{x}10^{6}\ \frac{\text{N}}{\text{m}^2}$$

$$K_{IC} = Y\sigma\sqrt{\pi a} = (1.1)(2\,\text{MPa})\sqrt{(\pi)(0.02\,\text{m})} = 0.55\ \text{MPa}.\sqrt{\text{m}}$$

Concrete

Concrete is the most common material in civil engineering. It is a composition of coarse-aggregate, fine-aggregate (sand), cement and water. Cement is the binding agent; it reacts with water, hydrates, and gives strength to whole mix. Certain amount of water is required for the cement to fully hydrate, and make the mix workable. However, excess water creates large bubbles inside the mix once evaporates, and makes the concrete weaker and non-durable. Today, three general types of portland cement are primarily used for agricultural applications:

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Type I cement is the normal, general purpose, and most common type. Unless an alternative is specified, Type I is usually used.
Type II cement is modified to release less heat during hydration, and is therefore, more suitable for projects involving large masses of concrete-heavy retaining walls, or deadmen for suspension bridges.
Type III cement produces concrete that gains strength very rapidly. It is very finely ground and sets rapidly, making it useful for slip-form construction and cold-weather jobs.

Good water is essential for quality concrete. It should be free of trash, organic matter and excessive chemicals and/or minerals. The strength and other properties of concrete are highly dependent on the amount of water and the water-cement ratio. Enough water is needed for full hydration, but too much water leads to shrinkage cracks, voids, low strength, and generally poor-quality concrete. If a mix is too stiff to handle well, and the water-cement ratio is not known, do not just add water. Either reject the batch, or add both cement and water. Re-proportion subsequent batches to produce the right consistency and workability.

While concrete mix design, it is assumed that aggregates are dry. The proportions of aggregates, cement, and water is determined. Then, depending on the absorption capacity of aggregate, and available in-situ moisture content, the final water required is calculated. This process is very important for the FE Exam. Let us see one problem.

Problem 8.15

A concrete mix design originally had the following quantities:

Cement = 700 lb/yd^3
Coarse Aggregate = 1,700 lb/yd^3
Fine Aggregate = 1,100 lb/yd^3
Water = 300 lb/yd^3

The aggregates were considered oven-dry condition. However, the coarse and fine field aggregates had 5% and 4% moistures respectively. The absorption rate for both types of aggregates is 0.5%. The amount of water to be used is most nearly:

A. 202 lb/yd^3 B. 185 lb/yd^3 C. 175 lb/yd^3 D. 165 lb/yd^3

Solution: B

$$\text{Excess water in Coarse-Aggregate} = \frac{(5\% - 0.5\%)}{100}(1{,}700) = 76.5 \text{ lb/yd}^3$$

$$\text{Excess water in Fine-Aggregate} = \frac{(4\% - 0.5\%)}{100}(1{,}100) = 38.5 \text{ lb/yd}^3$$

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$$\text{Amount of water to be used} = 300 - 76.5 - 38.5 = 185 \ \frac{\text{lb}}{\text{yd}^3}$$

The above problem had excess water at in-situ condition for both coarse aggregate and fine aggregate. This is why, we deducted it from the required except the water that will be absorbed. Some other problems related to concrete mix design are discussed below.

Problem 8.16

A concrete mix has cement : sand : aggregate = 1 : 1.5 : 2.5 by weight. The specific gravity of cement, sand, and course aggregate are 3.15, 2.62, and 2.65 respectively. The cement bought had 94 lb in each sack. 6 gallons of water was used for each sack. The mix design had 4% air void by volume. The amount of cement per cubic in pcf foot used is most nearly:

A. 21.3
B. 25.3
C. 29.3
D. 33.3

Solution: B

$$V = \frac{\text{Weight (W)}}{\text{Unit weigh t } (\gamma)} = \frac{W}{G\gamma_w}$$

Volume of 1 sack of cement, $\frac{W}{G\gamma_w} = \frac{94\ lb}{3.15(62.4)} = 0.478\ \text{ft}^3$

Volume of sand used in 1 sack of cement, $\frac{W}{G\gamma_w} = \frac{1.5 \text{ x } 94\ lb}{2.62(62.4)} = 0.86\ \text{ft}^3$

Volume of aggregate used in 1 sack of cement, $\frac{W}{G\gamma_w} = \frac{2.5 \text{ x } 94\ lb}{2.65(62.4)} = 1.42\ \text{ft}^3$

Added water in 1 sack of cement = 6 gal = 6/(7.48) ft^3 = 0.8 ft^3
Volume of cement, sand, aggregate, and water = 3.56 ft^3 (96% of volume, 4% air void)
Total volume = 3.56 ft^3/ 0.96= 3.71 ft^3
Quantity of cement used = 94 lb / 3.71 ft^3 = 25.3 pcf

Problem 8.17

In a concrete mix design, a student proposed 70% aggregate, 20% cement, and 10% water. If the amount of aggregate used was 20 kg, the amount of cement is most nearly:

A. 0.175 kg B. 5.71 kg C. 2.86 kg D. 8.97 kg

Solution: B

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70% is equivalent to 20 kg. So, 20% is equivalent to (20/70)*20 = 5.71 kg

Problem 8.18

Two concrete cylinders (6 in x 12 in) were tested in uniaxial compression test. The samples failed at 88.5 kips and 90.5 kips load. The average compressive strength of the samples is most nearly:

A. 89.5 ksi B. 3.17 ksi C. 0.80 ksi D. 3.71 ksi

Solution: B

$$\text{Strength of sample 1} = \frac{\text{Load}}{\frac{\pi D^2}{4}} = \frac{88.5 \text{ kips}}{\frac{\pi 6^2}{4}} = 3.13 \text{ ksi}$$

$$\text{Strength of sample 2} = \frac{\text{Load}}{\frac{\pi D^2}{4}} = \frac{90.5 \text{ kips}}{\frac{\pi 6^2}{4}} = 3.2 \text{ ksi}$$

Average = 3.17 ksi

Problem 8.19

Concrete strength means -

A. Cracking strength
B. Compressive strength
C. Tensile strength
D. Shear strength

Solution: B

Problem 8.20

Which of the following types is a general-purpose cement?

A. Type I B. Type II C. Type III D. Type IV

Solution: A

Problem 8.21

Which of the following is false?

A. Initial set occurs when the paste begins to stiffen
B. Final set occurs when cement has hardened to a point which can sustain load

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C. The specific gravity of portland cement is 3.15
D. None of the above

Solution: D

All the three statements are true.

Wood

Wood is an extremely versatile material with a wide range of physical and mechanical properties among the many species of wood. Freshly-cut tree (wood) is seasoned to reduce moisture to a suitable level. Because of the orientation of the wood fibers and the way a tree increases in diameter as it grows, properties vary along three mutually perpendicular axes: longitudinal, radial, and tangential. The longitudinal axis is parallel to the fiber (grain) direction, the radial axis is perpendicular to the grain direction and normal to the growth rings, and the tangential axis is perpendicular to the grain direction and tangent to the growth rings. Although timber and lumber are used in different meaning in different regions, it is mostly used as a same item with different English words. Timber/lumber is the processed wood to be used in engineering; lumber is the raw wood. Some problems from wood are mentioned below:

Problem 8.22

Which of the following materials is a combustible material?

A. Concrete B. Wood C. Sand D. Steel

Solution: B

Wood is a combustible material; it is very susceptible to fire earlier than others listed here.

Problem 8.23

Does wood contain moisture?

A. yes for all species
B. not at all
C. yes if it is from coastal areas
D. yes if it is from rainy areas

Solution: A

Wood contains moisture. Its properties are also dependent on water content.

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Problem 8.24

In which direction of wood, it is the strongest?

A. Transverse
B. Radial
C. Equal in any direction
D. Longitudinal direction

Solution: D

Wood is the strongest along the longitudinal direction.

Problem 8.25

Seasoning of wood performed:

A. To increase strength
B. To increase durability
C. To remove natural defects
D. To reduce moisture to a suitable level

Solution: D

This is a process by which moisture content in a freshly cut tree is reduced to a suitable level. By doing so the durability of lumber is increased.

Asphalt Concrete

Like portland cement concrete, asphalt concrete consists of coarse-aggregate, fine-aggregate, and asphalt binder as the cementing materials. Proper selection of binder type, and proportions of aggregates and binder are very important in asphalt mix design. Now-a-days, Superpave mix design is used replacing the old Marshall, Hveem, or other methods. Superpave mix design has the following process:

a) Selection of Aggregates
b) Selection of Asphalt Binder Type
c) Selection of Design Aggregate Structure
d) Determination of Design Binder Content
e) Evaluation of Moisture Susceptibility

Aggregate is selected in a way that it satisfies both the source and consensus requirements. Source properties, commonly defined by the owner/contractors includes Los Angeles abrasion, soundness,

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and deleterious materials testing. Consensus properties includes coarse aggregate angularity, fine aggregate angularity, flat and elongated particles, and sand equivalency testing.

Asphalt binder type is selected based on the average 7-daymaximum pavement temperature, the minimum pavement temperature of the pavement site, and the expected traffic volume.

The design aggregate structure requires the verification of Voids-in-Mineral Aggregate (VMA), Voids-Filled with Asphalt (VFA), Dust-to-Binder ration, and compaction density for the target traffic-volume at the specified compaction effort.

Once the aggregate structure is selected, the amount of binder or binder content is selected based on the amount which will produce optimum air void, VMA, VFA, etc.

The last step in the volumetric mix design process is to evaluate the moisture sensitivity of the design mixture. One batch of laboratory-compacted specimens is simulated for moisture-damage by subjecting to partial vacuum saturation followed by an optional freeze cycle, followed by a 24-hour heating cycle at 60 °C. Then, the tensile strength ratio of moisture-damaged samples and fresh samples is calculated. The minimum criteria for tensile strength ratio is 0.8 or 80 % but depends on agency to agency.

The Superpave mix design criteria is listed in Transportation Subsection of Civil Engineering Section in the NCEES Ref. Handbook. Considering the asphalt mix design is a long process, only few short problems from asphalt are mentioned below:

Problem 8.26

Which of the following materials typically undergoes fatigue loading?

A. Steel B. Concrete C. Asphalt D. Wood

Solution: C

Asphalt typically undergoes fatigue (repeated) loading. Consider traffic one after another.

Problem 8.27

Which of the following materials is a visco-elastic material?

A. Asphalt
B. Wood
C. Sand
D. Steel

Solution: A

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Asphalt is largely a visco-elastic material. If you sustain a load on it, it will continue having deflection until you remove the load. Therefore, in the asphalt parking lot, the deformation is higher just below the wheels.

Problem 8.28

Which of the following is a factor in asphalt mix design currently in practice?

A. Aggregate amount
B. Climate
C. Predicted traffic volume
D. All of the above

Solution: D

Currently, Superpave mix design is being used for asphalt road. This method uses all the factors listed here. The details can be found in the Transportation Engineering section.

"You must present a current government-issued ID upon arrival at the test center. Acceptable forms of ID must be government issued and must include:

- *a valid expiration date,*
- *your name and date of birth,*
- *a recognizable photo, and*
- *your signature.*
- *Your name must be in English. Valid U.S. military IDs that do not include a signature will be accepted. Student IDs will not be accepted.*
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9 FLUID MECHANICS

Fundamentals of Engineering (FE) CIVIL Exam Specification mentions, 4–6 questions may appear in the exam based on Fluid Mechanics as follows:

A. Flow measurement
B. Fluid properties
C. Fluid statics
D. Energy, impulse, and momentum equations

Basic Definitions

Density (ρ) of a fluid is defined as the mass of unit volume of the fluid. It is expressed in kg/m^3 or lb/ft^3, and can be written as:

$$\rho = \frac{m}{V}$$

$$\rho = \lim_{\Delta v \to 0} \frac{\Delta m}{\Delta V}$$

Specific Weight (γ) of a fluid is defined as the weight of unit volume of fluid. It is expressed in kN/m^3 or lb/ft^3, and can be written as:

$$\gamma = \frac{W}{V}$$

$$\gamma = \lim_{\Delta v \to 0} \frac{\Delta W}{\Delta V}$$

Specific Gravity (G) is the ratio of unit weight of a material and that of water. It has no unit and can be expressed as:

$$G = \frac{\gamma}{\gamma_w} = \frac{\rho}{\rho_w}$$

ρ = density (also called mass density)
Δm = mass of infinitesimal volume of the material considered
ΔV = volume of infinitesimal material considered
γ = specific weight = ρg
ΔW = weight of an infinitesimal volume

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G = specific gravity
ρ_w = density of water at standard conditions = 1,000 kg/m³ or 62.4 pcf
γ_w = specific weight of water standard conditions = 9,810 N/m³ or 62.4 pcf

Stress is defined as follows:

$$\tau(l) = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A}$$

$\tau(l)$ = surface stress vector at point, l
ΔF = force acting on infinitesimal area, ΔA
ΔA = infinitesimal area at point, l

Normal and tangential stresses are defined as:

$$\tau_n = -P$$

$$\tau_t = \mu \frac{dv}{dy}$$

τ_n and τ_t = normal and tangential stress components at point l, respectively
P = pressure at point l
μ = absolute dynamic viscosity of the fluid, N.s/m² [lb/ft.s]
dv = differential velocity
dy = differential distance, normal to boundary
v = velocity at boundary condition
y = normal distance, measured from boundary
v = kinematic viscosity = μ/ρ, m²/s [ft²/s]

For a thin Newtonian fluid film and a linear velocity profile, $v(y) = \frac{vy}{\delta}$ and $\frac{dv}{dy} = \frac{v}{\delta}$

where: v = velocity of plate on film, and δ = thickness of fluid film

For a power law (non-Newtonian) fluid, $\tau_t = K\left(\frac{dv}{dt}\right)^n$

K = consistency index
n = power law index
$n < 1$ = pseudo plastic
$n > 1$ = dilatant

Problem 9.1

A small flat plate of surface area of 10^{-3} m² is separated from a large plate by a fluid of 0.1-cm thickness. A force of 1.5×10^{-5} N is required to slide the small plate at a velocity of 10^{-2} m/s with respect to the large plate. The viscosity (Ns/m²) of the fluid is most nearly:

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A. 5.1×10^{-3}
B. 1.5×10^{-3}
C. 5.1×10^{-2}
D. 5.1×10^{-4}

Solution: B

Shear stress, $\tau = \mu \frac{dv}{dy}$

Shear stress, $\tau = \frac{F}{A}$

Combining, $F = A\mu \frac{dv}{dy}$

$$\mu = \frac{F}{A}\frac{dy}{dv} = \frac{1.5(10^{-5})\,\text{N}\,(10^{-3}\,\text{m})}{10^{-3}\,\text{m}^2\,(10^{-2}\,\text{ms}^{-1})} = 1.5x10^{-3}\ \text{N.s.m}^{-2}$$

Problem 9.2

A small flat plate of surface area of 10×10^{-3} m² is placed on a fluid of 2×10^{-3}-m thickness. Most nearly, how much horizontal force is required to slide the plate horizontally at a velocity of 0.05 m/s if the viscosity of the fluid is 1.55 Ns/m²?

A. 0.039 N
B. 0.39 N
C. 3.9 N
D. 93 N

Solution: B

Shear stress, $\tau = \mu \frac{dv}{dy}$

Shear stress, $\tau = \frac{F}{A}$

$$F = A\mu \frac{dv}{dy} = \frac{10(10^{-3})(1.55)(0.05)}{2(10^{-3})} = 0.39\ \text{N}$$

Problem 9.3

The viscosity of water is 500×10^{-6} N.s/m². The kinematic viscosity of the water is most nearly:

A. 500×10^{-6} m²/s B. 50×10^{-6} m²/s C. 5×10^{-6} m²/s D. 0.5×10^{-6} m²/s

Solution: D

$$\text{Kinematic viscosity, } \upsilon = \frac{\mu}{\rho} = \frac{500x10^{-6}\ \frac{\text{N.s}}{\text{m}^2}}{1{,}000\ \frac{\text{kg}}{\text{m}^3}} = 0.5x10^{-6}\ \frac{\text{m}^2}{\text{s}}$$

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Problem 9.4

What is the value of 'n' in the equation, $\tau_t = K\left(\frac{dv}{dt}\right)^n$, for Non-Newtonian pseudo plastic fluid?

A. $n = 0$ B. $n = 1$ C. $n < 1$ D. $n > 1$

Solution: C

From the NCEES FE Reference Handbook, Fluids Mechanics, for Non-Newtonian pseudo plastic fluid, $n < 1$.

Surface Tension and Capillarity

Surface tension (σ) is the tendency of a fluid surface to acquire the least surface area possible. It is expressed as the force per unit contact length:

$$\sigma = \frac{F}{L}$$

σ = surface tension, force/length (most of the text books use T instead of σ)
F = surface force at the interface
L = length of interface

Capillary action is the ability of a liquid to flow in narrow spaces without the assistance of gravity. The capillary rise, h is approximated as:

$$h = \frac{4\sigma \cos\beta}{\gamma d}$$

h = height of the liquid in the vertical tube
σ = surface tension
β = angle made by the liquid with the wetted tube wall ($\cos\beta \approx 1.0$)
γ = specific weight of the liquid
d = the diameter of the capillary tube

Problem 9.5

To raise a horizontal wire of length 0.05 m from the surface of water, a force of 7.28×10^{-3} N along with the weight of the wire is required. The surface tension of water is most nearly:

A. 7.28×10^{-3} N/m
B. 1.45×10^{-1} N/m
C. 7.28×10^{-2} N/m
D. 7.28×10^{-3} N

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Solution: C

$L = 2 \times 0.05$ m [2 times, as there is water on both sides of the wire]

Surface Tension, $T = \frac{F}{L} = \frac{7.28(10^{-3})\text{N}}{2(0.05)\text{ m}} = 7.28(10^{-2})\text{ N/m}$

Problem 9.6

To raise a horizontal wire of length 4 cm from the surface of water, most nearly what amount of force along with the weight of the wire is required? The surface tension of water is 72×10^{-3} N/m.

A. 2.88×10^{-2} N
B. 2.88×10^{-3} N
C. 5.76×10^{-3} N
D. 7.28×10^{-3} N

Solution: C

$L = 2 \times 0.04$ m [as there is water on both sides of the wire]
Force, $F = TL = 72 \times 10^{-3}$ N/m $(2 \times 0.04$ m$) = 5.76 \times 10^{-3}$ N

Problem 9.7

The diameter of a capillary tube is 0.2 mm. It is dipped in water of surface tension of 72×10^{-3} N/m and density of 10^3 kg/m^3. The height of water in the tube is most nearly:

A. 0.29 mm
B. 0.51 inch
C. 0.15 m
D. 1.2 m

Solution: C

Capillary rise, $h = \frac{4T}{\gamma d} = \frac{4(72x10^{-3}\text{ N/m})}{(9.81x10^{3}\text{ N/m}^3)(0.2x10^{-3}\text{ m})} = 0.15$ m .Note: $\cos\beta$ is very often neglected as it is close to one.

Characteristics of a Static Liquid

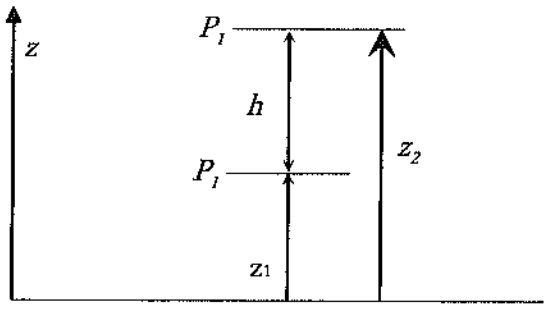
The Pressure Field in a Static Liquid

The difference in a pressure between two different points is-

$$P_2 - P_1 = -\gamma(z_2 - z_1) = -\gamma h = -\rho g h$$

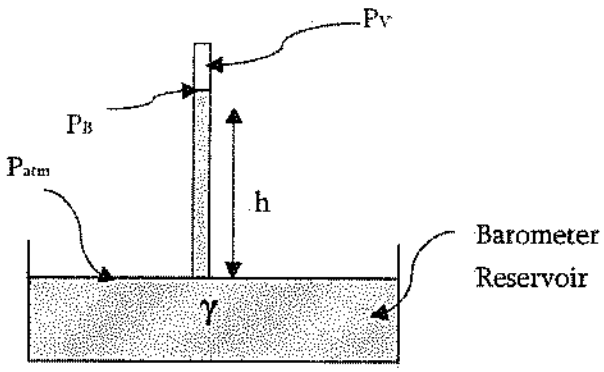
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Absolute pressure = atmospheric pressure + gage pressure reading
Absolute pressure = atmospheric pressure – vacuum gage pressure reading



Manometers

The equation to determine the pressure using manometer is discussed in the NCEES FE Ref. Handbook, Fluid Mechanics. Review it from the Handbook. Another device that works on the same principle as the manometer is the simple barometer.

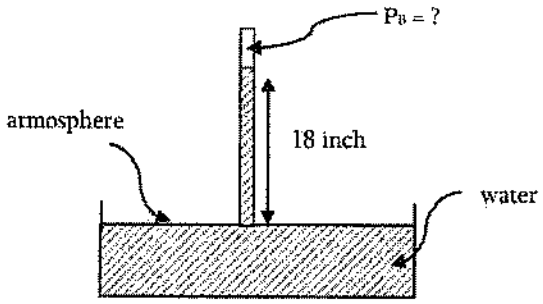


$$P_{atm} = P_A = P_v + \gamma h = P_B + \gamma h = P_v + \rho g h$$

P_v = vapor pressure of the barometer fluid

Problem 9.8

For the following barometer, what is the pressure, P_B? Consider the atmospheric pressure to be 101 kPa.



A. 110 kPa B. 101 kPa C. 91 kPa D. 96.5 kPa

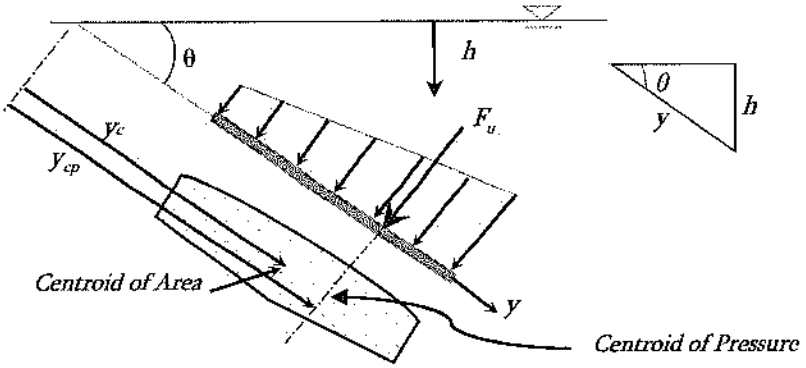
Solution: D

$$P_{atm} = P_B + \rho g h$$
$$P_B = P_{atm} - \rho g h = 101 \text{ kPa} - 9.81 \text{ kN/m}^3 \text{ (18 in)(1/39.36 m/in)} = 96.5 \text{ kPa}$$

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Forces on Submerged Surfaces and the Center of Pressure

The equation to determine the pressure/force in a submerged object is discussed in the NCEES FE Ref. Handbook. It is further discussed here. Let us consider an object that is submerged at an angle of θ with respect to the water surface.



$$y_c = \frac{h_c}{\sin\theta}$$

$$y_{cp} = y_c + \frac{I_{xc}}{y_c A}$$

Fig. Forces on submerged surfaces

The pressure on a point at a vertical distance h below the surface is: $P = P_{atm} + \rho g h$ for $h \geq 0$

P = pressure
P_{atm} = atmospheric pressure
P_c = pressure at the centroid of area
P_{cp} = pressure at center of pressure
y_c = slant distance from liquid surface to the centroid of area
h_c = vertical distance from liquid surface to the centroid of area
y_{cp} = slant distance from liquid surface to center of pressure
h_{cp} = vertical distance from liquid surface to center of pressure
θ = angle between liquid surface and edge of submerged surface
I_{xc} = moment of inertia about the centroidal x-axis

If atmospheric pressure acts above the liquid surface and on the non-wetted side of the submerged surface, then:

$$y_{cp} = y_c + \frac{I_{xc}}{y_c A}$$

$$y_{cp} = y_c + \frac{\rho g \sin\theta I_{xc}}{P_c A}$$

On the wetted side, $F_R = (P_{atm} + \rho g y_c \sin\theta)A$

P_{atm} acting both sides, $F_{R_{net}} = (\rho g y_c \sin\theta)A$

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Archimedes Principle and Buoyancy

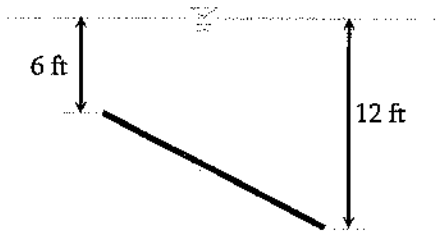
Archimedes Principle can be stated as:

1) The buoyant force exerted on a submerged or floating body is equal to the weight of the fluid displaced by the body.
2) A floating body displaces a weight of fluid equal to its own weight; i.e., a floating body is in equilibrium.

The center of buoyancy is located at the centroid of the displaced fluid volume. In the case of a body lying at the interface of two immiscible fluids, the buoyant force equals the sum of the weights of the fluids displaced by the body.

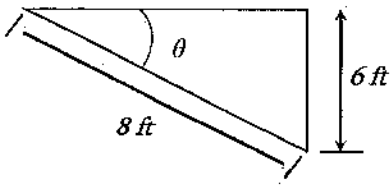
Problem 9.9

What is the slant distance from the liquid surface to the center of pressure for the following submerged plate? The plate dimension is 8 ft x 8 ft.



A. 8.44 ft B. 10.44 ft C. 12.44 ft D. 14.44 ft

Solution: C



$\text{Sin}\theta = 6/8$

$\theta = 48.6°$

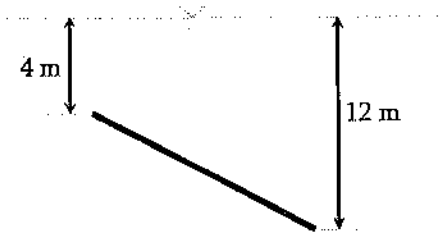
$h_c = 6\text{ ft} + 4\text{ ft}\,(\sin 48.6) = 9\text{ ft}$; $y_c = \dfrac{h_c}{\sin\theta} = 9\text{ ft} / \sin 48.6 = 12\text{ ft}$

$$y_{cp} = y_c + \frac{I_{xc}}{y_c A} = 12 + \frac{8(8)^3/12}{12(8x8)} = 12.44\text{ ft}$$

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Problem 9.10

What is the average vertical pressure on the following submerged plate? The plate dimension is 18 m x 18 m.



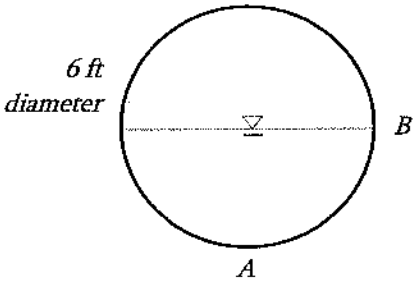
A. 500 psf B. 78.5 psf C. 78.5 kPa D. Cannot be determined

Solution: C

$$\text{Average pressure, } \bar{p} = \frac{p_1 + p_2}{2} = \frac{1}{2}(\gamma h_1 + \gamma h_2) = \frac{1}{2}(9.81\ \frac{\text{kN}}{\text{m}^2})(4+12)\,\text{m} = 78.5\ \text{kPa}$$

Problem 9.11

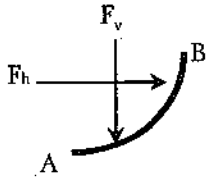
A 6-ft-diameter drainage conduit is half full of water. The magnitude of the resultant force (lb per ft of conduit) that the water exerts on the curved section AB of the wall is most nearly:



A. 281 B. 441 C. 523 D. 576

Solution: C

$$F_h = \frac{1}{2}\gamma h^2 = \frac{1}{2}(62.4\,\text{pcf})(3\,\text{ft})^2 = 281\ \text{ lbs/ft of conduit}$$



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$$F_v = \gamma(\text{volume of water}) = 62.4 \text{ pcf}\left(\frac{1}{4}\pi r^2\right) = 62.4\left(\frac{9\pi}{4}\right) = 441 \text{ lbs per ft of conduit}$$

$$F_R = \sqrt{(F_h)^2 + (F_v)^2} = \sqrt{281^2 + 441^2} = 523 \text{ lbs per ft of conduit}$$

Problem 9.12

Which of the following statements is not true?

A. The buoyant force exerted on a submerged or floating body is equal to the weight of the fluid displaced by the body
B. A floating body displaces a weight of fluid equal to its own weight; i.e., a floating body is in equilibrium.
C. The center of buoyancy is located at the centroid of the displaced fluid volume.
D. None of the above

Solution: D

Principles of One-Dimensional Fluid Flow

The Continuity Equation

If the flow (Q) is continuous, the continuity equation (as applied to one-dimensional flow) states that the flow passing two points in a stream is equal at each point. This can be written as follows:

$$Q = A_1 v_1 = A_2 v_2$$
$$\dot{m} = \rho Q = \rho A v$$

Q = volumetric flow rate
$\dot{m}$ = mass flow rate
A = cross-sectional area of flow
v = average flow velocity
ρ = the fluid density

For steady, one-dimensional flow, $\dot{m}$ is a constant. If, in addition, the density is constant, then Q is constant.

The Energy Equation

The energy equation for steady incompressible flow with no shaft device is –

$$\frac{P_1}{\gamma} + z_1 + \frac{v_1^2}{2g} = \frac{P_2}{\gamma} + z_2 + \frac{v_2^2}{2g} + h_f$$

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$$\frac{P_1}{\rho g} + z_1 + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + z_2 + \frac{v_2^2}{2g} + h_f$$

h_f = the head loss due to friction and all remaining terms are defined earlier. If the cross-sectional area and the elevation of the pipe are the same at both sections (1 and 2), then $z_1 = z_2$ and $v_1 = v_2$. The pressure drop, $P_1 - P_2$ is given by the following:

$$P_1 - P_2 = \gamma h_f = \rho g h_f$$

The Field Equation

The field equation is derived when the energy equation is applied to one-dimensional flows. Assuming no friction losses, and no pump or turbine exists between sections 1 and 2 in a system, the field equation can be written as:

$$\frac{P_1}{\gamma} + z_1 + \frac{v_1^2}{2g} = \frac{P_2}{\gamma} + z_2 + \frac{v_2^2}{2g}$$

$$\frac{P_1}{\rho} + z_1 g + \frac{v_1^2}{2} = \frac{P_2}{\rho} + z_2 g + \frac{v_2^2}{2}$$

P_1, P_2 = pressures at sections 1 and 2
v_1, v_2 = average velocities of the fluid at the sections
z_1, z_2 = the vertical distances from a datum to the sections (the potential energy)
γ = the specific weight of the fluid (ρg)
g = the acceleration of gravity
ρ = fluid density

Hydraulic Gradient (Grade Line)

Hydraulic grade line is the line connecting the sum of pressure and elevation heads at different points in conveyance systems. If a row of piezometers were placed at intervals along the pipe, the grade line would join the water levels in the piezometer water columns.

Energy Line (Bernoulli Equation)

The Bernoulli equation states that the sum of pressure velocity, and elevation head is constant. The energy line is this sum, or the total head line above a horizontal datum. The difference between the hydraulic grade line and the energy line is the $\frac{v^2}{2g}$ term.

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Problem 9.13

An incompressible fluid is flowing through a 4-inch diameter pipe with a velocity of 20 m/s. If the pipe is connected to a 3-inch diameter pipe, the increase in velocity (m/s) is most nearly:

A. 35.6 B. 15.6 C. 7.5 D. None of the above

Solution: B

$$v_1 = 20\ \frac{m}{s}$$

$$A_1 = \frac{\pi D_1^{\,2}}{4} = \frac{\pi(4)^2}{4} = 4\pi$$

$$A_2 = \frac{\pi D_2^{\,2}}{4} = \frac{\pi(3)^2}{4} = 2.25\pi$$

Continuity Equation: $A_1 v_1 = A_2 v_2$

$$v_2 = \frac{A_1 v_1}{A_2} = \frac{(4\pi)(20)}{(2.25\pi)} = 35.6\ \frac{\text{m}}{\text{s}}$$

Increase in velocity = 35.6 - 20 = 15.6 m/s

Problem 9.14

Which of the following statements is true?

A. Bernoulli equation states, the sum of pressure velocity and elevation head is constant.
B. The energy line is this sum, or the total head line above a horizontal datum.
C. The difference between hydraulic grade line and energy line is the $v^2/2g$ term.
D. All of the above

Solution: D

From the NCEES FE Ref. Handbook, Fluid Mechanics, all the three statements are true.

Reynolds Number

Reynolds number is used to characterize fluid flow, and is defined as the ratio of inertial forces to viscous forces. Mathematically, it can be shown as follows:

$$\text{Re} = \frac{vD\rho}{\mu} = \frac{vD}{\upsilon}$$

ρ = the mass density
D = the diameter of the pipe, dimension of the fluid streamline, or characteristic length
μ = the dynamic viscosity
ν = the kinematic viscosity
Re = the Reynolds number

The critical Reynolds number, $(Re)_c$ defined to be the minimum Reynolds number as a flow will turn turbulent.

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Flow through a pipe is generally characterized as:

a) laminar for Re < 2,100
b) fully turbulent for Re > 10,000
c) transitional flow for 2,100 < Re < 10,000

The velocity distribution for laminar flow in circular tubes or between planes is –

$$v(r) = v_{max}\left[1-\left(\frac{r}{R}\right)^2\right]$$

r = the distance (m) from the centerline
R = the radius (m) of the tube or half the distance between the parallel planes
v = the local velocity (m/s) at r
v_{max} = the velocity (m/s) at the centerline of the duct
$v_{max} = 1.18\bar{v}$, for fully turbulent flow
$v_{max} = 2\bar{v}$, for circular tubes in laminar flow and
$v_{max} = 1.5\bar{v}$, for parallel planes in laminar flow

where: $\bar{v}$ the average velocity (m/s) in the duct

The shear stress distribution can be presented as:

$$\frac{\tau}{\tau_w} = \frac{r}{R}$$

where: τ and τ_w are the shear stresses at radii r and R respectively.

Problem 9.15

The kinetic viscosity of a fluid is $5x10^{-5}$ m²/s in a 0.1-m diameter pipe. If the velocity of the flow is 1 m/s, the type of flow is most nearly:

A. Laminar flow
B. Fully turbulent flow
C. Transitional flow
D. Cannot be determined

Solution: A

$$\text{Re} = \frac{vD}{\upsilon} = \frac{1\ \frac{\text{m}}{\text{s}}(0.1\ \text{m})}{5x10^{-5}\ \frac{\text{m}^2}{\text{s}}} = 2{,}000$$

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Flow through a pipe is generally characterized as laminar for $Re < 2{,}100$ and fully turbulent for $Re > 10{,}000$, and transitional flow for $2{,}100 < Re < 10{,}000$. So, this flow is laminar.

Problem 9.16

A 2-ft diameter circular tube has a laminar flow with the maximum velocity of 5 m/s. The velocity (m/s) of the flow close to the periphery of the tube is most nearly:

A. 2 B. 5 C. 0 D. 3.49

Solution: C

Radius, $R = 1$ ft
Point of concern, $r \approx 1$ ft
The velocity distribution for laminar flow in circular tubes or between planes is

$$v(r) = v_{max}\left[1-\left(\frac{r}{R}\right)^2\right] = 5\ \frac{\text{m}}{\text{s}}\left[1-\left(\frac{1\,\text{ft}}{1\,\text{ft}}\right)^2\right] = 0\ \text{m/s}$$

Problem 9.17

A 2-ft diameter circular tube has a laminar flow with the maximum velocity of 5 m/s. The velocity (m/s) of the flow at the centerline of the tube is most nearly:

A. 2 B. 5 C. 0 D. 3.49

Solution: B

Radius, $R = 1$ ft
Point of concern, $r = 0$ ft
The velocity distribution for laminar flow in circular tubes or between planes is

$$v(r) = v_{max}\left[1-\left(\frac{r}{R}\right)^2\right] = 5\ \frac{\text{m}}{\text{s}}\left[1-\left(\frac{0\,\text{ft}}{1\,\text{ft}}\right)^2\right] = 5\ \text{m/s}$$

Problem 9.18

The average velocity of a fully turbulent flow is 5 m/s. The maximum velocity (m/s) of the flow is most nearly:

A. 10 B. 7.5 C. 5.9 D. 5.0

Solution: C

$$v_{max} = 1.18\bar{v} = 1.18(5\,\text{m/s}) = 5.9\ \text{m/s}$$

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Problem 9.19

The shear stress in a flow in a circular tube at the 12-inch from the centerline is 10 Pa. The shear stress (Pa) at 24-inch from the centerline of the tube is most nearly:

A. 10 B. 20 C. 5 D. 0

Solution: B

From the NCEES FE Ref. Handbook, Fluid Mechanics, $\frac{\tau}{\tau_w} = \frac{r}{R}$

$$\Rightarrow \quad \tau_w = \frac{R}{r}\tau = \frac{24}{12}(10 \text{ Pa}) = 20 \text{ Pa}$$

Head Loss Due to Flow

Frictional Loss

One of the laws of friction is that there is resistance opposite to the movement of a body. Therefore, when fluid moves in a conduit, the frictional force causes decrease in pressure or velocity. The decrease in pressure is termed as head loss. The larger the roughness inside the conduit, the larger the head loss. Darcy–Weisbach equation relates the head loss, or pressure loss, due to friction along a given length of pipe to the average velocity of the fluid flow. It is presented below:

$$h_f = f\frac{L}{D}\frac{v^2}{2g}$$

f = the Moody, Darcy, or Stanton friction factor
D = diameter of the pipe
L = length of the flow for which the head is calculated

A chart that gives f versus Re for various values of roughness, known as a Moody, Darcy, or Stanton diagram, is available in the Fluid Mechanics section of the NCEES FE Ref. Handbook. It is required to find out the friction factor for different roughness conditions for different flow conditions.

Minor Losses in Pipe Fittings, Contractions, and Expansions

Head losses also occur as the fluid flows through pipe fittings (i.e., elbows, valves, couplings, etc.) and sudden pipe contractions and expansions. It can be determined using the following equation:

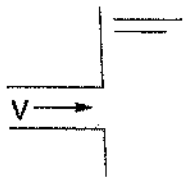
$$h_{f,\,fitting} = C\frac{v^2}{2g}$$

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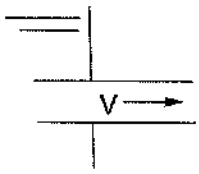
Specific fittings have characteristic values of C, which is expected to be provided in the problem statement. A generally accepted nominal value for head loss in well-streamlined gradual contractions is as follows:

$$h_{f,fitting} = 0.04\frac{v^2}{2g}$$

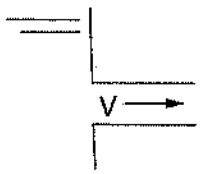
The head loss at either an entrance or exit of a pipe from or to a reservoir is also given by the $h_{f,fitting}$ equation. Values for C for various cases are shown below:



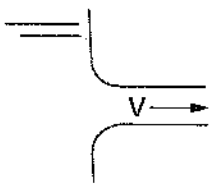
Sharp Exit: C = 1.0



Protruding Entrance: C = 0.8



Sharp Entrance: C = 0.5



Round Entrance: C = 0.1

Pressure Drop for Laminar Flow

The equation for Q in terms of the pressure drop ΔP_f is the Hagen-Poiseuille equation. This relation is valid only for flow in the laminar region. The equation is given below:

$$Q = \frac{\pi R^4 \Delta P_f}{8\mu L} = \frac{\pi D^4 \Delta P_f}{128\mu L}$$

Drag Force

The drag force, F_D on objects immersed in a large body of flowing fluid or objects moving through a stagnant fluid is presented by:

$$F_D = \frac{C_D \rho v^2 A}{2}$$

C_D = the drag coefficient. It can be found in Fluid Mechanics Section of the NCEES FE Ref. Handbook.
v = the velocity of the flowing fluid or moving object
A = the projected area of blunt objects
ρ = fluid density

For flat plates placed parallel with the flow:

$$C_D = \frac{1.33}{(\mathrm{Re})^{0.5}} \quad \left(10^4 < \mathrm{Re} < 5\mathrm{x}10^5\right)$$

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$$C_D = \frac{0.031}{(\mathrm{Re})^{1/7}} \quad (10^6 < \mathrm{Re} < 10^9)$$

Flow in Noncircular Conduits

Analysis of flow in conduits having a noncircular cross section uses the hydraulic radius, R_H or the hydraulic diameter, D_H as follows:

$$R_H = \frac{\text{Cross-sectional area}}{\text{Wetted perimeter}} = \frac{D_H}{4}$$

Problem 9.20

The hydraulic diameter of a noncircular conduit is 40 inch. The hydraulic radius of that conduit if it has full flow is most nearly:

A. 80 in B. 20 in C. 10 in D. 5 in

Solution: C

$$R_H = \frac{D_H}{4} = \frac{40}{4} = 10 \text{ in}$$

Problem 9.21

A fluid is flowing at 1.5 m/s in a 0.02-m diameter conduit. The length of the conduit is 50 m. If the Moody factor for the conduit is 0.02. The head loss of the flow is most nearly:

A. 1.37 m B. 5.73 m C. 2.37 m D. 6.98 m

Solution: B

$$h_f = f\frac{L}{D}\frac{v^2}{2g} = 0.02\frac{50 \text{ m}}{0.02 \text{ m}}\frac{(1.5 \text{ m/s})^2}{2(9.81)} = 5.73 \text{ m}$$

Problem 9.22

The Reynolds Number of a flow through a pipe is 200,000. The relative roughness of the pipe is 0.008. The friction factor of the pipe flow is most nearly:

A. 0.016 B. 0.026 C. 0.036 D. 0.082

Solution: C

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From the NCEES FE Ref. Handbook, Fluid Mechanics,
For Reynolds Number = 200,000 and Relative roughness = 0.008, f= 0.036

Problem 9.23

The Reynolds Number of a flow through a pipe is 2,000. The relative roughness of the pipe is 0.008. The friction factor of the pipe flow is most nearly:

A. 0.016 B. 0.026 C. 0.032 D. 0.082

Solution: C

From the NCEES FE Ref. Handbook, Fluid Mechanics,
Re < 2,100; So, it is a laminar flow.
For laminar flow, f= 64/Re = 64/2,000 = 0.032

Problem 9.24

The Reynolds Number of a flow is 2. The drag coefficient of a spherical object inside the flow is most nearly:

A. 12 B. 6 C. 8 D. 10

Solution: A

From the NCEES FE Ref. Handbook, Fluid Mechanics, for Reynolds Number < 10,
$C_D = 24/Re$
So, $C_D = 24/Re = 24/2 = 12$

Problem 9.25

A spherical object is placed inside a water flow with Re = 8 and velocity of 1 m/s. The projected area of the spherical object perpendicular to the flow is 0.01 m². The drag force exerted on the spherical body is most nearly:

A. 5 N B. 10 N C. 15 N D. 20 N

Solution: C

From the NCEES FE Ref. Handbook, Fluid Mechanics, for Re < 10, $C_D = 24/Re$
So, $C_D = 24/8 = 24/8 = 3$

From the NCEES FE Ref. Handbook (Fluid Mechanics), the drag force, F_D on objects immersed in a large body of flowing fluid or object moving through a stagnant fluid is

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$$F_D = \frac{C_D \rho v^2 A}{2} = \frac{3(1{,}000\ \frac{\text{kg}}{\text{m}^3})(1\ \frac{\text{m}}{\text{s}})^2 (0.01\ \text{m}^2)}{2} = 15\ \text{N}$$

Problem 9.26

What is most nearly the drag coefficient for flat plate placed parallel to the flow with Re = 100,000?

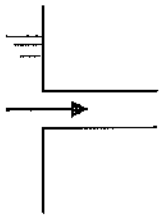
A. 0.0042 B. 0.042 C. 0.42 D. 4.2

Solution: A

$$C_D = \frac{1.33}{(\text{Re})^{0.5}} = \frac{1.33}{(100{,}000)^{0.5}} = 0.0042$$

Problem 9.27

The head loss in the following sharp entrance fitting if the velocity of the flow is 1 m/s is most nearly:



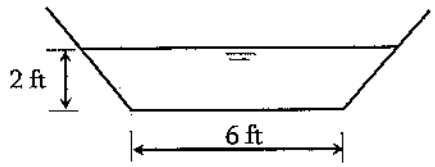
A. 0.02 m B. 0.026 m C. 0.03 m D. 0.035 m

Solution: B

From the FE Ref. Handbook, Fluid Mechanics, $h_{f,fitting} = C\frac{v^2}{2g} = 0.5\frac{(1\ \text{m/s})^2}{2(9.81)} = 0.0255\ \text{m}$

Problem 9.28

The hydraulic radius (ft) of the following trapezoidal channel with 1:1 side slopes is most nearly:



A. 2.12

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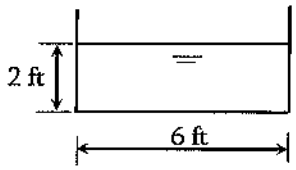
B. 1.37
C. 1.92
D. 3.22

Solution: B

$$R_H = \frac{\text{Wetted Area}}{\text{Wetted Perimeter}} = \frac{A}{P} = \frac{(6+2)(2\text{ ft})}{6+2\sqrt{2^2+2^2}} = \frac{16}{11.66} = 1.37\text{ ft}$$

Problem 9.29

The hydraulic radius (ft) of the following rectangular channel is most nearly:



A. 1.5 B. 1.2 C. 1.3 D. 1.4

Solution: B

$$R_H = \frac{\text{Wetted Area}}{\text{Wetted Perimeter}} = \frac{A}{P} = \frac{(6)(2\text{ ft})}{6+2+2} = \frac{12}{10} = 1.2\text{ ft}$$

Characteristics of Selected Flow Configurations

Open-Channel Flow and/or Pipe Flow of Water

Manning's Equation: In open-channel flow (gravity driven), Manning's equation is used to determine the velocity or flow rate through a channel whose longitudinal slope and hydraulic radius are known. The equation is shown below:

$$v = \frac{K}{n} R_H^{2/3} S^{1/2}$$

v = velocity
K = 1.0 for SI units, 1.486 for USCS units
n = roughness coefficient
R_H = hydraulic radius
S = slope of energy grade line

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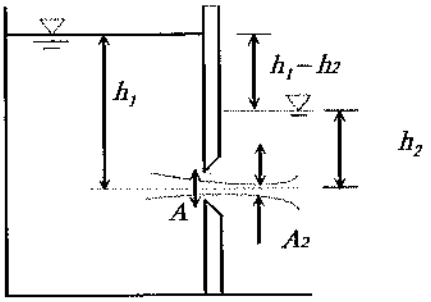
Hazen-Williams Equation: Hazen-Williams Equation is used to determine the velocity or flow rate through a channel whose longitudinal slope and hydraulic radius are known. The equation is shown below:

$$v = k_1 C R_H^{0.63} S^{0.54}$$

k_1 = 0.849 for SI units, 1.318 for USCS units
C = roughness coefficient, as tabulated in the Civil Engineering section (Hydrology/Water Resources of the NCEES FE Ref. Handbook) Handbook. Other symbols are defined earlier.

Submerged Orifice operating under steady-flow conditions:

An orifice is a device for measuring flow rate. It uses the principle of Bernoulli's Equation which states that there is a relationship between the pressure of the fluid and the velocity of the fluid. For the submerged condition, the flow through an orifice can presented as follows:



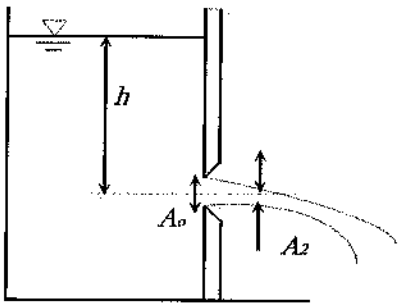
$$Q = A_2 v_2 = C_c C_v A \sqrt{2g(h_1 - h_2)} = CA\sqrt{2g(h_1 - h_2)}$$

In which the product of Cc and Cv is defined as the coefficient of discharge of the orifice (C).
v_2 = velocity of fluid exiting orifice.

Orifice Discharging Freely into Atmosphere:

If the orifice discharges freely into atmosphere, the flow through an orifice can presented as follows:

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$$Q = CA_o\sqrt{2gh}$$

h = water head measured from the liquid surface to the centroid of the opening
Q = volumetric flow
A_o = cross-sectional area of flow
g = acceleration due to gravity
h = height of fluid above orifice

Problem 9.30

Water is flowing in a 4-foot wide open channel of rectangular cross section with a flow depth of 2 feet. The channel has a constant bottom slope of 0.3%. If the Manning's coefficient is 0.012, the flow rate (cfs) of water in that channel is most nearly:

A. 40.2
B. 54.3
C. 37.2
D. 49.7

Solution: B

$R_H = A/P = (4x2)/(2+4+2) = 1$
$n = 0.012$
From the NCEES FE Ref. Handbook (Civil Engineering – Hydrology/Water),

$$Q = \frac{1.486}{n}AR_H^{2/3}S^{1/2} = \frac{1.486}{0.012}(4x2)\left(1^{2/3}\right)\left(0.003^{1/2}\right) = 54.3\ \text{ft}^3/\text{s}$$

Problem 9.31

Water is flowing in a 4-foot wide open channel of rectangular cross section with a flow depth of 2 feet. The channel has a constant bottom slope of 0.3%. If the Manning's coefficient is 0.012, density of water is 1.94 slugs/ft^3, and viscosity of water is 2.73x10^{-5} lb.s/ft^2, the Reynolds number of the flow is most nearly:

A. 480

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B. 4,800
C. 480,000
D. 1,930,000

Solution: D

From the NCEES FE Ref. Handbook (Fluid Mechanics), $\mathrm{Re} = \frac{vD\rho}{\mu}$

D can be replaced by $4R_H$

$R_H = A/P = (4x2)/(2+4+2) = 1$

$n = 0.012$

From the NCEES FE Ref. Handbook, (Civil Engineering – Hydrology/Water),

$$v = \frac{1.486}{n} R_H^{2/3} S^{1/2} = \frac{1.486}{0.012}\left(1^{2/3}\right)\left(0.003^{1/2}\right) = 6.8 \text{ ft/s}$$

$$\mathrm{Re} = \frac{vD\rho}{\mu} = \frac{(6.8\ \frac{\text{ft}}{\text{s}})(4x1\ \text{ft})(1.94\ \frac{\text{slug}}{\text{ft}^3})}{2.73(10^{-5})\ \text{lb.s/ft}^2} = 1,927,956$$

Problem 9.32

The flow rate (cfs) in a 36-inch diameter storm sewer with longitudinal slope of 0.002 and n of 0.012, flowing full under gravity is most nearly:

A. 12.3
B. 22.3
C. 32.3
D. 42.3

Solution: C

For flowing full, $R_H = D/4 = 36/4 = 9$ in $= 0.75$ ft

From the NCEES FE Ref. Handbook (Fluid Mechanics, Civil Engineering – Hydrology/Water),

$$Q = \frac{1.486}{n} A R_H^{2/3} S^{1/2} = \frac{1.486}{0.012}\left(\frac{\pi(3)^2}{4}\right)\left((0.75)^{2/3}\right)\left(0.002^{1/2}\right) = 32.3 \text{ ft}^3/\text{s}$$

Problem 9.33

What is most nearly the flow rate in a 36-inch diameter new cast iron storm sewer with slope = 0.002, when it is flowing full under gravity? Use Hazen-Williams Equation.

A. 15 cfs
B. 25 cfs

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C. 35 cfs
D. 45 cfs

Solution: C

For flowing full, $R_H = D/4 = 36/4 = 9$ in $= 0.75$ ft
From the NCEES Ref. Handbook, (Fluid Mechanics, Civil Engineering–Hydrology/Water),
Hazen-Williams Coefficient *C for* new cast iron pipe = 130

$$Q = 1.318CAR_H^{0.63}S^{0.54} = 1.318(130)\left(\frac{\pi(3)^2}{4}\right)\left((0.75)^{0.63}\right)\left(0.002^{0.54}\right) = 35.2\,\text{ft}^3/\text{s}$$

The Impulse-Momentum Principle

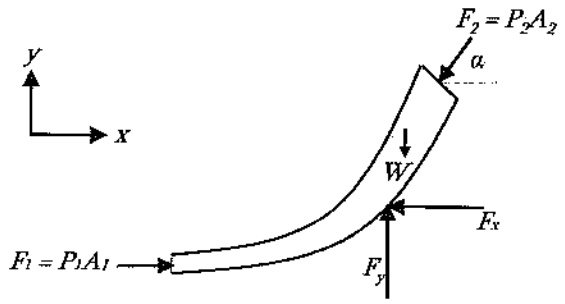
The Impulse-Momentum Principle can be used to determine the impulse/force exerted by fluid flow on to the conduit. The resultants force in a given direction acting on the fluid equals the rate of change of momentum of the fluid. It can be expressed as follows:

$$\sum F = \sum Q_2\rho_2 v_2 - \sum Q_1\rho_1 v_1$$

where

$\sum F$ = the resultant of all external forces acting on the control volume
$\sum Q_1\rho_1 v_1$ = the rate of momentum of the fluid flow entering the control volume in the same direction of the force
$\sum Q_2\rho_2 v_2$ = the rate of momentum of the fluid flow leaving the control volume in the same direction of the force

The force exerted by a flowing fluid on a bend, enlargement, on contraction in a pipeline may be computed using the impulse-momentum principle.



$\sum F_x = 0$ => $P_1A_1 - P_2A_2\cos\alpha - F_x = Q\rho(v_2\cos\alpha - v_1)$

$\sum F_y = 0$ => $F_y - W - P_2A_2\sin\alpha = Q\rho(v_2\sin\alpha - 0)$

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F = the force exerted by the bend on the fluid (the force exerted by the fluid on the bend is equal in magnitude and opposite in sign)

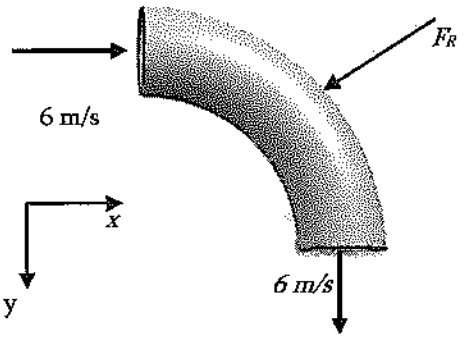
F_x and F_y are the x-component and y-component of the force, $F = \sqrt{F_x^2 + F_y^2}$ and $\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$

P = the internal pressure in the pipe line
A = the cross-sectional area of the pipe line
W = the weight of the fluid
v = the velocity of the fluid flow
α = the angle the pipe bend makes with the horizontal
ρ = the density of the fluid
Q = the quantity of fluid flow

Problem 9.34

Water is flowing through a 90° curved uniform vane with a cross-section of 0.01 m² as shown below. The resultant reaction force, F_R exerted by vane on the water is most nearly:

A. 210 N
B. 310 N
C. 410 N
D. 510 N



Solution: D

$Q = Av = 0.01\ \text{m}^2\ (6\ \text{m/s}) = 0.06\ \text{m}^3/\text{s}$

$$F_{Rx} = \rho Q(v_{2x} - v_{1x}) = 1{,}000\frac{kg}{m^3}(0.06\ \frac{m^3}{s})(0-6) = -360\ N$$

$$F_{Ry} = \rho Q(v_{2y} - v_{1y}) = 1{,}000\frac{kg}{m^3}(0.06\ \frac{m^3}{s})(-6-0) = -360\ N$$

Resultant, $F_R = \sqrt{(F_{Rx})^2 + (F_{Ry})^2} = \sqrt{(-360)^2 + (-360)^2} = 509.1\ \text{N}$

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Problem 9.35

A jet of water, 3.0-inch in diameter is moving to the right and impinges on a flat plate held normal to its axis. For the velocity of 80 fps, what force will keep the plate in equilibrium?

A. 609 lb
B. 309 lb
C. 1,300 lb
D. 20,000 lb

Solution: A

This is a 1D problem.

$$F_{Rx} = \rho Q(v_{2x} - v_{1x}) = \rho A v_x (v_x - 0) = \left(\frac{62.4\ \frac{\text{lb}}{\text{ft}^3}}{32.2}\right)\left(\frac{\pi}{4}\left(\frac{3.0}{12}\ \text{ft}\right)^2\right)\left(80\ \frac{\text{lb}}{\text{sec}}\right)\left(80\ \frac{\text{lb}}{\text{sec}}\right) = 609\ \text{lb}$$

Problem 9.36

The force exerted by a 25-mm diameter stream of water against a flat plate held normal to the stream's axis is 645 N. The flow velocity is most nearly:

A. 12.6 m/s
B. 18.9 m/s
C. 26.4 m/s
D. 36.3 m/s

Solution: D

$$F_{Rx} = \rho Q(v_{2x} - v_{1x})$$

$$645 = \rho A v_x^2$$

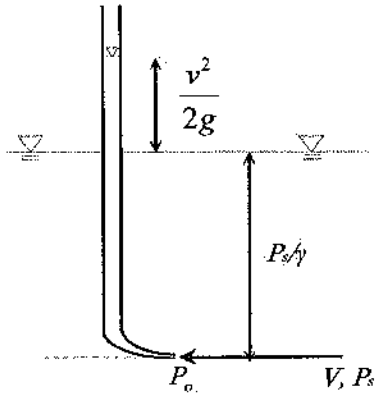
$$v_x = \sqrt{\frac{645\ \text{N}}{\left(1{,}000\ \frac{\text{kg}}{\text{m}^3}\right)\left(\frac{\pi}{4}(0.025)^2\ \text{m}^2\right)}} = 36.3\ \frac{\text{m}}{\text{sec}}$$

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Fluid Flow Measurement

The Pitot Tube

A pitot tube is a pressure measurement instrument used to measure fluid flow velocity. The stagnation pressure equation for an incompressible fluid can be determined as:



$$v = \sqrt{\frac{2(P_o - P_s)}{\rho}} = \sqrt{\frac{2g(P_o - P_s)}{\gamma}}$$

v = the velocity of the fluid
P_o = the stagnation pressure
Ps = the static pressure of the fluid at the elevation where the measurement is taken

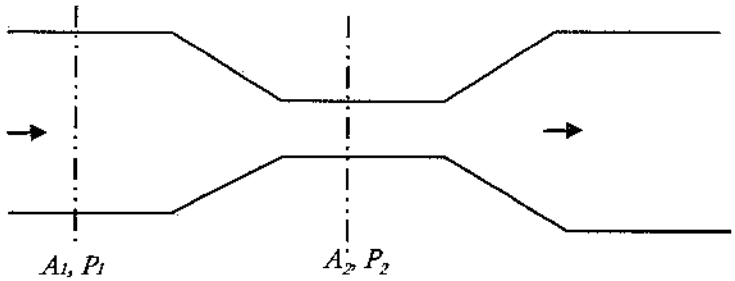
Venturi Meters

Venturi meter is used to measure flow rate. It uses a converging section of pipe to give an increase in the flow velocity and a corresponding pressure drop. Then, the flow rate can be determined as follows:

$$Q = \frac{C_v A_2}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2}} \sqrt{2g\left(\frac{P_1}{\gamma} + z_1 - \frac{P_2}{\gamma} - z_2\right)}$$

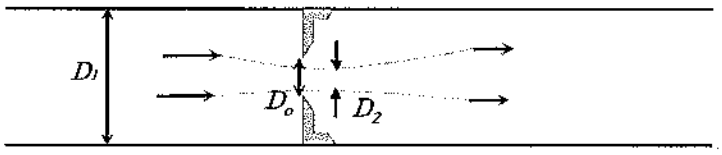
Q = volumetric flow rate
Cv = the coefficient of velocity
A = cross-sectional area of flow
P = pressure
z_1 = elevation of venturi entrance
z_2 = elevation of venturi throat

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Orifices

An orifice is an opening (as a vent, mouth, or hole) through which fluid can pass. Then, using Bernoulli's principle, the flow rate can be measured as follows:



$$Q = CA_o\sqrt{2g\left(\frac{P_1}{\gamma} + z_1 - \frac{P_2}{\gamma} - z_2\right)}$$

The cross-sectional area at the vena contracta, A_2 is characterized by a coefficient of contraction C_c and given by C_cA. The coefficient of the meter (orifice coefficient), C is given by:

$$C = \frac{C_V C_C}{\sqrt{1 - C_C^2\left(\frac{A_o}{A_1}\right)^2}}$$

As reported in the NCEES FE Ref. Handbook, Fluid Mechanics, the values of C for sharp edge, rounded, short tube, and borda orifices are 0.61, 0.98, 0.8, and 0.51 respectively. During the exam, instead of C, the coefficients C_c and C_v may be given. Then, you need to determine C.

Problem 9.37

A pitot tube has been used to measure the velocity of air. It shows a pressure of 0.53 m of Hg. Assume that the air has a static pressure of 70 kPa and a density 1.2 kg/m³. The specific gravity of Hg is 13.6. The velocity (m/s) of the flow is most nearly:

A. 24.3
B. 29.3
C. 34.4
D. 38.3

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Solution: C

$$P_o = h\rho g = (0.53\text{ m})\,(13{,}600\text{ kg/m}^3)\,(9.81) = 70{,}710\text{ Pa}$$

$$\text{Velocity, } v = \sqrt{\frac{2(P_o - P_s)}{\rho}} = \sqrt{\frac{2(70{,}710 - 70{,}000)\text{ Pa}}{1.2\ \frac{\text{kg}}{\text{m}^3}}} = 34.4\ \text{m/s}$$

Problem 9.38

A submerged orifice has a cross-sectional area of 0.1 m². The upstream and the downstream water depths are 7 m and 2 m, respectively. The flow rate (m³/s) if the discharge coefficient is 0.8 is most nearly:

A. 0.08
B. 2.8
C. 1.8
D. 0.8

Solution: D

$$\text{Flow rate, } Q = CA\sqrt{2g(h_1 - h_2)} = 0.8(0.1\text{m}^2)\sqrt{2(9.81)(7\text{ m} - 2\text{ m})} = 0.8\ \frac{\text{m}^3}{\text{s}}$$

Problem 9.39

An orifice discharging water freely into atmosphere has a cross-sectional area of 0.1 m². The upstream water depth is 7 m. What is most nearly the flow rate if the orifice coefficient is 0.8?

A. 0.94 m³/s B. 2.84 m³/s C. 1.84 m³/s D. 0.8 m³/s

Solution: A

$$\text{Flow rate, } Q = CA\sqrt{2gh} = 0.8(0.1\text{m}^2)\sqrt{2(9.81)(7\text{ m})} = 0.94\ \frac{\text{m}^3}{\text{s}}$$

Dimensional Homogeneity

Dimensional Analysis

A dimensionally homogeneous equation has the same dimensions on the left and right sides of the equation. Dimensional analysis involves the development of equations that relate dimensionless groups of variables to describe physical phenomena.

Buckingham Pi Theorem: The number of independent dimensionless groups that may be employed to describe a phenomenon known to involve n variables is equal to $(n - \bar{r})$, where $\bar{r}$ is the number of basic dimensions. (e.g., M, L, T) needed to express the variables dimensionally.

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Similitude

To use a model to simulate the conditions of the prototype, the model must be geometrically, kinematically, and dynamically like the prototype system. To obtain dynamic similarity between two flow pictures, all independent force ratios must be the same in both the model and the prototype. Thus, dynamic similarity between two flow pictures (when all possible forces are acting) is expressed in the five simultaneous equations below.

$$\left[\frac{F_I}{F_P}\right]_p = \left[\frac{F_I}{F_P}\right]_m = \left[\frac{\rho v^2}{P}\right]_p = \left[\frac{\rho v^2}{P}\right]_m$$

$$\left[\frac{F_I}{F_V}\right]_p = \left[\frac{F_I}{F_V}\right]_m = \left[\frac{vl\rho}{\mu}\right]_p = \left[\frac{vl\rho}{\mu}\right]_m = [\mathrm{Re}]_p = [\mathrm{Re}]_m$$

$$\left[\frac{F_I}{F_G}\right]_p = \left[\frac{F_I}{F_G}\right]_m = \left[\frac{v^2}{\lg}\right]_p = \left[\frac{v^2}{\lg}\right]_m = [Fr]_p = [Fr]_m$$

$$\left[\frac{F_I}{F_E}\right]_p = \left[\frac{F_I}{F_E}\right]_m = \left[\frac{\rho v^2}{E_v}\right]_p = \left[\frac{\rho v^2}{E_v}\right]_m = [Ca]_p = [Ca]_m$$

$$\left[\frac{F_I}{F_T}\right]_p = \left[\frac{F_I}{F_T}\right]_m = \left[\frac{\rho l v^2}{\sigma}\right]_p = \left[\frac{\rho l v^2}{\sigma}\right]_m = [We]_p = [We]_m$$

where:

subscripts p and m stand for prototype and model respectively
F_I = inertia force
F_P = pressure force
F_V = viscous force
F_G = gravity force
F_E = elastic force
F_T = surface tension force
Re = Reynolds number
We = Weber number
Ca = Cauchy number
Fr = Froude number
l = characteristic length
v = velocity
ρ = density
σ = surface tension
E_v = bulk modulus
μ = dynamic viscosity
P = pressure
g = acceleration of gravity

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Problem 9.40

A water body was scaled down to a model with a characteristic length ratio of 15:1. The velocity of the real body is 10 m/s. What is most nearly the velocity (m/s) in the model if they have equal Froude numbers?

A. 1.5
B. 6.67
C. 2.59
D. 150

Solution: C

$$\left[\frac{F_I}{F_G}\right]_p = \left[\frac{F_I}{F_G}\right]_m = \left[\frac{v^2}{\lg}\right]_p = \left[\frac{v^2}{\lg}\right]_m = [Fr]_p = [Fr]_m$$

$$\frac{v_m^2}{l_m g} = \frac{v_p^2}{l_p g}$$

$$\Rightarrow \quad v_m^2 = v_p^2 \frac{l_m}{l_p} = (10 \text{ m/s})^2 \frac{1}{15} = 6.67$$

$$\Rightarrow \quad v_m = 2.59 \text{ m/s}$$

Problem 9.41

A water body was scaled down to a model with a characteristic length ratio of 15:1. The velocity of the real body is 10 m/s. The velocity (m/s) in the model if they have equal Reynolds numbers is most nearly:

A. 1.5
B. 6.67
C. 2.59
D. 150

Solution: D

$$\left[\frac{F_I}{F_V}\right]_p = \left[\frac{F_I}{F_V}\right]_m = \left[\frac{vl\rho}{\mu}\right]_p = \left[\frac{vl\rho}{\mu}\right]_m = [\text{Re}]_p = [\text{Re}]_m$$

$$\Rightarrow \quad \frac{v_m l_m \rho}{\mu} = \frac{v_p l_p \rho}{\mu}$$

$$\Rightarrow \quad v_m = v_p \frac{l_p}{l_m} = (10 \text{ m/s}) \frac{15}{1} = 150 \text{ m/s}$$

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Properties of Water

Properties of water both in SI units and US units are listed in the Fluid Mechanics of the NCEES FE Ref. Handbook. Please review it and solve the following problem.

Problem 9.42

The density (kg/m^3) of water at 100 °C is most nearly:

A. 1,000
B. 977.8
C. 965.3
D. 958.4

Solution: D

From the NCEES FE Ref. Handbook, Fluid Mechanics, the density of water at 100 °C is 958.4 kg/m^3.

"The NCEES FE Reference Handbook *is the only reference material that can be used during the exam. You will be provided with an electronic reference handbook during the exam. For access prior to your exam, you may either purchase a hard copy or download a free electronic copy. Register or log in to MyNCEES to download your free copy of the* FE Reference Handbook.*" Visit http://ncees.org/engineering/fe/ for more information.*

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10 ENGINEERING ECONOMICS

Fundamentals of Engineering (FE) CIVIL Exam Specification mentions, 4–6 questions may appear in the exam based on Engineering Economics as follows:

A. Discounted cash flow (e.g., equivalence, PW, equivalent annual worth, FW, rate of return)
B. Cost (e.g., incremental, average, sunk, estimating)
C. Analyses (e.g., breakeven, benefit-cost, life cycle)
D. Uncertainty (e.g., expected value and risk)

Table 10.1 lists all the formulas to convert one quantity to another. Instead of using these formulas, which is time consuming, Factor Tables from the NCEES FE Ref. Handbook may be used. One of the Factor Tables for 6% is mentioned in Table 10.2.

Table 10.1 Factor and Formula List

Factor Name	Converts	Symbol	Formula
Single Payment Compound Amount	to F given P	$\left(\frac{F}{P}, i\%, n\right)$	$(1+i)^n$
Single Payment Present Worth	to P given F	$\left(\frac{P}{F}, i\%, n\right)$	$(1+i)^{-n}$
Uniform Series Sinking Fund	to A given F	$\left(\frac{A}{F}, i\%, n\right)$	$\frac{i}{(1+i)^n-1}$
Capital Recovery	to A given P	$\left(\frac{A}{P}, i\%, n\right)$	$\frac{i(1+i)^n}{(1+i)^n-1}$
Uniform Series Compound Amount	to F given A	$\left(\frac{F}{A}, i\%, n\right)$	$\frac{(1+i)^n-1}{i}$
Uniform Series Present Worth	to P given A	$\left(\frac{P}{A}, i\%, n\right)$	$\frac{(1+i)^n-1}{i(1+i)^n}$

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Table 10.2 Factor Table for 6% interest rate

Factor Table - $i = 6.00\%$

n	P/F	P/A	P/G	F/P	F/A	A/P	A/F	A/G
1	0.9434	0.9434	0.0000	1.0600	1.0000	1.0600	1.0000	0.0000
2	0.8900	1.8334	0.8900	1.1236	2.0600	0.5454	0.4854	0.4854
3	0.8396	2.6730	2.5692	1.1910	3.1836	0.3741	0.3141	0.9612
4	0.7921	3.4651	4.9455	1.2625	4.3746	0.2886	0.2286	1.4272
5	0.7473	4.2124	7.9345	1.3382	5.6371	0.2374	0.1774	1.8836
6	0.7050	4.9173	11.4594	1.4185	6.9753	0.2034	0.1434	2.3304
7	0.6651	5.5824	15.4497	1.5036	8.3938	0.1791	0.1191	2.7676
8	0.6274	6.2098	19.8416	1.5938	9.8975	0.1610	0.1010	3.1963
9	0.5919	6.8017	24.5768	1.6895	11.4913	0.1470	0.0870	3.6133
10	0.5584	7.3601	29.6023	1.7908	13.1808	0.1359	0.0759	4.0220
11	0.5268	7.8869	34.8702	1.8983	14.9716	0.1268	0.0668	4.4213
12	0.4970	8.3838	40.3369	2.0122	16.8699	0.1193	0.0593	4.8113
13	0.4688	8.8527	45.9629	2.1329	18.8821	0.1130	0.0530	5.1920
14	0.4423	9.2950	51.7128	2.2609	21.0151	0.1076	0.0476	5.5635
15	0.4173	9.7122	57.5546	2.3966	23.2760	0.1030	0.0430	5.9260
16	0.3936	10.1059	63.4592	2.5404	25.6725	0.0990	0.0390	6.2794
17	0.3714	10.4773	69.4011	2.6928	28.2129	0.0954	0.0354	6.6240
18	0.3505	10.8276	75.3569	2.8543	30.9057	0.0934	0.0324	6.9597
19	0.3305	11.1581	81.3062	3.0256	33.7600	0.0896	0.0296	7.2867
20	0.3118	11.4699	87.2304	3.2071	36.7856	0.0872	0.0272	7.6051
21	0.2942	11.7641	93.1136	3.3996	39.9927	0.0850	0.0250	7.9151
22	0.2775	12.0416	98.9412	3.6035	43.3923	0.0830	0.0230	8.2166
23	0.2618	12.3034	104.7007	3.8197	46.9958	0.0813	0.0213	8.5099
24	0.2470	12.5504	110.3812	4.0489	50.8156	0.0797	0.0197	8.7951
25	0.2330	12.7834	115.9732	4.2919	54.8645	0.0782	0.0182	9.0722
30	0.1741	13.7648	142.3588	5.7435	79.0582	0.0726	0.0126	10.3422
40	0.0972	15.0463	185.9568	10.2857	154.7620	0.0665	0.0065	12.3590
50	0.0543	15.7619	217.4574	18.4202	290.3359	0.0634	0.0034	13.7964
60	0.0303	16.1614	239.0428	32.9877	533.1282	0.0619	0.0019	14.7909
100	0.0029	16.6175	272.0471	339.3021	5,638.3681	0.0602	0.0002	16.3711

Nomenclature

A = Uniform amount per interest period
BV = Book value
C = Cost
d = Inflation adjusted interest rate per interest period
D_j = Depreciation in year j
F = Future worth, value, or amount
f = General inflation rate per interest period
i = Interest rate per interest period
i_e = Annual effective interest rate
n = Number of compounding periods; or the expected life of an asset
P = Present worth, value, or amount
r = Nominal annual interest rate
S_n = Expected salvage value in year n

Subscripts

j = at time j; n = at time n years

Problem 10.1

A sum of $1,200 is deposited in a savings account with 6% annual interest. If no money is withdrawn for six years, most neatly the account balance after 6 years is –

A. $1,520 B. $1,590 C. $1,702 D. $1,202

Solution: C

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Using Equation: Future worth, $F = P(1+i)^n = 1{,}200(1+0.06)^6 = 1{,}200(1.4185) = 1{,}702$
Using Factor Table from NCEES FE Ref. Handbook, (or Table 10.2 from this book): The F/P factor for 6% interest rate after 6 years (*F/P, 6%, 6*) is 1.4185.
F = 1.4185 (1,200) = 1,702. Therefore, the future value is equal in both ways. So, any option can be used.

Risk

Risk is the chance of an outcome other than what is planned to occur or expected in the analysis.

Break-Even Analysis

By altering the value of any one of the variables in a situation, holding all the other values constant, it is possible to find a value for that variable that makes the two alternatives equally economical. This value is the break-even point.

Break-even analysis is used to describe the percentage of capacity of operation for a manufacturing plant at which income just covers expenses. The payback period is the period required for the profit or other benefits of an investment to equal the cost of the investment.

Inflation

To account for inflation, the dollars are deflated by the general inflation rate per interest period f, and then they are shifted over the time scale using the interest rate per interest period i. Use an inflation adjusted interest rate per interest period d for computing P.

The formula for d is, $d = i + f + (i \times f)$

Problem 10.2

The current price of a load cell is $6,000, which was bought 4 years back. The annual maintenances cost is $300 for its total life time. During the first 4 years, the rate of inflation is 5%, and the effective annual rate of interest is 6%. The uninflated worth of the equipment during the first year is most nearly –

A. $3,200 B. $3,300 C. $3,400 D. $3,900

Solution: D

The first four years had an inflation. Therefore, the interest rate needs to be adjusted for inflation.

$$d = i + f + (i \times f) = 0.06 + 0.05 + (0.06)(0.05) = 0.113$$

Present worth, $P = F(1+d)^{-4} = 6{,}000(1+0.113)^{-4} = 6{,}000(0.6517) = 3{,}910 \approx 3{,}900$

Factor Table from the NCEES FE Ref. Handbook, cannot be used as the factor is not available for the adjusted interest rate.

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Depreciation

Straight Line

In straight line method, depreciation is assumed linear with service life. For example, the initial cost of a product is \$5,000 and expected salvage value of after 10 years is \$1,000. Then, the depreciation is \$4,000 (\$5,000 - \$1,000). The depreciation per year is \$4,000/10 = \$400 per year. Therefore, the depreciation per year can be expressed mathematically as follows:

$$D_j = \frac{C - S_n}{n}$$

S_n = Expected salvage value in year n
C = initial cost
D_j = Depreciation in j year

Modified Accelerated Cost Recovery System (MACRS)

MACRS is the new system which allows for greater accelerated depreciation over longer time periods. Although the MACRS depreciation method is used only for income tax purposes, this book uses it for solving other types of business to show how this method works. In this method, the depreciation in j year is presented by:

$$D_j = (\text{factor})C$$

A table of MACRS factors is provided below.

MACRS FACTORS				
Year	Recovery Period (Years)			
	3	5	7	10
	Recovery Rate (Percent)			
1	33.33	20.00	14.29	10.00
2	44.45	32.00	24.49	18.00
3	14.81	19.20	17.49	14.40
4	7.41	11.52	12.49	11.52
5		11.52	8.93	9.22
6		5.76	8.92	7.37
7			8.93	6.55
8			4.46	6.55
9				6.56
10				6.55
11				3.28

Book Value

Whichever method is used to calculate the depreciation, the book value (BV) is calculated by deducting the cumulative depreciation from the initial cost, which is shown below:

$$\text{BV} = \text{Initial Cost} - \sum D_j$$

Problem 10.3

A construction company purchased a front loader with an initial cost of \$9,000. The annual interest rate is 2%. The annual maintenance cost is \$350 and is considered to be paid at the end of each

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year. Using the MACRS depreciation system, what is most nearly the depreciation for the front loader in the first year of operation assuming a 10 years of recovery period?

A. $640 B. $670 C. $720 D. $900

Solution: D

The MACRS factor for the first year with 10-year recovery period is 10%.
Depreciation at the first year = 10% of $9,000 = $ 900

Problem 10.4

A construction company purchased a front loader with an initial cost of $9,000. The annual interest rate is 2%. The annual maintenance cost is $350 and is considered to be paid at the end of each year. Using the MACRS depreciation system, what is most nearly the depreciation for the front loader in the first 3 years of operation assuming a 10 years of recovery period?

A. $2,600 B. $6,200 C. $4,200 D. $3,800

Solution: D

The MACRS factor with 10-year recovery period for the first, second, and third years are 10%, 18% and 14.4%, respectively.
Depreciation at the first year = 10% of $9,000 = $ 900
Depreciation at the second year = 18% of $9,000 = $ 1,620
Depreciation at the third year = 14.4% of $9,000 = $ 1,296
Total depreciation = $ 3,816 ≈ 3,800

Problem 10.5

A construction company purchased a front loader with an initial cost of $9,000. The annual interest rate is 2%. The annual maintenance cost is $350 and is considered to be paid at the end of each year. Using the MACRS depreciation system, what is most nearly the book value for the front loader after the first 3 years of operation assuming a 10 years of recovery period?

A. $6,200 B. $5,200 C. $4,200 D. $3,800

Solution: B

The MACRS factor with 10-year recovery period for the first, second, and third years are 10%, 18% and 14.4%, respectively.
Depreciation at the first year = 10% of $9,000 = $ 900
Depreciation at the second year = 18% of $9,000 = $ 1,620
Depreciation at the third year = 14.4% of $9,000 = $ 1,296

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Total depreciation = $ 3,816
BV = Initial Cost - $\sum D_j$ = $ 9000 – $ 3816 = $ 5,184 ≈ $ 5,200

Problem 10.6

A drilling kit with a useful life of 10 years has an initial cost of $3,600 and a salvage value of $1,500. The interest rate is 2%. Using the straight-line method, the total depreciation of the equipment for the first 5 years is most nearly-

A. $1,260 B. $810 C. $1,050 D. $920

Solution: C

$$\text{Depreciation Rate} = \frac{\text{Initial Cost} - \text{Salvage Value}}{\text{Service Life}} = \frac{3,600 - 1,500}{10} = 210 \text{ per year}$$

Total depreciation in 5 years = 5 ($ 210) = $ 1,050.

Problem 10.7

A computer with a useful life of 15 years has an initial cost of $3,500 and a salvage value of $350. The interest rate is 6%. Using the modified accelerated cost recovery system (MACRS) method of depreciation and a 7 years recovery period, what is the most nearly the book value of the computer after the second year?

A. $1,900 B. $2,150 C. $2,300 D. $2,400

Solution: B

The MACRS factor with 7-year recovery period for the first and second years are 14.29%, and 24.49% respectively.
Depreciation at the first year = 14.29% of $3,500 = $ 500
Depreciation at the second year = 24.49% of $3,500 = $ 857
Total depreciation = $ 1,357
BV = Initial Cost – $\sum D_j$ = $ 3,500 – $ 1,357 = $ 2,143 ≈ $ 2,150

Problem 10.8

Based on the straight-line method of deprivation, the book value at the end of the 7th year for a dodger having an initial cost of $80,000, and a salvage value of $15,000 at the end of its expected life of 10 years is most nearly:

A. $27,500
B. $29,500
C. $32,500
D. $34,500

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Solution: D

$$\text{Depreciation per year} = \frac{\$80{,}000\text{-}\$15{,}000}{10\,\text{year}} = \$6{,}500$$

Book value after 7th year = \$80,000 – 7 (\$6,500) = \$34,500

Taxation

Income taxes are paid at a specific rate on taxable income. Taxable income is total income less depreciation and ordinary expenses. Expenses do not include capital items, which should be depreciated.

Capitalized Costs

Capitalized costs are present worth values using an assumed perpetual period.

$$\text{Capitalized Costs} = P = \frac{A}{i}$$

Problem 10.9

An apple sauce factory was purchased with an initial lump-sum payment of \$100,000. It was found that profits from this factory is \$8,000 each year. What is most nearly the interest rate earned on that investment?

A. 8.0% B. 9.0% C. 10% D. 12%

Solution: A

$$P = \frac{A}{i} \quad \Rightarrow \quad i = \frac{A_{profit}}{P_{cost}} = \frac{8{,}000}{100{,}000} = 0.08 \approx 8\%$$

Bonds

Bond value equals the present worth of the payments the purchaser (or holder of the bond) receives during the life of the bond at some interest rate i. Bond yield equals the compound interest rate of the bond value when compared with the bond cost.

Rate-of-Return

The minimum acceptable rate-of-return (MARR) is that interest rate that one is willing to accept, or the rate one desires to earn on investments. The rate-of-return on an investment is the interest rate that makes the benefits and costs equal.

Benefit-Cost Analysis

In a benefit-cost analysis, the benefits, B of a project should exceed the estimated costs, C.

$$B - C \geq 0, \text{ or } B/C \geq 1$$

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Problem 10. 10

A laser printer was purchased with an initial cost of $1,200 and an annual maintenance cost of $200/yr which continues indefinitely. The interest rate is 6%. The present worth of all expenditure is most nearly what?

A. $2,000 B. $4,500 C. $4,000 D. $5,000

Solution: B

As the annual maintenance cost continues indefinitely, $P_{\text{maintenance}} = \frac{A}{i} = \frac{200}{0.06} = 3333$

The present worth, $P_{\text{total}} = P_{\text{initial}} + P_{\text{annual}} = 1,200 + 3,333 = 4,533 \approx 4,500$

Problem 10.11

An official sound system has a useful life of 15 years has an initial cost of $5,000. The salvage value is $300, and the annual maintenance is $200/yr. The interest rate is 6%. What is most nearly the present worth of the costs of the sound system?

A. $5,200 B. $5,300 C. $6,800 D. $5,700

Solution: C

$$P_{\text{total}} = P_{\text{initial}} + P_{\text{maintenance}} - P_{\text{salvage}}$$

Annual Maintenance:
P/A factor for 6% interest rate for 15 years = 9.7122
$P_{maintenance}$ =$ 200 (9.7122) = $ 1,942
Salvage Value:
P/F factor for 6% interest rate for 15 years = 0.4173
$P_{salvage}$ = $300 (0.4173) = $125
The present worth, $P_{total} = P_{initial} + P_{annual} - P_{salvage} = 5,000 + 1,942 - 125 = \$6,814 \approx \$6,800$

Problem 10.12

Which of the following programs is not a varied cash flow program?

A. Stepped B. Gradual C. Delayed D. Uniform

Solution: D

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Problem 10.13

A back-loader construction equipment has a useful life of 10 years has an initial cost of $50,000. The salvage value is $6,000, and the annual maintenance is $2,000. The interest rate is 8%. What is most nearly the present worth of the costs of the back-loader?

A. $52,050 B. $53,060 C. $60,640 D. $57,020

Solution: C

$$P_{total} = P_{initial} + P_{maintenance} - P_{salvage}$$

Annual Maintenance:

P/A factor for 8% interest rate for 10 years = 6.7101

$P_{maintenance}$ =$ 2,000 (6.7101) = $ 13,420

Salvage Value:

P/F factor for 8% interest rate for 10 years = 0.4632

$P_{salvage}$ = $6,000 (0.4632) = $2,779

The present worth,

$$P_{total} = P_{initial} + P_{annual} - P_{salvage} = 50,000 + 13,420 - 2,779 = \$60,641 \approx \$60,640$$

Problem 10.14

The need for a large capacity water supply system is forecast to occur 4 years from now. At that time the system required is estimated to cost $40,000. If an account earns 12% per year compounded annually, the amount that must be placed in the account at the end of each year in order to accumulate the necessary purchase price is most nearly;

A) $8,000
B) $8,370
C) $9,000
D) $10,000

Solution: B

$A=F(A/F,i,n)$ = 40,000(A/F,12%,4) = $8,369 per year

Problem 10.15

A project has the estimated cash flows shown below:

Year End	0	1	2	3	4
Cash Flow	-$800	-$400	+$1,00	+$1,00	+$1,00

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Using an interest rate of 12% per year compounded annually, the annual worth of the project is most nearly:

A. $325
B. $226
C. $320
D. $262

Solution: A

$P = -800 - 400(P/F, 12\%, 1) + 1{,}000(P/F, 12\%, 2) + 1{,}000(P/F, 12\%, 3) + 1{,}000(P/F, 12\%, 4)$
$= -1{,}100 - 400(0.8929) + 1{,}000(0.7972) + 1{,}000(0.7118) + 1{,}000(0.6355)$
$= 987.34$
$A = P(A/P, 12\%, 4) = 987.34(0.3292) = \$325$ per year

The break-even point (BEP)

The break-even point (BEP) is the point at which total cost and total revenue are equal. There is no net loss or gain, and one has "broken even," though opportunity costs have been paid and capital has received the risk-adjusted, expected return. One example here:

Problem 10.16

A company can manufacture a product using hand tools with tool cost of $1,000, and cost per unit is $1.50. As an alternative, an automated system will cost $26,000 with a manufacturing cost per unit of $0.50. With an anticipated annual volume of 5,000 units and neglecting interest, the break-even point (years) is most nearly:

A. 3 years
B. 4 years
C. 5 years
D. 6 years

Solution: C

Hand Tools: $1.50 x 5000 units = $7,500
Automated System: $0.50 x 5,000 = $2,500

Annual Savings = $7,500 - $2,500 = $5,000
Additional Investment = $26,000 - $1,000 = $25,000

Payback time = $25,000/$5,000 = 5.0 years

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11 COMPUTATIONAL TOOLS

Fundamentals of Engineering (FE) CIVIL Exam Specification mentions, 4–6 questions may appear in the exam based on Computational Tools as follows:

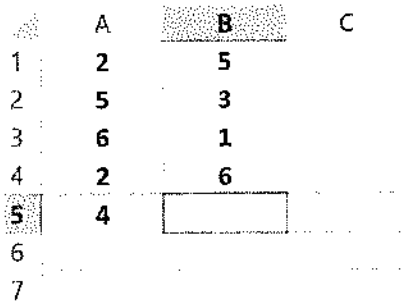
A. Spreadsheet computations
B. Structured programming (e.g., if-then, loops, macros) or Computer Application

Please be advised that there is no specific syllabus or guidelines given in the NCEES Ref. Handbook regarding this topic. The author of this manual is recommending using Microsoft Office Excel Worksheet with a desktop/laptop computer. This will give you general knowledge on computer and MS Microsoft Office Excel Worksheet. Some of the problems are discussed here to give you an idea. In addition, make sure that the NCEES exam does not want you to be an expert in this area. However, general knowledge on this area is good enough. Go through the following problems for your understanding.

Spreadsheet computations

Problem 11.1

For the following spreadsheet, what will be the value in B5, if the command in this box is "sum(A1:B3)"?

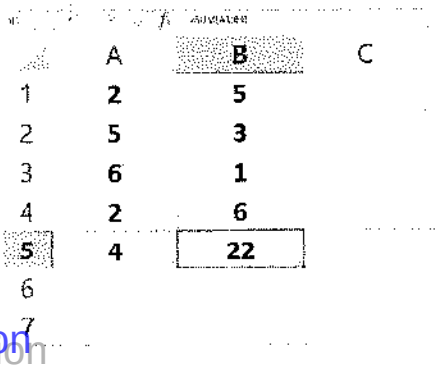


	A	B	C
1	2	5	
2	5	3	
3	6	1	
4	2	6	
5	4		
6			
7			

A. 22
B. 30
C. 15
D. None of the above

Solution: A

"sum(A1:B3)" means the summation of values from A1 to B3.



	A	B	C
1	2	5	
2	5	3	
3	6	1	
4	2	6	
5	4	22	
6			
7			

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Problem 11.2

For the following spreadsheet, if the command at C3 is copied and pasted into C6, what is the value at C6? The command at C3 is "sum(A1:A4)".

	A	B	C	D
1	2	5	6	
2	5	3	2	
3	6		=sum(A1:A4)	
4	2	6		
5	4	3		
6	2	1		
7				

A. 14
B. 15
C. 8
D. Error

Solution: C

The command changes from "sum(A1:A4)" to "sum(A4:A7)"

	A	B	C	D
1	2	5	6	
2	5	3	2	
3	6	2	15	
4	2	6		
5	4	3		
6	2	1	8	
7				
8				

Problem 11.3

How many cells are in the range of B5:Y50?

A. 2,375 B. 2,496 C. 95 D. 1,104

Solution: D

No. of column = Y – B + 1 = 25 – 2 + 1 =24
No. of row = 50 – 5 + 1 = 46
Total = 46 x 24 = 1,104

Problem 11.4

What is the next column after Z in a spreadsheet?

A. A B. Z1 C. AA D. AB

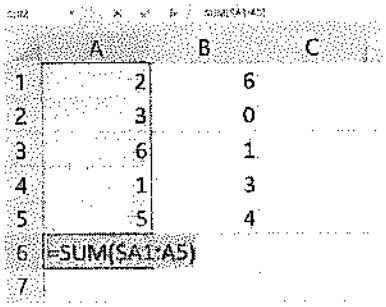
Solution: C

X Y Z AA AB AC

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Problem 11.5

For the following spreadsheet, what will be the value in B6, if the command in A6 copied and pasted in B6? The command at A6 is "=sum($A1:A5)".

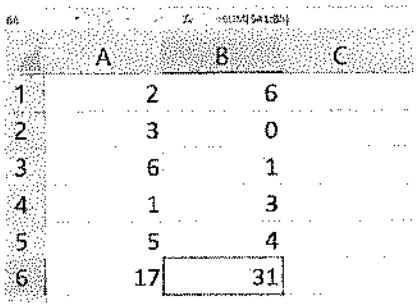


	A	B	C
1	2	6	
2	3	0	
3	6	1	
4	1	3	
5	5	4	
6	=SUM($A1:A5)		
7			

A. 17 B. 30
C. 14 D. 31

Solution: D

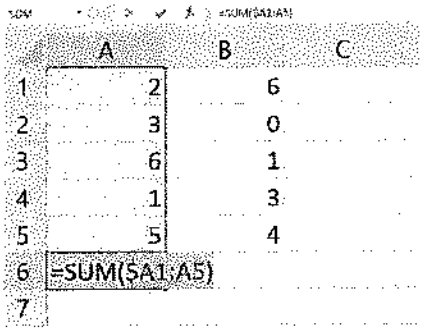
The command at A6 is "Sum($A1:A5)". When it is copied from A6 and pasted into B6, the A1 does not move due to the $ sign, but A5 moves to B5. So, the result is the summation from A1 to B5.



	A	B	C
1	2	6	
2	3	0	
3	6	1	
4	1	3	
5	5	4	
6	17	31	

Problem 11.6

For the following spreadsheet, what will be the value in A7, if the command in A6 copied and pasted in A7? The command at A6 is "=sum($A1:A5)".

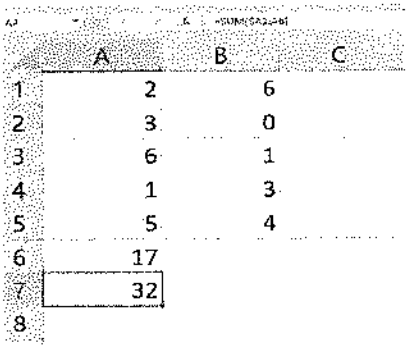


	A	B	C
1	2	6	
2	3	0	
3	6	1	
4	1	3	
5	5	4	
6	=SUM($A1:A5)		
7			

A. 17 B. 30
C. 34 D. 32

Solution: D

The command at A6 is "Sum($A1:A5)". When it is copied and pasted into A7, the A1 is moving to A2. Similarly, A5 is moving to A6. The dollar sign before 'A' fixes 'A' only, not the row number. So, the result is the summation from A2 to A6.

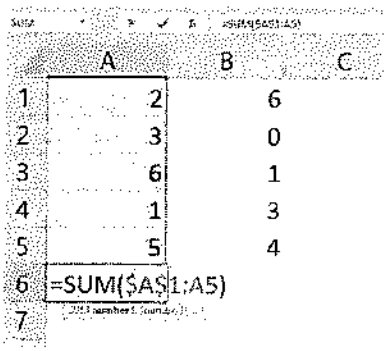


	A	B	C
1	2	6	
2	3	0	
3	6	1	
4	1	3	
5	5	4	
6	17		
7	32		
8			

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Problem 11.7

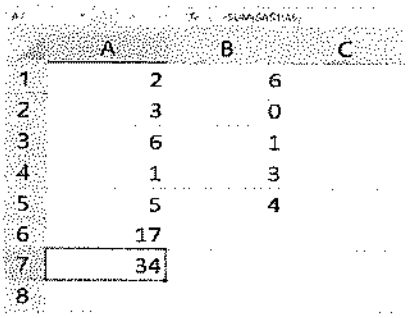
For the following spreadsheet, what will be the value in A7, if the command in A6 copied and pasted in A7? The command at A6 is "Sum($A$1:A5)".



A. 17 B. 34
C. 14 D. 32

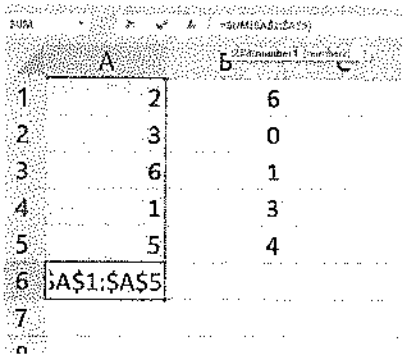
Solution: B

The command at A6 is "Sum($A$1:A5)". When it is copied and pasted into A7, the A1 does not move in any direction. The dollar sign before 'A' fixes 'A'; the dollar sign before 1 fixes 1. So, the result is the summation from A1 to A6.



Problem 11.8

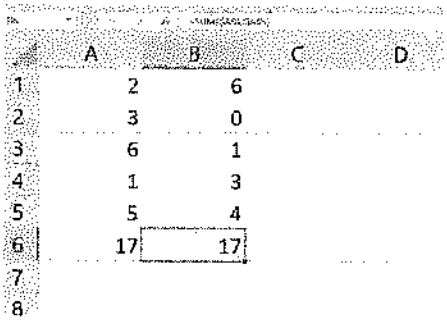
For the following spreadsheet, what will be the value in B6, if the command in A6 copied and pasted in B6? The command at A6 is "Sum($A$1:$A$5)".



A. 17 B. 34
C. 14 D. 31

Solution: A

The command at A6 is "Sum($A$1:$A$5)". When it is copied and pasted into B6, the A1 and A5 will not move in any direction. The dollar sign before 'A' will fix 'A'; the dollar sign before 1 will fix 1. It is similar for A5. So, the result is the summation from A1 to A5.



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Problem 11.9

A spreadsheet cell is located at the intersection of 5th row and 11th column. This address can be expressed as:

A. L5 B. E11 C. K5 D. 11-5

Solution: C

Structured Programming

A flowchart is a diagram using different shapes (for different purposes) and arrows to demonstrate workflow, a process, or an algorithm. A flowchart may be quite simple or extremely complex. When I was a teacher, I had my students use flowcharts to plan a writing assignment (depending on the type of writing). I still use them at times when deciding. An algorithm is a finite set of procedures to be followed order to solve a mathematical or computer problem. Algorithms are often written out in flowchart form for visual understanding, but these are usually far more complex than the average business flowchart. Now that we have defined flowcharts and algorithms, the question is: What is the difference between flowcharts and algorithms? Flowcharts can be used to organize processes for all sorts of purposes: business, personal, educational, and certainly for algorithms. We've established that algorithms are used to organize finite processes for mathematical and computer purposes. Algorithms usually use flowcharts for a visual form of organization (so in a manner of speaking, algorithms could often be a type of flowchart).

Problem 11.10

A computer programming is shown below. What is the value of Y after the program is executed?

Set Y = 2
X = 2
Do while Y ≤ 8
Y = X² + Y
X = Y
End while

A. 8 B. 80 C. 16 D. 42

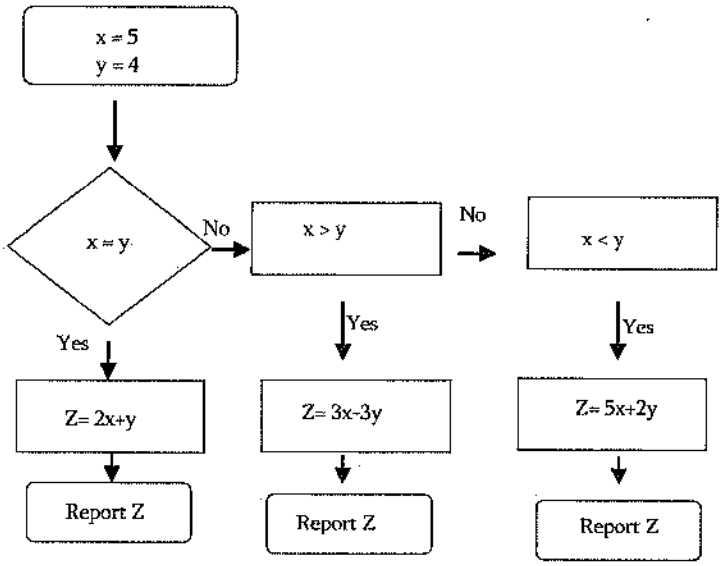
Solution: D

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First execution:	Second execution:	Third execution:
Set Y = 2	Set Y = 6	Set Y = 42 (not, Y ≤ 8)
X = 2	X = 6	No execution
$Y = X^2 + Y = 4 + 2 = 6$	$Y = X^2 + Y = 6^2 + 6 = 42$	
	X = Y = 42	

Problem 11.11

For the following flowchart, what is the value of 'Z'?



A. 3 B. 14 C. 24 D. 33

Solution: A

The second column wins here. So, Z = 3x – 3y = 3(5) – 3(4) = 3

Problem 11.12

1 kilobyte =

A. 1,000 bytes B. 1,024 bytes C. 10^3 bytes D. 10^6 bytes

Solution: B

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Problem 11.13

What is the value of the following computer program?

```
A = 1
B = 2
Do while A < B
C = B
A = 2B – 2
B = B + 1
End do
Output C
```

A. 3 B. 4 C. 5 D. 6

Solution: A

First Execution:	Second Execution:	Third execution:
A = 1	A = 2	A = 4
B = 2	B = 3	B = 4
C = B = 2	C = B = 3	A = B (so no execution)
A = 2B – 2 = 2(2) – 2 = 2	A = 2B – 2 = 2(3) – 2 = 4	
B = B + 1 = 2 + 1 = 3	B = B + 1 = 3 + 1 = 4	

Problem 11.14

Which of the following devices is an input device?

A. Scanner B. Printer C. Fax D. Monitor

Solution: A

Problem 11.15

How many cells are in the range of A5:Z100?

A. 2,470 B. 2,496 C. 95 D. 26

Solution: B

No. of column = Z – A + 1 = 26 – 1 + 1 =26
No. of row = 100 – 5 + 1 = 96
Total = 96 x 26 = 2,496

Problem 11.16

Which of the following keys represents the multiply sign in computer coding?

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A. # B. * C. X D. ()

Solution: B

Problem 11.17

How is X^Y written in computer coding?

A. X^Y B. XY C. X(Y) D. X^Y

Solution: A

Problem 11.18

Which of the following buttons represents the division sign in computer programming?

A. # B. * C. / D. %

Solution: C

Problem 11.19

A flash drive has 500 Mbytes of data. The average transfer rate is 25 Mbps (mega bit per second). How much time will it take to transfer all the flash drive data to a computer?

A. 20 sec B. 81 sec C. 123 sec D. 168 sec

Solution: D

1 Mbyte = 2^{20} bytes = $8x2^{20}$ bits
25 Mbps = $25x10^6$ bits/sec
T = Size/rate = 500 Mbyte/25 Mbps = $500x8x2^{20}$ bits / $25x10^6$ bits/sec = 167.8 sec

Problem 11.20

How many digits are there in binary numerical system?

A. 2 B. 4 C. 8 D. 10

Solution: A

Binary system has 2 digits: 0 and 1.

Problem 11.21

Which of the following devices is not an input device?

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A. Scanner B. Keyboard C. Printer D. Mouse

Solution: C

Problem 11.22

RAM in computer means -

A. Read only memory
B. Read access memory
C. Radom only memory
D. Random access memory

Solution: D

RAM is an acronym for "random access memory," while ROM stands for "read-only memory."

Problem 11.23

Core i3 has three cores and core i5 has five cores. True or false?

A. True
B. False

Solution: B

Core i7 does not have seven cores nor does Core i3 has three cores. The numbers are simply indicative of their relative processing powers.

Problem 11.24

The output of the following computer programming is most nearly:
Note: x++ means x = x + 1

```
int x = 5;
int sum = 0;

loopStart:
if (x <= 10 )
{
    sum = sum + x;

    x++;
    goto loopStart;
}
```

A. 11 B. 26 C. 35 D. 45

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Solution: D

First Iteration: sum = sum + x = 0 + 5 = 5
Then, x = x + 1 = 5 + 1 = 6
Second Iteration: sum = sum + x = 5 + 6 = 11
Then, x = x + 1 = 6 + 1 = 7
Third Iteration: sum = sum + x = 11 + 7 = 18
Then, x = x + 1 = 7 + 1 = 8
Fourth Iteration: sum = sum + x = 18 + 8 = 26
Then, x = x + 1 = 8 + 1 = 9
Fifth Iteration: sum = sum + x = 26 + 9 = 35
Then, x = x + 1 = 9 + 1 = 10
Sixth Iteration: sum = sum + x = 35 + 10 = 45

But the while function says x <= 10, i.e., will execute if x is equal or less than 10.

Problem 11.25

The output of the following computer programming is most nearly:

```
#include<stdio.h>

int main(void)
{
    int i=1;

    while(i<=100)
    {
        printf("%d\n\n", i);
        i++;
    }

    return 0;
}
```

A. Sum of 1 to 100
B. Print 1 to 100
C. Print 1 to 100, then 100 to 0
D. None of the above

Solution: C

The program says print i = 1 while i <= 100 meaning printing 1 to 100.

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12 GEOTECHNICAL ENGINEERING

Fundamentals of Engineering (FE) CIVIL Exam Specification mentions, 9-14 questions may appear in the exam based on Geotechnical Engineering as follows:

A. Geology
B. Index properties and soil classifications
C. Phase relations (air-water-solid)
D. Laboratory and field tests
E. Effective stress (buoyancy)
F. Stability of retaining walls (e.g., active pressure/passive pressure)
G. Shear strength
H. Bearing capacity (cohesive and non-cohesive)
I. Foundation types (e.g., spread footings, deep foundations, wall footings, mats)
J. Consolidation and differential settlement
K. Seepage/flow nets
L. Slope stability (e.g., fills, embankments, cuts, dams)
M. Soil stabilization (e.g., chemical additives, geosynthetics)
N. Drainage systems
O. Erosion control

Introduction

Soil is defined as the upper layer of earth, typically consisting of a mixture of organic remains, rock particles, air, and water. The particles are the result of weathering (disintegration and decomposition) of rocks and decay of vegetation. By the way, rock can be of several types such igneous, metamorphic, etc. The soil origin and formation can be summarized as follows:

1. Residual Soils- Soil deposited in place after being weathered from parent rocks
2. Transported Soils – Transported soil from one place to another. It can be of 4 types.
    a. *Gravity Deposits.* Gravity deposits are soil deposits transported by the effect of gravity.
    b. *Alluvial Deposits.* Transported by moving water near rivers.
    c. *Glacial Deposits.* Glacial deposits result of the action of glaciers, enormous sheets of ice.

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d. *Wind Deposits.* Wind deposits (also known as *aeolian deposits*) obviously have wind as the transporting agent.

Broadly, soil can be classified as – coarse, fine, and organic. Organic soil is black in color, often has unpleasant odor, and has a very minimal or negligible load carrying capacity. Hence, organic soil is not usable in construction. Now, our topic is coarse soil. However, fine soil is also discussed very briefly to understand the difference between coarse and fine soils. Soils may be divided into several categories based on the particle size. This is mentioned below:

1. *Boulder – Greater than 12 inch (300 mm)*
2. *Cobbles – Greater than 3 inch (75 mm); less than 12 inch (300 mm)*
3. Coarse-grained or *granular* soils
   a) *Gravel – greater than 4.75 mm (#4 sieve); less than 3 inch (75 mm)*
      i. *Coarse Gravel – 75 mm to 19 mm*
      ii. *Fine Gravel – 19 mm to 4.75 mm*
   b) *Sand – greater than* 0.075 mm (# 200 sieve); smaller than 4.75 mm *(#4 sieve)*
      i. *Coarse Sand – 4.75 mm to 2.0 mm (Sieve # 4 to 10)*
      ii. *Medium Sand – 2.0 mm to 0.425 mm (Sieve # 10 to 40)*
      iii. *Fine Sand – 0.425 mm to 0.075 mm (Sieve # 40 to 200)*
4. *Fines - Smaller than* 0.075 mm (# 200 sieve); no minimum size
   a) *Silt – Cohesionless*
   b) *Clay – Cohesive (sieving cannot separate clay)*

Fine soils may be separated into two categories based on physical property:

1. *Cohesionless – Gravel, Sand, and Silt*
2. *Cohesive - Clay*

Problem 12.1

Which of the following soils has the cohesive property?

A. Silt B. Clay C. Sand D. gravel

Solution: B

Only clay has the cohesive or putty-like (sticky) property.

Phase Relationships

A soil mass consists of solid, water, and air as shown below. All the three parts occupy some volumes and weights although the weight of air is neglected.

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Soil Compaction

Compaction means densification of soil by applying compactive load on soil. It removes the air from the soil and rearrange the soil particles. The degree of compactness or the relative density (D_r) can be determined as follows:

$$D_r = \frac{e_{max} - e}{e_{max} - e_{min}}(100)\%$$

$$D_r = \frac{\left[\gamma_{D(field)} - \gamma_{D(min)}\right]\gamma_{D(max)}}{\left[\gamma_{D(max)} - \gamma_{D(min)}\right]\gamma_{D(field)}}(100)\%$$

Relative compaction (RC) means the current compaction level compared to the maximum compaction possible. For any construction job, RC is specified by the design agency, typically greater than 80%, varies depending on the job type. It is determined as follows:

$$RC = \frac{\gamma_{D(field)}}{\gamma_{D(max)}}(100)\%$$

Problem 12.2

The unit weight of a 0.12- m^3 soil-solid sample is 26 kN/m^3. The specific gravity of the soil is most nearly:

A. 2.4 B. 2.52 C. 2.65 D. 2.72

Solution: C

$$G_S = \left(\frac{W_S}{V_S}\right)/\gamma_W = \gamma_s/\gamma_W = 26/9.81 = 2.65$$

Problem 12.3

If a soil sample is kept on drying days after days, which of the following would not change?

A. Moist unit weight
B. Dry unit weight
C. Saturated unit weight
D. Submerged unit weight

Solution: B

Dry unit of any soil does not change even drying or wetting.

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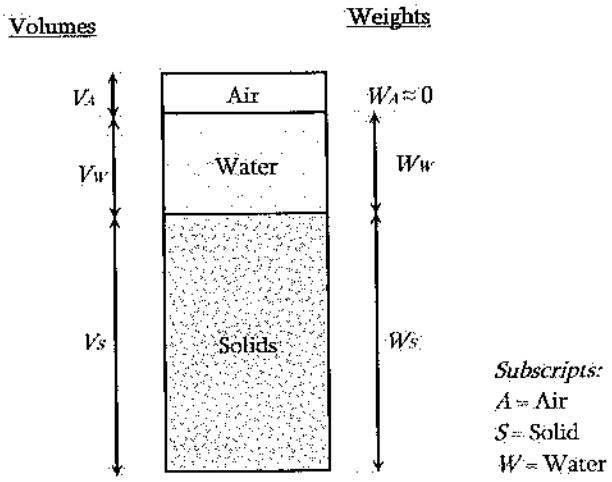


Fig. Phase diagram of a soil

Based on the above diagram, the following formulations can be made:

Volume of voids: $V_V = V_A + V_W$

Total unit weight: $\gamma = W/V$

Saturated unit weight: $\gamma_{sat} = \dfrac{(G_S + e)\gamma_W}{(1+e)}$

Effective (submerged) unit weight: $\gamma' = \gamma_{sat} - \gamma_w$

Unit weight of soil solids: $\gamma_S = \dfrac{W_S}{V_S}$

Dry unit weight: $\gamma_D = \dfrac{W_S}{V}$

Water content (%): $w = \left(\dfrac{W_W}{W_S}\right)$x100

Specific gravity of soil solids: $G_S = \dfrac{\left(\dfrac{W_S}{V_S}\right)}{\gamma_W} = \dfrac{\gamma_S}{\gamma_W}$

where, γ_s is the unit weight of soil solid, and γ_w is the unit weight of water. The γ_w is used as 9.81 kN/m^3 or 62.4 pcf if temperature is not a concern.

Void ratio: $e = \dfrac{V_V}{V_S}$

Porosity: $n = \dfrac{V_V}{V} = \dfrac{e}{1+e}$

Degree of saturation (%): $S = \left(\dfrac{V_W}{V_V}\right)$x100 ; $S = \dfrac{wG_S}{e}$

Make sure that you know the differences between γ_d and γ_s, and S and w.

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Problem 12.7

The saturated unit weight of a soil is 20 kN/m³. The submerged (effective) unit weight of the soil is most nearly:

A. 9.81 kN/m³
B. 19.21 kN/m³
C. 20.0 kN/m³
D. 10.2 kN/m³

Solution: D

$$\gamma' = \gamma_{sat} - \gamma_w = 20 - 9.81 = 10.2 \text{ kN/m}^3$$

Problem 12.8

If the void ratio of a soil is 1.1, the porosity of that soil is most nearly:

A. 0.32 B. 0.42 C. 0.52 D. 0.62

Solution: C

$$n = \frac{e}{1+e} = \frac{1.1}{1+1.1} = 0.52$$

Problem 12.9

A proposed dam will contain 4,000,000 m³ of earth. Soil to be taken from a borrow pit will be compacted to a void ratio of 0.70. The void ratio of the soil in the borrow pit is 1.10. The volume of soil (m³) that must be excavated from the borrow pit is most nearly:

A. 4,250,000
B. 4,950,000
C. 5,550,000
D. 3,250,000

Solution: B

For dam

$$\frac{V_V}{V_S} = 0.70 \quad \Rightarrow V_V = 0.7V_S$$

Again, $V_V + V_S = 4{,}000{,}000$

$\Rightarrow 0.7V_S + V_S = 4{,}000{,}000$

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Problem 12.4

A chunk of moist soil weighs 90 lb with a volume of 0.82 ft³. After oven drying, the weight decreases to 39 lb. The unit weight of the soil is most nearly:

A. 90 pcf B. 100 pcf C. 110 pcf D. 115 pcf

Solution: C

Unit weight means moist unit weight; $\gamma = \frac{W}{V} = \frac{90\ \text{lb}}{0.82\ \text{ft}^3} = 109.8\ \text{pcf} \approx 110\ \text{pcf}$

Problem 12.5

A chunk of moist soil weighs 49 lb with a volume of 0.46 ft³. After oven drying, the weight decreases to 38 lb. The specific gravity and void ratio of soil are 2.64 and 1.1, respectively. The degree of saturation of the soil is most nearly:

A. 39%
B. 49%
C. 59%
D. 69%

Solution: D

$$w = \frac{W_W}{W_S}100\% = \frac{W - W_S}{W_S}100\% = \frac{49-38}{38}100\% = 28.9\% = 0.289$$

$$Se = wG_S$$

$$S = \frac{wG_S}{e} = \frac{0.289(2.64)}{1.1} = 0.69 = 69\%$$

Problem 12.6

The void ratio and specific gravity of a soil are 0.9 and 2.7 respectively. The saturated unit weight of the soil is most nearly:

A. 98 pcf B. 108 pcf C. 118 pcf D. 122 pcf

Solution: C

$$\gamma_{sat} = \frac{(G_S + e)\gamma_W}{(1+e)} = \frac{(2.7+0.9)62.4}{1+0.9} = 118\ \text{pcf}$$

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Again, $V_V + V_S = 1$

$\Rightarrow$ $0.8V_S + V_S = 1$

$\Rightarrow$ $V_S = 0.56\ ft^3$ and $V_V = 0.44\ ft^3$

Again, $\frac{V_W}{V_V} = 0.35$

$\Rightarrow$ $V_W = 0.35\ V_V = 0.35\ (0.44) = 0.154\ ft^3$

To fully saturate, V_W and V_V must be equal; So, (0.44 - 0.154) = 0.286 ft^3 of water must be added for each ft^3 of water.

Problem 12.12

A soil sample with void ratio of 0.9 can have the densest void ratio of 0.7 and the loosest void ratio of 1.2. The relative density of the soil is most nearly:

A. 40% B. 50 % C. 55% D. 60%

Solution: D

$$D_r = \frac{e_{max} - e}{e_{max} - e_{min}}(100)\% = \frac{1.2 - 0.9}{1.2 - 0.7}(100)\% = 60\%$$

Problem 12.13

A soil can be compacted to the maximum dry density of 120 pcf. After field testing, it was found that the compacted dry density of the soil is 110 pcf. The relative compaction of the soil is most nearly:

A. 92% B. 82% C. 72% D. 62%

Solution: A

$$RC = \frac{\gamma_{D(field)}}{\gamma_{D(max)}}(100)\% = \frac{110}{120}(100)\% = 91.67\% \approx 92\%$$

Problem 12.14

A soil can be compacted to the maximum dry density of 120 pcf and the minimum dry density of 95 pcf. After field testing, it was found that the compacted dry density of the soil is 105 pcf. The relative density of the soil is most nearly:

A. 41% B. 46% C. 49% D. 53.2%

Solution: B

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V_s = 2,352,941 m³ (This solid volume is also true for borrow pit)

For borrow pit

$$\frac{V_V}{V_S} = 1.10 \Rightarrow V_V = 1.1V_S$$

Again, $V_V + V_S = V$

$\Rightarrow 1.1V_S + V_S = V$

$V = 2.1\ V_s = 2.1\ (2{,}352{,}941) = 4{,}941{,}176\ m^3 \approx 4{,}95{,}000\ m^3$

Problem 12.10

A soil sample with unit weight of 130 pcf and water content of 15% was partially dried to a unit weight of 120 pcf without changing its void ratio. Its current water content is most nearly:

A. 3.9% B. 4.9% C. 5.9% D. 6.2%

Solution: D

Let's work with 1 ft³ of soil.

Before drying

$$\frac{W_W}{W_S} = 0.15$$

So, $W_W = 0.15\ W_S$

Again, $W_W + W_S = 130$ lb

$0.15W_S + W_S = 130$

$W_S = 113$ lb

After drying

Drying reduces the weight of water by: 130 - 120=10 lb

Weight of water before drying = 130 – 113 = 17 lb

Weight of water after drying = 17 – 10 = 7 lb

New water content: $\frac{W_W}{W_S}(100)\% = \frac{7}{113}(100)\% = 6.2\%$

Problem 12.11

A soil sample with void ratio of 0.8 is 35% saturated. The amount of water is to be added to fully saturate the soil is most nearly:

A. 0.154 ft³
B. 0.44 ft³
C. 0.14 ft³
D. 0.286 ft³

Solution: D

Let's work with 1 ft³ of soil.

$$\frac{V_V}{V_S} = 0.80 \Rightarrow V_V = 0.8V_S$$

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$$D_r = \frac{\left[\gamma_{D(field)} - \gamma_{D(min)}\right]\gamma_{D(max)}}{\left[\gamma_{D(max)} - \gamma_{D(min)}\right]\gamma_{D(field)}}(100)\% = \frac{(105-95)120}{(120-95)105}(100)\% = 45.7\% \approx 46\%$$

Problems, 12.15 to 12.21 are based on the following phase diagram. So, read the diagram carefully.

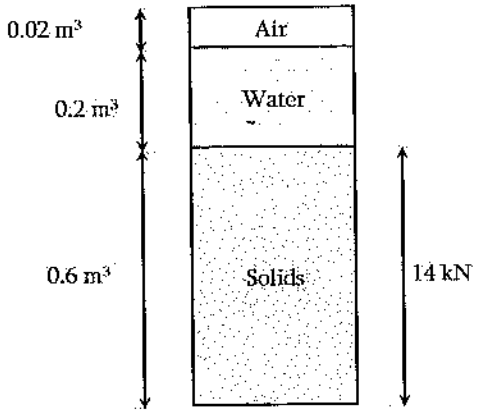


Fig. Phase diagram for problems 12.15 to 12.21

Problem 12.15

The weight of the water for the soil whose phase diagram is shown above is most nearly:

A. 0.2 kN B. 1.96 kN C. 1.69 kN D. 2.96 kN

Solution: B

$$W_W = V_W \gamma_W = 0.2\ \text{m}^3\ (9.81\ \text{kN/m}^3) = 1.96\ \text{kN}$$

Problem 12.16

The water content of the soil whose phase diagram is shown above is most nearly:

A. 8% B. 10% C. 12% D. 14%

Solution: D

$$W_W = V_W \gamma_W = 0.2\ \text{m}^3\ (9.81\ \text{kN/m}^3) = 1.96\ \text{kN}$$

$$w = \frac{W_W}{W_S}(100)\% = \frac{1.96}{14}(100)\% = 14\%$$

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Problem 12.17

The porosity of the soil whose phase diagram is shown above is most nearly:

A. 0.41 B. 0.37 C. 0.27 D. 0.17

Solution: C

$$n = \frac{V_V}{V} = \frac{V_W + V_A}{V_V + V_W + V_A} = \frac{0.2 + 0.02}{0.6 + 0.2 + 0.02} = 0.268 \approx 0.27$$

Problem 12.18

The void ratio of the soil whose phase diagram is shown above is most nearly:

A. 0.32 B. 0.34 C. 0.37 D. 0.40

Solution: C

$$e = \frac{V_V}{V_S} = \frac{V_W + V_A}{V_S} = \frac{0.2 + 0.02}{0.6} = 0.37$$

Problem 12.19

The degree of saturation of the soil whose phase diagram is shown above is most nearly:

A. 86% B. 88% C. 91% D. 94%

Solution: C

$$S = \frac{V_W}{V_V} = \frac{V_W}{V_V} = \frac{0.2}{0.22} = 0.91 = 91\%$$

Problem 12.20

The specific gravity of the soil solids whose phase diagram is shown above is most nearly:

A. 2.32 B. 2.38 C. 2.442 D. 2.48

Solution: B

$$G_S = \frac{W_S}{V_S} \frac{1}{\gamma_W} = \frac{14}{0.6}\left(\frac{1}{9.81}\right) = 2.38$$

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Problem 12.21

The unit weight of the soil solid in kN/m³ whose phase diagram is shown above is most nearly:

A. 9.81
B. 15.63
C. 19.46
D. 20.32

Solution: C

Unit weight means the moist unit weight; Weight of air is neglected.

$$\gamma = \frac{W}{V} = \frac{W_S + W_W}{V_S + V_W + V_A} = \frac{14 + 1.96}{0.6 + 0.2 + 0.02} = 19.46 \text{ kN/m}^3$$

Index Property and Soil Classification

Grain-Size Analysis



In the previous section, the soil type, size, and its origin were discussed. Again to remind you, any soil larger than 0.075 mm (#200 sieve size) is course grained. Coarse gain soil includes sand, gravel, boulder, and so on. Any soil smaller than 0.075 mm or passing #200 sieve is silt or clay depending on the cohesive property. Soil is sieved using different sieves staking the larger size above as shown below. While soil is passed through a sieve, soil particles smaller than the opening size of the sieve passes through, whereas those bigger than the opening size is to be retained. After shaking, the masses retained in different sieves are measured and recorded carefully. Then, a graph 'Cumulative % Passing' versus 'Sieve Size' is plotted.

From this curve, two parameters, Coefficient of Curvature (C_c) and Coefficient of Uniformity (C_u) are determined as follows:

$$C_u = \frac{D_{60}}{D_{10}} \qquad \text{and} \qquad C_c = \frac{(D_{30})^2}{(D_{60})(D_{10})}$$

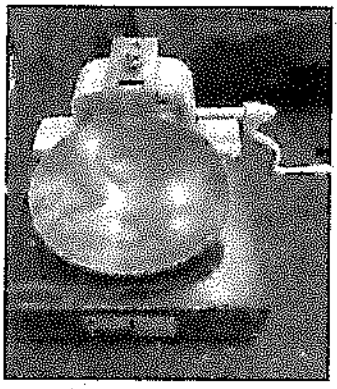
D_{10} = size corresponding to cumulative 10% passing (also called *effective size)*
D_{30} = size corresponding to cumulative 30% passing
D_{60} = size corresponding to cumulative 60% passing

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Table 12.1. Different sieves and their sizes

Sieve No.	Size (mm)
4	4.75
10	2
20	0.85
40	0.425
100	0.15
200	0.075

Atterberg Limits



For fine soil, liquid limit (LL) and plastic limit (PL) are of interest. These two limits are also called Atterberg limits. The LL is the water content at the plastic-liquid boundary. It is determined using casagrande device shown below:

The basic steps of determining LL are mentioned below:

1) Take 200 grams of soil passing #40 sieve
2) Mix water and let it cure
3) Spread the soil in the cup to a depth of 10 mm at the deepest point of the device
4) Form a groove in the soil by drawing the tool perpendicular to the surface
5) Lift and drop the cup 2 drops per second
6) Record the number of blows required to close the groove
7) After several trails, determine the water content needing 25 blows to close the groove

PL is the water content at the plastic-semisolid boundary. It is determining by rolling a sample to make a thread of 3.2 mm diameter. The basic steps are mentioned below:

1) Take about 20 grams of soil passing #40 sieve
2) Thoroughly mix small amount of water and cure it for at least 16 minutes
3) Take 2 grams of wet soil
4) Roll by hand between palm and glass plate
5) If it breaks before reaching the thread diameter of 3.2 mm, mix more water and repeat steps (1) through (4).
6) If the diameter can be reached, break it into several pieces and reroll.

Plasticity Index is the difference between the LL and the PL. If LL is equal or greater than 50%, the soil is called high plastic soil; otherwise it is low plastic soil.

Soil Classification Systems

Unified Soil Classification System (USCS) is used for general engineering purpose other than roads and highways. The basic steps to classify soil using this system are mentioned below:

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Step 1: Collect the Grain-Size Curve (C_c and C_u may not be required)
Step 2: Collect the LL and PL of the soil
Step 3: Use the USCS chart from the NCEES FE Ref. Handbook

AASHTO soil classification system is usually used for roads and highway. Please read the AASHTO Soil Classification Chart to classify the soil and then use the following equation to determine the group index (GI). Remember, GI can be positive round number only. If your value is negative use zero in the parenthesis.

$$GI = (F-32)[0.2+0.005(LL-40)]+0.01(F-15)(PI-10)$$

F = % passing #200 sieve
$PI = LL - PL$

Problem 12.22

Which one of the following values can be the group index (GI) of a soil?

A. 7.5 B. -2 C. 10 D. 2.5

Solution: C

GI can be positive round number only. If your value is negative use zero.

Problem 12.23

The highest sieve size among the following sieves is most nearly:

A. #4 B. #40 C. #100 D. #200

Solution: A

4 has the highest sieve size. In other words, #4 means dividing 1.0 inch into 4 parts by the wire fabric; #200 means dividing 1.0 inch into 200 parts by the wire fabric.

Problem 12.24

For a soil sample, 40% of the material passes through a #200 sieve. The type of the soil is most nearly:

A. A-1 B. A-2 C. A-3 D. A-4

Solution: D

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From the NCEES FE Ref. Handbook, 35% or more of the material passes through a #200 sieve falls in A-4 to A-7.

Problem 12.25

For a gravel soil, $C_u = 5$ and $C_c = 2$. Which of the following can be the type of the soil?

A. SW B. GW C. GP D. GS

Solution: B

From the NCEES FE Ref. Handbook, for GW, $Cu \geq 4$ and $1 \leq Cc \leq 3$.

Problem 12.26

Which of the following soil types is the best (highest strength) for roads and highway?

A. A-8 B. A-7 C. A-3 D. A-1

Solution: D

A-1 is the best soil for roads and highway. Usually higher the number, worse the soil is, except for A-2 and A-3. A-8 is the worst soil for engineering.

Problem 12.27

Effective size of soil particle means what?

A. Median size of a soil group
B. Size at which 10% soil passes
C. Size at which 30% soil passes
D. Size at which 60% soil passes

Solution: B

Size at which 10% soil is finer (or, 10% passing) is called the effective size of soil.

Problem 12.28

For a soil, the size corresponding to 60% passing is 10 mm and the size corresponding to 10% passing is 2 mm. The uniformity coefficient of that soil is most nearly:

A. 0.2 B. 5 C. 20 D. Cannot be determined

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Solution: B

$D_{60} = 10\,\text{mm}$, $D_{10} = 2\,\text{mm}$, $C_u = \dfrac{D_{60}}{D_{10}} = \dfrac{10\,\text{mm}}{2\,\text{mm}} = 5$. Make sure you know the difference between C_c and C_u.

Problem 12.29

In a LL test, 20 blows are required to close the groove at 20% moisture content, and 30 blows are required to close the groove at 16% moisture content. The LL of the test soil is most nearly:

A. 18 B. 14 C. 20 D. 10

Solution: A

By interpolation, the water content which needs 25 blows to close the groove is 18%. Even, by visual inspection, it can be found that there is no other logical answer.

Problem 12.30

In a plastic limit test, the required/desired thread diameter is most nearly:

A. 3.2 mm B. 2.3 mm C. 0.5 in D. 0.5 mm

Solution: A

The required thread diameter is 3.2 mm, or 1/8 inch.

Problem 12.31

Depending on the water content, soil may be at liquid state, plastic state, and so on. For this statement, which of the following options is correct?

A. True, higher the water content softer the soil is
B. True, lower the water content softer the soil is
C. False
D. None of the above

Solution: A

Depending on the water content, soil may be at liquid state, plastic state, semi-solid and solid.

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Problem 12.32

USCS soil classification is used in general engineering practice. True or False?

A. True
B. False

Solution: A

USCS soil classification is used in general engineering practice. AASHTO soil classification is used in roads and highway.

Problem 12.33

Group index in AASHTO soil classification system cannot be -

A. Negative B. Rounded C. Fraction D. Positive

Solution: A, C

Group index in AASHTO soil classification system cannot be negative and fraction. During the FE exam, if multiple answer is possible, choose the best one or the first one.

Problem 12.34

Median size of soil particles means:

A. Median size of a soil group
B. Size at which 50% soil is finer
C. Size at which 30% soil is finer
D. Size at which 60% soil is finer

Solution: B

Size at which 50% soil is finer (or, 50% passing) is called the median size of soil.

Problem 12.35

More than 50% of a soil is retained in a #200 sieve. The soil is most nearly:

A. Sand-Silt
B. Gravel-Clay
C. Sand-Gravel
D. Fine Soil

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Solution: C

See the USCS soil classification chart.

Problem 12.36

In a clay soil, the LL is 30. The soil is most nearly:

A. High-liquid limit
B. High plastic
C. Low plastic
D. None of the above

Solution: C

Low-plastic soil is that which has less than LL of 50.

Problem 12.37

Liquid limit is determined using-

A. the Casagrande Device
B. threading a wet soil
C. oven method
D. None of the above

Solution: A

Liquid limit is determined using the Casagrande Device.

Problem 12.38

Plastic limit is determined using -

A. the Casagrande Device
B. threading a soil
C. oven method
D. None of the above

Solution: B

Plastic limit is determined using threading a soil.

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Problem 12.39

"M" means what in soil classification system?

A. Median soil
B. Moisture
C. Silt
D. Moderate Plastic Soil

Solution: C

Permeability and Seepage

Hydraulic conductivity (also called, coefficient of permeability), k is a measure of flow of water through a porous soil. In the laboratory, two types of tests are performed to determine the permeability of soil (k): Constant Head Permeability and Falling Head Permeability.

From constant head test:

$$k = \frac{Q}{iAt_t} = \frac{QL}{hAt_t}$$

$$i = \frac{dh}{dL}$$

From falling head test:

$$k = 2.303 \frac{aL}{At_t} \log_{10}\left(\frac{h_1}{h_2}\right)$$

where:

Q = total quantity of water
A = cross-sectional area of test specimen perpendicular to flow
a = cross-sectional area of reservoir tube
t_t = elapsed time
h_1 = head at time $t = 0$
h_2 = head at time $t = t_t$
L = length of soil column

Discharge velocity, $v = ki$

Flow Nets: The flow rate can also be determined by drawing a flow net. The following equation is used to determine the flow rate using a flow net:

$$q = \frac{khN_f}{N_d}$$

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q = flow per unit time
N_f = number of flow channels
N_d = number of equipotential drops
H = total hydraulic head differential

Factor of safety against seepage liquefaction can be determined as follows:

$$FS = \frac{i_c}{i_e} = \frac{\text{Critical Gradient}}{\text{Seepage Exit Gradient}}$$

where: $i_c = \dfrac{\gamma - \gamma_w}{\gamma_w}$

Problem 12.40

A water flow is flowing through a soil whose length is 2 m. If the hydraulic head difference is 0.2 m, the hydraulic gradient is most nearly:

A. 0.1 B. 0.4 C. 10 D. None of the above

Solution: A

$$i = \frac{h}{L} = \frac{0.2m}{2m} = 0.1$$

Problem 12.41

Water flows through a soil whose cross-sectional area is 0.15 m^2. The hydraulic gradient is 0.12 and the coefficient of permeability is 5×10^{-4} m/s. The flow rate (m^3/s) is most nearly:

A. 9×10^{-5}
B. 9×10^{-6}
C. 5×10^{-9}
D. None of the above

Solution: B

$$q = kiA = (5\times10^{-4}\ \text{m/s})(0.12)(0.15\ \text{m}^2) = 9\times10^{-6}\ \ \text{m}^3/\text{s}$$

Problem 12.42

Water flows through a soil whose cross-sectional area is 0.15 m^2. The hydraulic gradient is 0.12 and the coefficient of permeability is 5×10^{-4} m/s. If water flows for 10 minutes, the total flow (m^3) is most nearly:

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A. 5.4×10^{-3}
B. 6.2×10^{-3}
C. 4.5×10^{-3}
D. None of the above

Solution: A

$$Q = kiAt_E = (5\times10^{-4}\ \text{m/s})(0.12)(0.15\ \text{m}^2)(600\ s) = 5.4\times10^{-3}\ \text{m}^3$$

Problem 12.43

In a constant head permeability test, a total of 500 m^3 of water is discharged in 1 minute. The length and the diameter of the soil sample are 10 cm and 8 cm respectively. If the head difference is 6 cm, the permeability (m/s) of that soil is most nearly:

A. 2,435
B. 2,654
C. 2,763
D. None of the above

Solution: C

$$k = \frac{Q}{iAt_E} = \frac{QL}{hAt_E} = \frac{(500\,\text{m}^3)(0.1\,\text{m})}{(0.06\,\text{m})\left(\frac{\pi(0.08)^2}{4}\right)(60\,\text{s})} = 2,763.11\ \ \text{m/s}$$

Problem 12.44

If the unit weight of a moist soil is 100 pcf, its critical gradient is most nearly:

A. 0.12
B. 0.60
C. 0.84
D. None of the above

Solution: B

$$i_c = \frac{\gamma - \gamma_w}{\gamma_w} = \frac{100\ \text{pcf} - 62.4\ \text{pcf}}{62.4\ \text{pcf}} = 0.60$$

Problem 12.45

The seepage exit gradient of a soil with a moist unit of 100 pcf is 0.1. The factor of safety against the seepage is most nearly:

A. 5 B. 10 C. 6 D. None of the above

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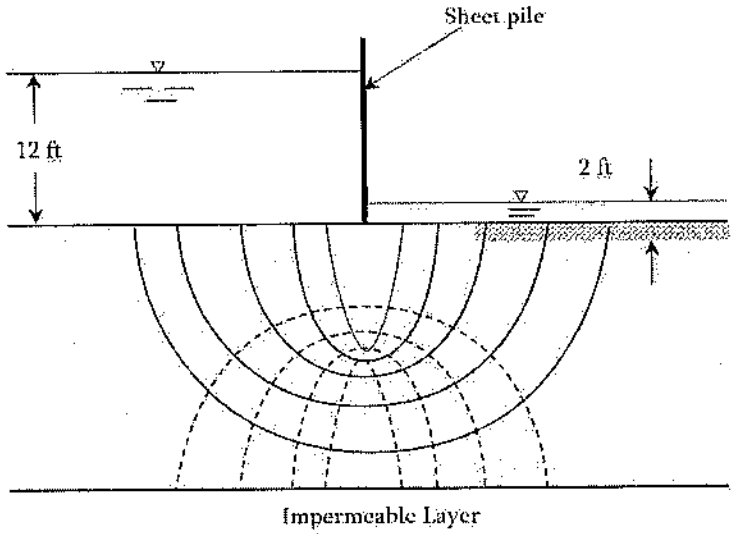
Solution: C

$$i_c = \frac{\gamma - \gamma_w}{\gamma_w} = \frac{100 \text{ pcf} - 62.4 \text{ pcf}}{62.4 \text{ pcf}} = 0.60$$

$$FS = \frac{i_c}{i_E} = \frac{0.6}{0.1} = 6$$

Problem 12.46

For the flow net shown below, what is the total rate of seepage per unit width of sheet pile through the permeable stratum? The coefficient of permeability of the soil is 2×10^{-3} ft/sec.



a) 1.11×10^{-2} ft³/s
b) 1.33×10^{-2} ft³/s
c) 13.3×10^{-2} ft³/s
d) None of the above

Solution: B

$$q = \frac{khN_f}{N_d} = \frac{2 \times 10^{-3} \text{ (ft/s)(10 ft)6}}{9} = 1.33 \times 10^{-2} \text{ ft}^3/\text{s}$$

Problem 12.47

Capillary rise occurs due to -

A. Pore media
B. Surface tension
C. Tendency to rise
D. None of the above

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Solution: B

Capillary rise occurs due to surface tension, and depends on surface tension, radius of the tube, unit weight of the fluid, and the contact angle. As the contact angle is very small, it is very often neglected. Capillary rise can be determined as $z = \frac{2T}{r\gamma}$.

Problem 12.48

Pressure exerted by soil water is called -

A. effective pressure
B. total pressure
C. pore water pressure
D. None of the above

Solution: C

Pressure exerted by soil water is called pore water pressure. Pressure exerted by unsaturated soil or dry soil is called effective pressure. The sum of pore water pressure and the effective pressure is called the total pressure

Problem 12.49

Field permeability test can be conducted in –

A. confined aquifer
B. unconfined aquifer
C. both confined and unconfined aquifers
D. None of the above aquifer

Solution: C

Field permeability test can be conducted in both confined and unconfined aquifers. This is why, we have two different equations for determining the field permeability depending on confined or unconfined aquifer.

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Soil Consolidation and Settlement

Settlement in sand or sandy soil is not included in the NCEES FE Ref. Handbook. However, settlement in clay is included and occupies a big space. Settlement in clay is of three types-

1) Immediate Settlement
2) Primary Settlement
   a) Normally consolidated Clay
   b) Over-consolidated Clay
3) Secondary Settlement

Primary consolidation occurs in clay due to expulsion of water with time. It takes several years to settle completely. The other two types (immediate and secondary) are very small and may be neglected. The primary settlement topic is very briefly described here. The primary settlement is calculated using the following *e-logp* curve.

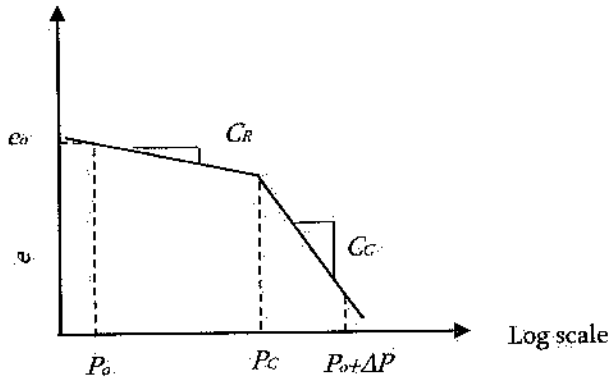


Fig. Soil Consolidation Curve, Pressure (Log_{10} Scale)

e_o = initial void ratio (prior to consolidation)
Δe = change in void ratio
p_o = initial effective consolidation stress, σ'_o
p_c = past maximum consolidation stress, σ'_c
Δp = induced change in consolidation stress at center of consolidating stratum

Compression index

In virgin compression range: $C_C = \dfrac{\Delta e}{\Delta \log p}$

By correlation to liquid limit: $C_c = 0.009\,(LL - 10)$

Recompression/Swell index

In recompression range: $C_R = \dfrac{\Delta e}{\Delta \log p}$

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By correlation to compression index, $C_R = \frac{C_C}{6}$

Δp can be calculated as: $\Delta p = I q_s$

I = Stress influence value at center of consolidating stratum
q_s = Applied surface stress causing consolidation

Depending on pressure level, consolidation settlement or change in thickness of clay layer (ΔH) can be calculated as follows:

a) If (p_o and $p_o + \Delta p$) < p_c, then $\Delta H = C_R \left(\frac{H_o}{1+e_o}\right) \log\left(\frac{P_o + \Delta P}{P_o}\right)$

b) If (p_o and $p_o + \Delta p$) > p_c then $\Delta H = C_C \left(\frac{H_o}{1+e_o}\right) \log\left(\frac{P_o + \Delta P}{P_o}\right)$

c) If $p_o < p_c < (p_o + \Delta p)$, then

$$\Delta H = C_R \left(\frac{H_o}{1+e_o}\right) \log\left(\frac{P_C}{P_O}\right) + C_C \left(\frac{H_o}{1+e_o}\right) \log\left(\frac{P_o + \Delta P}{P_C}\right)$$

Ultimate consolidation settlement in soil layer: $S_{ULT} = \varepsilon_V H_s$

H_s = thickness of soil layer: $\varepsilon_V = \frac{\Delta e_{TOT}}{(1+e_o)}$

where: Δe_{TOT} = total change in void ratio due to recompression and virgin compression

Approximate settlement (at time $t = t_c$)

$S_t = U_{AV} S_{ULT}$
U_{AV} = average degree of consolidation
t_c = elapsed time since application of consolidation load

The time required to settle certain amount settlement can also be determined. The steps are as follows:

Step 1. Find the Time Factor (T_v) for any degree of settlement. Few factors are mentioned below.

Step 2. Collect the coefficient of consolidation (C_v); usually it is determined by laboratory testing.

Step 3. Find the hydraulic drainage path (H_d). If the concern clay layer is underlain by solid bedrock which is impervious, then the thickness of the clay layer is the drainage path. If the concern clay layer has porous layers on both above and below the clay layer, then the hydraulic drainage path is half of the clay layer's thickness.

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Step 4. Calculate the time required to settle, $t = \frac{T_v}{C_v}\left(H_{dr}^2\right)$

Table 12.2 Variation of time factor with degree of consolidation

U%	T_v
0	0
10	0.00785
20	0.0314
30	0.0707
40	0.126
50	0.197
60	0.286
70	0.403
80	0.567
90	0.848
99	1.781
100	∞

Problem 12.50

The Liquid Limit of a soil is 30. The Recompression Index (C_R) of that soil is most nearly:

A. 0.18 B. 0.03 C. 0.33 D. 0.06

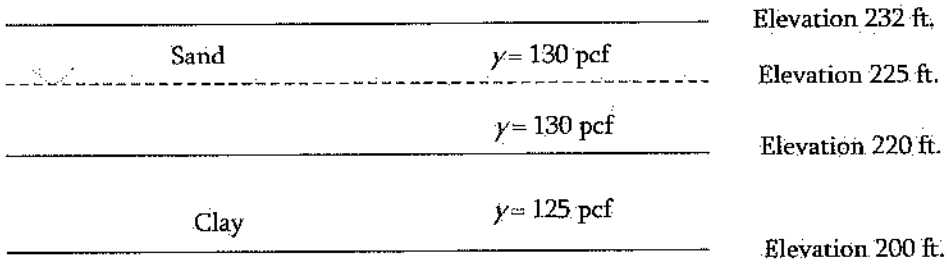
Solution: B

$$C_c = 0.009\,(LL - 10) = 0.009\,(30 - 10) = 0.18$$

$$C_R = \frac{C_c}{6} = \frac{0.18}{6} = 0.03$$

Problem 12.51

The effective overburden pressure at the mid-height of the clay layer for the soil shown below is most nearly:



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A. 1,274 psf
B. 1,474 psf
C. 1,674 psf
D. 1,874 psf

Solution: D

$$P_o = (130\,\text{pcf})(7\,\text{ft}) + (130 - 62.4\,\text{pcf})(5\,\text{ft}) + (125 - 62.4\,\text{pcf})(10\,\text{ft}) = 1{,}874\,\text{psf}$$

Problem 12.52

The compression index of a soil is 0.3. The increase in vertical stress caused by a foundation on the clay is twice of the overburden stress. The decrease in void ratio of this clay soil due to this increase in stress is most nearly:

A. 0.08
B. 0.12
C. 0.014
D. 0.022

Solution: C

The increase in vertical stress is twice. This means the final stress is three times of the initial stress.

$$C_c = \frac{\Delta e}{\Delta \log P}$$

$$\Delta e = C_c \log \frac{P_2}{P_1} = (0.03) \log \left(\frac{3P_1}{P_1} \right) = 0.014$$

Problem 12.53

A 10 ft. layer has a void ratio of 1.1. Upon increase in vertical stress by a building, the void ratio decreases by 0.02. The settlement of the clay layer is most nearly:

A. 0.84 in
B. 0.94 in
C. 1.04 in
D. 1.14 in

Solution: D

$$\Delta H = H \frac{\Delta e}{1 + e} = (10\,\text{ft}) \left(\frac{0.02}{1 + 1.1} \right) = 0.095\,\text{ft} = 1.14\,\text{inch}$$

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A foundation is to be constructed at a site whose soil profile is shown below. The initial void ratio of the over-consolidated clay layer is 0.9. The compression index of the clay layer is 0.3 and the swell index is 0.05. The consolidation pressure due to the foundation is calculated to be 60 kPa at the mid-height of the clay layer.

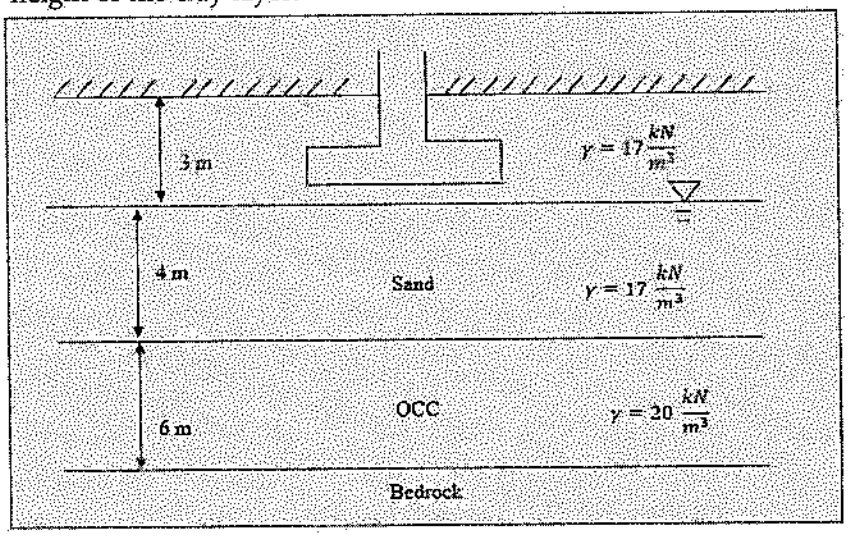


Fig. Soil profile for next Problems, 12.54-12.58

Problem 12.54

For the above soil, the effective over-burden pressure at the mid-height of the clay layer is most nearly:

A. 90 kPa B. 100 kPa C. 110 kPa D. 120 kPa

Solution: C

$$P_o = \left(17 \frac{kN}{m^3}\right)(3m) + \left(17 - 9.81 \frac{kN}{m^3}\right)(4m) + \left(20 - 9.81 \frac{kN}{m^3}\right)(3m) = 110.33 kPa$$

Problem 12.55

The total effective pressure at the mid-height of the clay layer is most nearly:

A. 110 kPa B. 170 kPa C. 230 kPa D. 420 kPa

Solution: B

$$P = P_o + \Delta P = 110.33 + 60 = 170.33 \text{ kPa}$$

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Problem 12.56

If the over-consolidated pressure of the clay layer is 130 kPa, the primary consolidation settlement for the clay layer is most nearly:

A. 8.2 cm B. 10.2 cm C. 12.2 cm D. 14.2 cm

Solution: C

$P_o = 110.33$ kPa; $P = P_o + \Delta P = 170.33$ kPa; $P_C = 130$ kPa

$P_o + \Delta P > P_C > P_O$

Consolidation settlement, $\Delta H = C_R\left(\frac{H_o}{1+e_o}\right)\log\left(\frac{P_C}{P_O}\right) + C_C\left(\frac{H_o}{1+e_o}\right)\log\left(\frac{P_o+\Delta P}{P_C}\right)$

$$\Delta H = (0.05)\left(\frac{6}{1+0.9}\right)\log\left(\frac{130}{110.33}\right) + (0.3)\left(\frac{6}{1+0.9}\right)\log\left(\frac{170.33}{130}\right)$$

$$= 0.0112 + 0.111 \quad = 0.122 \text{ m} = 12.2 \text{ cm}$$

Problem 12.57

For the above problem, if the over-consolidation pressure is 100 kPa, the primary consolidation settlement in cm for the clay layer is most nearly:

A. 18 B. 20 C. 1.8 D. 0.18

Solution: A

$P_o = 110.33$ kPa; $P = P_o + \Delta P = 170.33$ kPa; $P_C = 100$ kPa

$P_o > P_C$;

$P_o + \Delta P > P_C$

Both P_o and $P_o+\Delta P$ are greater than P_C. So, consolidation settlement,

$$\Delta H = C_C\left(\frac{H_o}{1+e_o}\right)\log\left(\frac{P_o+\Delta P}{P_o}\right)$$

$$= (0.3)\left(\frac{6}{1+0.9}\right)\log\left(\frac{170.33}{110.33}\right) = 0.179 \text{ m} = 17.9 \text{ cm} \approx 18 \text{ cm}$$

Problem 12.58

For the above problem, if the over-consolidation pressure is 200 kPa, the primary consolidation settlement (cm) for the clay layer is most nearly:

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A. 28 B. 3 C. 30 D. 0.03

Solution: B

$P_o = 110.33$ kPa ; $P = P_o + \Delta P = 170.33$ kPa ; $P_C = 200$ kPa

$P_o < P_C$; $P_o + \Delta P < P_C$

Both P_o and $P_o+\Delta P$ are smaller than P_C. So, consolidation settlement,

$$\Delta H = C_R\left(\frac{H_o}{1+e_o}\right)\log\left(\frac{P_o+\Delta P}{P_o}\right)$$

$$\Delta H = (0.05)\left(\frac{6}{1+0.9}\right)\log\left(\frac{170.33}{110.33}\right) = 0.03 \text{ m} = 3 \text{ cm}$$

Problem 12.59

Upon constructing a foundation, the total change in void ratio due to re-compression and virgin compression is 0.1. The initial void ratio of the soil is 1.3. If the thickness of the soil later is 5 m, the ultimate consolidation settlement (cm) in the soil layer is most nearly:

A. 21.7
B. 24.1
C. 28.8
D. 26.3

Solution: A

$$S_{ult} = \varepsilon_v H_S = \left(\frac{\Delta e_{TOT}}{1+e_o}\right)H_S = \left(\frac{0.1}{1+1.3}\right)(5 \text{ m}) = 0.217 \text{ m} = 21.7 \text{ cm}$$

Problem 12.60

The coefficient of consolidation (C_v) for a clay is 3.1 x 10^{-3} in^2/min. If the clay layer is underlain by impermeable bed rock, most nearly how much time will it take to settle 10% of the layer probable to total primary settlement? The clay layer is 8 feet.

A. 10 d B. 12 d C. 14.2 d D. 16.2 d

Solution: D

$T_v = 0.00785$ for $U = 10\%$

$$t = \frac{T_v}{c_v}\left(H_{dr}^2\right) = \frac{0.00785}{3.1\text{x}10^{-3}\ \frac{\text{in}^2}{\text{min}}}(96 \text{ in})^2 = 16.2 \text{ days for } 10\% \text{ settlement}$$

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Problem 12.61

The coefficient of consolidation (C_v) for a clay is 3.1 x 10^{-3} in²/min. If the clay layer is underlain by permeable sand, most nearly how much time will it take to settle 10% of the layer probable to total primary settlement? The clay layer is 8 feet.

A. 4.1 d B. 5.2 d C. 2.9 d D. 1.2 months

Solution: A

$$T_v = 0.00785 \text{ for } U = 10\%$$

$$t = \frac{T_v}{c_v}\left(H_{dr}^2\right) = \frac{0.00785}{3.1\text{x}10^{-3}\ \frac{\text{in}^2}{\text{min}}}\left(\frac{96}{2}\ \text{in}\right)^2 = 5834\ \text{min} = 4.1\ \text{days}$$

Problem 12.62

Secondary consolidation occurs in -

A. clay b. sand c. silt d. rock

Solution: A

Primary/secondary consolidation occurs in clay only.

Problem 12.63

Normal-consolidation means what?

A. Soil is too much compacted by roller
B. Soil has past pressure which is less than the current pressure
C. Soil can withstand greater pressure compared to current overburden pressure
D. Too much compaction may destroy the soil

Solution: B

Normal-consolidation means, soil has past pressure which is equal or less than the current pressure. The third option is true for over-consolidation soil. But, it is not the definition.

Problem 12.64

Consolidation settlement is a function of what?

A. Plastic limit

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B. Plasticity Index
C. Shrinkage index
D. Liquid limit

Solution: D

Coefficient of consolidation is a function of Liquid limit. Thus, consolidation settlement is a function of LL.

Problem 12.65

In the past, a sandy soil had 500 kPa overburden pressure. After constructing a foundation, the total pressure is 400 kPa. The soil is most nearly:

A. Sand
B. Normally consolidated clay
C. Over consolidated clay
D. Normally consolidated sand

Solution: A

Normal-consolidation means, soil has the past pressure which is equal or less than the current pressure. But it is for clay. The current soil is sand.

Problem 12.66

Solid bedrock is -

A. porous media B. non-porous C. clay type D. drainage

Solution: B

Problem 12.67

Which one of the following has higher value?

A. Compression index
B. Swell index
C. Both are equal
D. Depends on soil type

Solution: A

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Swell index is always smaller than the Compression index. It is about 1/6 of the Compression Index.

Problem 12.68

Which one of the following has higher value?

A. Primary settlement
B. Secondary settlement
C. Both are almost equal
D. None of the above

Solution: A

Problem 12.69

Which one of the following relationships is true?

A. $C_c = 0.009\ (LL - 10)$
B. $C_c = 0.09\ (LL - 10)$
C. $C_c = 0.009\ (LL - 20)$
D. None of the above

Solution: A

Normal and Shear Stresses in Soil

Two types of soil strengths are of interest in foundation design. The first one is the unconfined compression strength. It is determined by applying load to a cylindrical soil until failure. The failure force over the cross-sectional area of soil is defined as the unconfined compressive strength, q_u. Another soil strength, called shear strength of soil is determined by direct shear test. In this test a soil sample is applied certain normal stress first, and then shear stress is applied until failure. This step is repeated at least twice with a different normal stress. Then, shear stress-normal stress diagram is plotted with a straight line. The slope of the line is called the angle of internal friction (ϕ) and expressed in degree. The shear stress corresponding to zero normal stress is called the cohesion (c). Sand/silt has no cohesion; whereas, clay has almost zero angle of internal friction.

Normal stress: $\sigma_N = \dfrac{P}{A}$

Shear stress: $\tau_F = \dfrac{T}{A}$

Shearing stress at failure: $\tau_F = c + \sigma_N \tan\phi$

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P = normal force
A = cross-sectional area over with force acts
T = shearing force
c = cohesion
ϕ = angle of internal friction

Problem 12.70

In a shear test on sand, a normal stress of 150 kPa was applied. The specimen failed when the shear stress reached to 80 kPa. The sand's angle of internal friction is most nearly:

A. 22°
B. 28°
C. 32°
D. 38°

Solution: B

$$\tau_F = \sigma_N \tan\phi$$

$$\phi = \tan^{-1}\left(\frac{\tau_F}{\sigma_N}\right) = \tan^{-1}\left(\frac{80}{150}\right) = 28^o \text{ (Angle of internal friction)}$$

Problem 12.71

Two shear tests were performed on a clay soil specimen until the sample failed. The data are presented below.

Normal Stress (psf)	Shear Stress (psf)
100	220
400	460

The soil's angle of internal friction is most nearly:

A. 22°
B. 28°
C. 32°
D. 39°

Solution: D

$$\phi = \tan^{-1}\left(\frac{\Delta\tau_F}{\Delta\sigma_N}\right) = \tan^{-1}\left(\frac{460-220}{400-100}\right) = \tan^{-1}\left(\frac{240}{300}\right) = \tan^{-1}(0.8) = 39^o$$

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Problem 12.72

Two direct shear tests were conducted on a soil sample until failure with the following data:

Normal Stress (psf)	Shear Stress (psf)
0	120
100	230

The cohesion (psf) of the dry soil is most nearly:

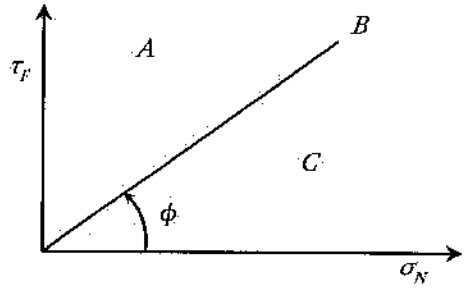
A. 120
B. 240
C. 170
D. Cannot be determined

Solution: A

c is the shear stress when normal stress = 0 ; c =120 psf

Problem 12.73

The shear stress and the normal stress curve of the sand is shown below. Which of the following is true?



A) A: Above failure, not possible point
B: Failure plane
B) C: Failure plane, soil is supposed to fail.
C) A: shear stress for densified condition
C: Below the failure, soil is safe against failure
D) None of the above

Solution: A

A: Above failure, not possible point
B: Failure plane
C: Below the failure, soil is safe against failure

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Effective stress, Pore Water Pressure, and total Pressure

Effective stress means the stress applied by the moist or dry soil itself. The soil can be moist. It can be determined as:

$$P_o = \gamma h$$

P_o = effective stress
γ = Unit weight of soil
h = height of soil above the concern point

For example, the effective stress at 7 ft below a soil with unit weight of 130 pcf is 130 pcf (7 ft) = 780 psf. This soil can be dry or moist. However, if the soil is fully saturated then, the effective stress can be determined as:

$$P_o = \gamma' h$$

γ' = Submerged unit weight of soil = $\gamma - \gamma_w$
γ_w = Unit weight of water (62.4 pcf or 9.81 kN/m^3)

The stress exerted by water is called pore water pressure which can be determined as:

$$U = \gamma_w h$$

The pore water pressure at a point inside a soil is equal vertically and horizontally. The total vertical pressure at a point can be calculated as –

$$P_v = P_o + U$$

Horizontal Stress Profiles and Forces

The vertical stress (P_V) due to the overburden soil is determined by the product of the unit weight of soil and the depth of the soil as described above. The horizontal stress (P_H) is calculated by multiplying earth pressure coefficient (K) if there is no saturated soil layer. Horizontal pressure at a point can be calculated as –

$$P_H = K P_V$$

Earth pressure can be of three types which are discussed here.

Earth Pressure at Rest: This concept is not discussed in the NCEES FE Ref. Handbook.

Active Pressure on Retaining Wall: When soil tries to move, and push the retaining wall, active earth pressure is exerted by soil on the wall. Then, the coefficient, K in the above equation is replaced by K_A.

K_A = Rankine active earth pressure coefficient $= \tan^2\left(45 - \frac{\phi}{2}\right)$

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Passive Pressure on Retaining Wall: When retaining wall tries to move and push the retaining wall, passive earth pressure is exerted by soil on the wall. Then, the coefficient, K in the above equation is replaced by K_P.

K_P = Rankine passive earth pressure coefficient $= \tan^2\left(45 + \frac{\phi}{2}\right)$

The above discussed horizontal stresses good if there is no saturated soil layer. If such exists, the effective vertical stress and the pore water pressure are separated. Then, the horizontal stress is determined by multiplying the effective vertical stress by the earth coefficient and then adding the pore pressure. It is worthy to note again that the pore water pressure is equal in both vertical direction and horizontal directions. The vertical and the horizontal stresses phenomenon are discussed below using a soil profile.

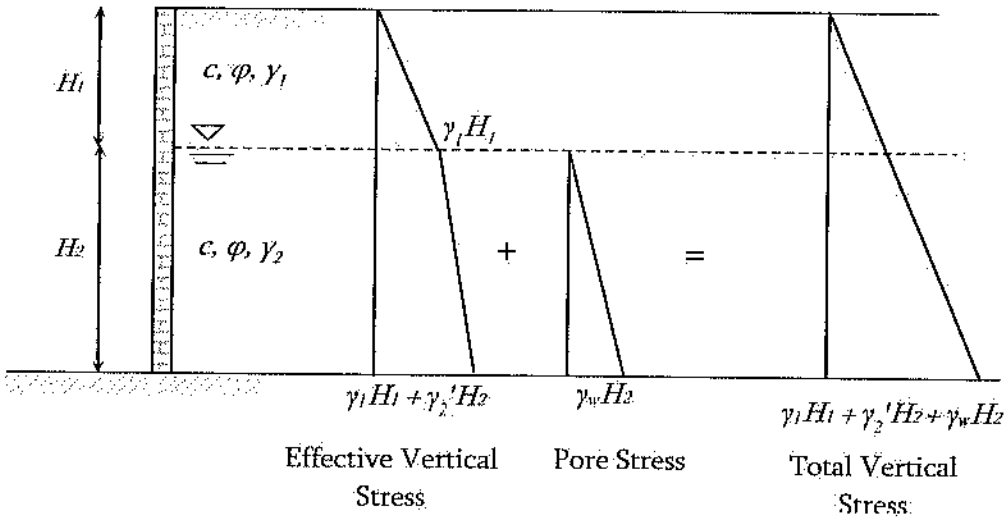


Fig. Vertical stress calculation

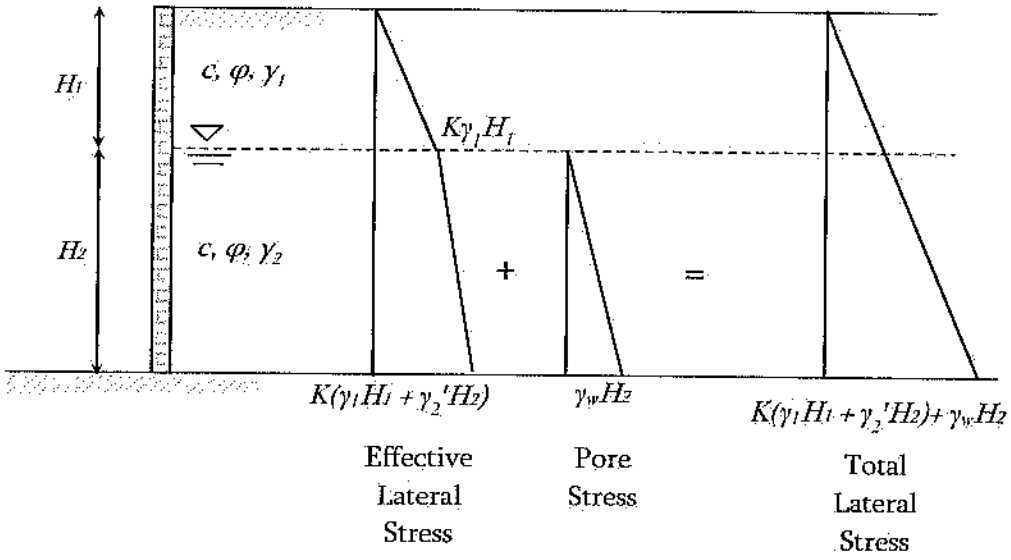


Fig. Horizontal stress calculation

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A retaining wall is supporting a soil as shown below. There is a ground water table 5 m below the surface.

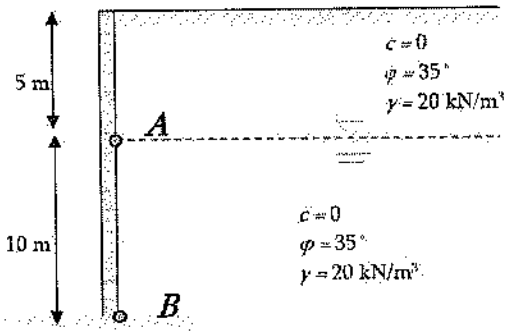


Fig. Soil profile for Problems 12.74 – 12.83

Problem 12.74

The vertical pressure at A is most nearly:

A. 90 kPa B. 100 kPa C. 110 kPa D. 120 kPa

Solution: B

$$\text{Vertical pressure at A} = \gamma h = \left(20\ \frac{\text{kN}}{\text{m}^3}\right)(5\ \text{m}) = 100\ \text{kPa}$$

Problem 12.75

The pore water pressure (kPa) at A is most nearly:

A. 0 B. 98.1 C. 196 D. Cannot be determined

Solution: A

The soil above the point, A is unsaturated. So, there is no pore water pressure at A.

Problem 12.76

The pore water pressure (kPa) at B is most nearly:

A. 0 B. 98.1 C. 196 D. Cannot be determined

Solution: B

$$U_B = \gamma_w h = \left(9.81\ \frac{\text{kN}}{\text{m}^3}\right)(10\ \text{m}) = 98.1\ \text{kPa}$$

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Problem 12.77

The vertical effective pressure (kPa) at B is most nearly:

A. 182 B. 202 C. 222 D. 242

Solution: B

$$\sigma_V' = (5\ \text{m})\left(20\ \frac{\text{kN}}{\text{m}^3}\right) + 10\ \text{m}(20 - 9.81)\frac{\text{kN}}{\text{m}^3} = 201.9\ \text{kPa} \approx 202\ \text{kPa}$$

Effective pressure means, we need to deduct the pore water pressure if the layer is fully saturated.

Problem 12.78

The total vertical pressure (kPa) at B is most nearly:

A. 180 B. 200 C. 220 D. 300

Solution: D

$$\sigma_V = (5\,\text{m})\left(20\,\frac{\text{kN}}{\text{m}^3}\right) + 10\,\text{m}\left(20\,\frac{\text{kN}}{\text{m}^3}\right) = 300\ \text{kPa}$$

This can also be calculated as, Total Pressure = Effective Pressure + Pore Water Pressure

Problem 12.79

The Rankine active earth pressure coefficient for the soil is most nearly:

A. 0.22 B. 0.27 C. 0.33 D. 0.39

Solution: B

$$K_A = \tan^2\left(45 - \frac{\phi}{2}\right) = \tan^2\left(45 - \frac{35}{2}\right) = 0.27$$

Problem 12.80

The lateral earth pressure (kPa) at A is most nearly:

A. 100 B. 67 C. 27 D. 7

Solution: C

$$k_A = \tan^2\left(45 - \frac{\phi}{2}\right) = \tan^2\left(45 - \frac{35}{2}\right) = 0.27$$

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$$\sigma_h \text{ at A} = K_A\left(\text{vertical pressure at A}\right) = K_A\,\gamma h = (0.27)\left(20\,\frac{\text{kN}}{\text{m}^3}\right)(5\text{ m}) = 27\text{ kPa}$$

Problem 12.81

The lateral pore water pressure at B is most nearly:

A. 981 kPa B. 2.7 kPa C. 19.6 kPa D. 9.81 kPa

Solution: D

Pore-water pressure at any point is equal in horizontal and in vertical directions.

$$U_B = \gamma_w h = \left(9.81\,\frac{\text{kN}}{\text{m}^3}\right)(10\text{ m}) = 9.81\text{ kPa}$$

Problem 12.82

The lateral effective pressure at B is most nearly:

A. 45.5 kPa B. 54.5 kPa C. 65.5 kPa D. 75 kPa

Solution: B

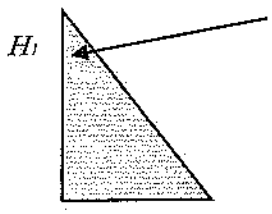
$$\sigma'_h \text{ at B} = K_A\left(\text{effective vertical pressure at B}\right) = K_A\left(\gamma_1 H_1 + \gamma'_2 H_2\right)$$
$$= (0.27)\left(\left(20\,\frac{\text{kN}}{\text{m}^3}\right)(5\text{m}) + (20-9.81)(10\text{ m})\right) = 54.5\text{ kPa}$$

Problem 12.83

The total lateral force and its location at the wall due to the top 5m soil is most nearly:

A. 58.5 kN/m, 3.35 m from the surface
B. 68.8 kN/m, 1.67 m from the surface
C. 67.5 kN/m, 3.35 m from the surface
D. 78.4 kN/m, 5.35 m from the surface

Solution: C



$$P_{A1} = \text{Area of the pressure diagram}$$
$$= \frac{1}{2}\left(K_A \gamma_1 H_1\right)(H_1)$$
$$= \frac{1}{2} K_A \gamma_1 H_1^2$$
$$= \frac{1}{2}(0.27)\left(20\,\frac{\text{kN}}{\text{m}^3}\right)(5\text{ m})^2$$

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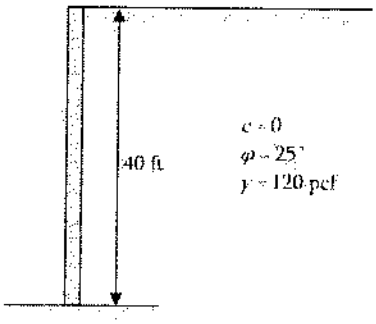
$$= 67.5 \text{ kN}$$

$$\text{Location } \frac{2}{3}H_1 \text{ from the surface} = \frac{2}{3}(5 \text{ m}) = 3.35 \text{ m}$$

Problem 12.84

The active earth force and its point of action at the wall for the soil shown below is most nearly:

A. 20 kip/ft, 13.3 ft from the surface
B. 30 kip/ft, 13.33 ft from the surface
C. 40 kip/ft, 27 ft from the surface
D. 50 kip/ft, 27 ft from the surface



Solution: C

$$K_A = \tan^2\left(45 - \frac{\phi}{2}\right) = 0.41$$

$$P_A = \frac{1}{2}K_A \gamma H^2 = \frac{1}{2}(0.41)(120 \text{ pcf})(40 \text{ ft})^2 = 39{,}360 \frac{\text{lb}}{\text{ft}} \approx 40 \frac{\text{kip}}{\text{ft}}$$

$$\text{Location, } \frac{2}{3}\text{H from the surface} = \frac{2}{3}(40 \text{ ft}) = 26.8 \text{ ft} \approx 27 \text{ ft}$$

Problem 12.85

If the wall is pushed towards the soil for the soil shown in the previous problem (Problem 12.103), the pressure exerted back by the soil to the wall is most nearly:

A. 210 kip/ft, 13.3 ft from the surface
B. 198 kip/ft, 13.33 ft from the surface
C. 237 kip/ft, 27 ft from the surface
D. 323 kip/ft, 27 ft from the surface

Solution: C

$$K_P = \tan^2\left(45 + \frac{\phi}{2}\right) \quad = \tan^2\left(45 + \frac{25}{2}\right) = 2.46$$

$$P_P = \frac{1}{2}K_P \gamma H^2 = \frac{1}{2}(2.46)(120 \text{ pcf})(40 \text{ ft})^2 = 236{,}536 \frac{\text{lb}}{\text{ft}} = 237 \frac{\text{kip}}{\text{ft}}$$

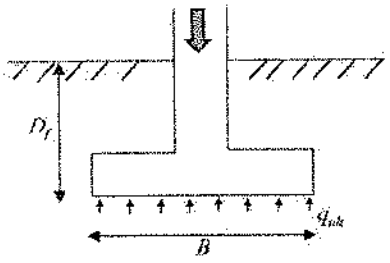
$$\text{Location of force} = \frac{2}{3}(40 \text{ ft}) = 26.8 \text{ ft} \approx 27 \text{ ft from the surface}$$

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Foundation Design

Foundation support structural members and transfer loads to the soil; structural members are usually columns or walls. This is a common area for geotechnical engineers and structural engineers. Structural design, size and shape of the footing is designed by the structural engineers. Geotechnical engineers determine the allowable soil pressure based on laboratory and field testing.

Foundation (sometimes called footing) can be of different types which are mentioned below. Wall, square, combined, and mat foundation are termed as shallow foundation and the last one, plie foundation is termed as deep foundation considering the depth of the foundation. Computation of wall footing only is included in the NCEES FE Ref. Handbook. Some basic concepts/items of foundation are also discussed here using examples.



Ultimate Bearing Capacity of wall footing, $q_{ult} = cN_c + \gamma' D_f N_q + 0.5\gamma' B N_\gamma$

N_c = bearing capacity factor for cohesion
N_q = bearing capacity factor for depth
N_γ = bearing capacity factor for unit weight
D_f = depth of footing below ground surface
B = width of strip footing
γ' = effective unit weight

The allowable bearing pressure can be calculated as: $q_{all} = \frac{q_{ult}}{F.S.}$

Problem 12.86

The member carries greater load in a structure is most nearly:

A. Slab B. Wall C. Beam D. Column

Solution: D

Column carries load from slab, beam, girder, etc.

Problem 12.87

Type of failure occurs in soil under the foundation is most nearly:

A. Compressive B. Shear C. Bearing D. Tension

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Solution: B

Shear failure occurs in foundation. Upon applying column loading, soil beneath the foundation slips laterally.

Problem 12.88

Which of the following value(s) is/are required to calculate the ultimate bearing pressure of soil under the foundation?

A. c B. ϕ C. ρ D. All the above

Solution: D

Cohesion (c), angle of internal friction (ϕ) (needed to find Bearing Factors), and density (needed to determine the unit weight, γ) are required to calculate the ultimate bearing pressure of soil under the foundation.

Problem 12.89

Immediate settlement occurs under foundation due to -

A. elastic distortion
B. expulsion of soil water
C. movement of soil water
D. All the above

Solution: A

Immediate settlement occurs under foundation due to elastic distortion. Consolidation settlement occurs due to expulsion of water which takes several years to complete.

Problem 12.90

Do the Terzaghi bearing capacity factors depend on internal friction angle?

A. Not at all
B. Yes
C. Depends on site
D. None of the above

Solution: B

Terzaghi bearing capacity factors depend on internal friction angle.

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Problem 12.91

Foundations installed to bypass unsatisfactory soils and transfers load to a suitable bearing stratum is called --

A. Mat foundation
B. Spread footing
C. Raft foundation
D. None of the above

Solution: D

Foundations installed to bypass unsatisfactory soils and transfers load to a suitable bearing stratum is called deep foundations. Piles, piers, etc. are the examples of deep foundation.

Problem 12.92

The groundwater table affects the pressure distribution under the foundation. True or false?

A. True for any soil type
B. False for any soil type
C. Depends on the soil type
D. Depends on foundation type

Solution: A

Problem 12.93

Deep pile supports the load in two ways: end-point bearing and what?

A. Lateral pressure
B. Water pressure
C. Side friction
D. None of the above

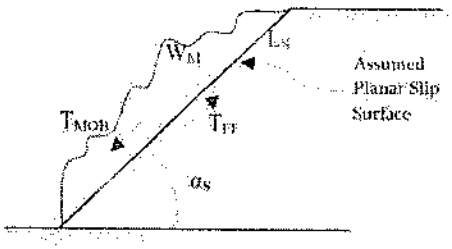
Solution: C

Pile supports the load in two ways: side friction and end-point bearing. The end-point bearing means the end resistance of the pile.

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Slope Failure Along Planar Surface

Analysis of slope stability is important in areas where inclined surface exists. The basic concept is that mobilized shear force (T_{MOB}) occurs along the assumed slip surface. This is developed due to the component of soil mass (W_M) along the slip surface. This mobilized shear force (T_{MOB}) is resisted by the available shear resistance (T_{FF}) which is developed due to frictional force along the slip surface and cohesive attraction. When T_{MOB} is greater than T_{FF}, slip occurs.



FS = factor of safety against slope instability $= \dfrac{T_{FF}}{T_{MOB}}$

T_{FF} = available shearing resistance along slip surface $= cL_S + W_M \cos\alpha_S \tan\phi$

T_{MOB} = mobilized shear force along the slip surface $= W_M \sin\alpha_s$

Ls = length of assumed planar slip surface

W_M = weight of soil above slip surface

α_S = angle of assumed slip surface with respect to horizontal

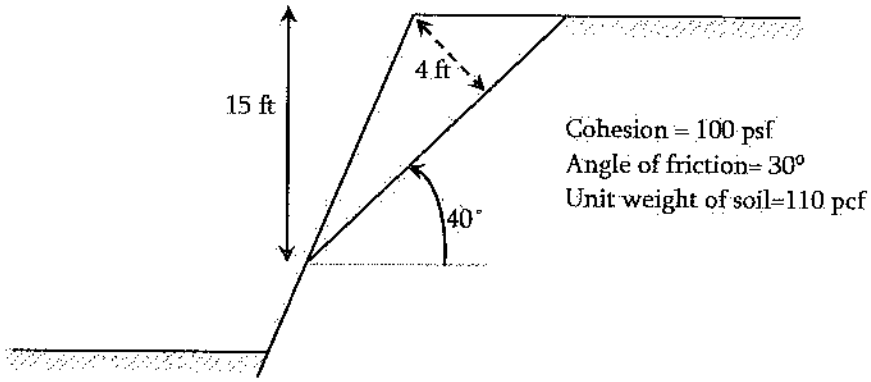
Problem 12.94

When does sliding occur in soil slope?

A. When shear stress developed in soil exceed the shear strength
B. When normal stress in soil exceeds normal strength
C. When shear stress developed in soil exceed normal strength
D. None of the above

Solution: A

Problems 12.95 – 12.99 are based on the following figure:



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Problem 12.95

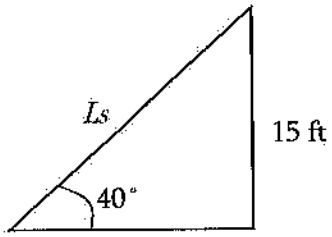
The length of planar slip surface for the above soil is most nearly:

A. 34.2 ft B. 57.1 ft C. 26.3 ft D. 23.3 ft

Solution: D

$$\frac{15}{L_S} = \sin 40$$

$$L_S = \frac{15}{\sin 40} = 23.33 \text{ ft}$$



Problem 12.96

The weight of the soil (lb/ft) mentioned above that may slide is most nearly:

A. 4,050
B. 4,550
C. 5,060
D. 5,550

Solution: C

$$W_M = \left(\frac{1}{2} L_S h\right)\gamma = \left(\frac{1}{2}(23\text{ft})(4\text{ft})\right)(110\text{pcf}) = 5{,}060 \ \frac{\text{lb}}{\text{ft}}$$

Problem 12.97

The mobilized shear force along the slip surface for the above-mentioned soil is most nearly:

A. 2,253 lb/ft B. 2,453 lb/ft C. 2,853 lb/ft D. 3,253 lb/ft

Solution: D

$$T_{MOB} = W_M \sin\alpha_s = 5{,}060(\sin 40) = 3{,}253 \text{ lb/ft}$$

Problem 12.98

The available shearing resistance in lb/ft along the slip surface is most nearly:

A. 2,251 B. 2,422 C. 4,538 D. 3,251

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Solution: C

$$T_{FF} = cL_S + W_M \cos\alpha_S \tan\varphi = 100\,\text{psf}(23\,\text{ft}) + \left(5{,}060\,\frac{\text{lb}}{\text{ft}}\right)\cos 40(\tan 30) = 4{,}538\ \frac{\text{lb}}{\text{ft}}$$

Problem 12.99

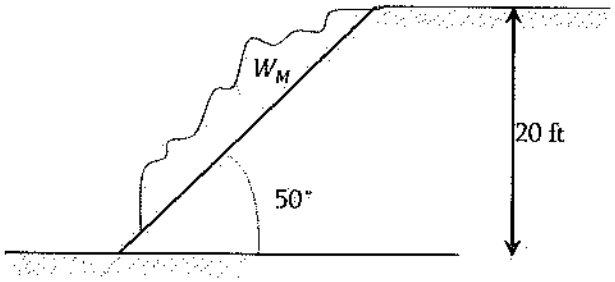
The factor of safety against a slope instability is most nearly:

A. 0.9 B. 1.1 C. 1.3 D. 1.4

Solution: D

$$FS = \frac{T_{FF}}{T_{MOB}} = \frac{4{,}538\ \frac{\text{lb}}{\text{ft}}}{3{,}253\ \frac{\text{lb}}{\text{ft}}} = 1.4$$; You need to calculate T_{FF} and T_{MOB} first to solve this problem, which are shown in the previous 2 problems.

Problems 12.100 – 12.103 are based on the following figure:



c= 160 psf
γ= 120 pcf
Ø= Angle of friction=25°
W_M=Weight of soil above slip surface =4000 $^{lb}/_{ft}$

Problem 12.100

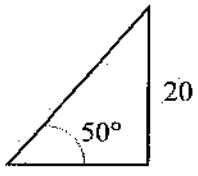
The length of planar slip surface is most nearly:

A. 34 ft
B. 57 ft
C. 26 ft
D. 23 ft

Solution: C

$$\frac{20}{L_S} = \sin 50^\circ$$

$$L_S = \frac{20}{\sin 50} = 26\ \text{ft}$$



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Problem 12.101

The mobilized shear force along the slip surface is most nearly:

A. 3 kip/ft B. 4 kip/ft C. 5 kip/ft D. 6 kip/ft

Solution: A

$$T_{MOB} = W_M \sin\alpha_s = 4{,}000(\sin 50) = 3{,}064 \text{ lb/ft} \approx 3 \text{ kip/ft}$$

Problem 12.102

The available shearing resistance along the slip surface is most nearly:

A. 5,833 lb/ft
B. 6,833 lb/ft
C. 5,359 lb/ft
D. 8,833 lb/ft

Solution: C

$$\tau_{FF} = cL_S + W_M \cos\alpha_S \tan\varphi = 160 \text{ psf}(26 \text{ ft}) + \left(4{,}000 \ \frac{\text{lb}}{\text{ft}}\right)(\cos 50)(\tan 25) = 5{,}359 \ \frac{\text{lb}}{\text{ft}}$$

Problem 12.103

The factor of safety against slope instability is most nearly:

A. 1.0
B. 1.2
C. 1.3
D. 1.75

Solution: D

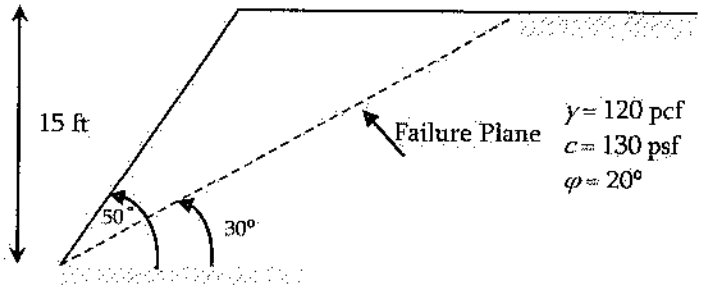
$$FS = \frac{T_{FF}}{T_{MOB}} = \frac{5{,}359 \ \frac{\text{lb}}{\text{ft}}}{3{,}064 \ \frac{\text{lb}}{\text{ft}}} = 1.75;$$

You need to calculate T_{FF} and T_{MOB} first to solve this problem, which are shown in the previous 2 problems.

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Problem 12.104

For the soil shown below, the weight of the soil above the slip surface is most nearly:



A. 8.3 kips/ft
B. 10.3 kips/ft
C. 12.1 kips/ft
D. 14.7 kips/ft

Solution: C

$$\text{Length of failure plane, } L_S = \frac{15\text{ ft}}{\sin\alpha_S} = \frac{15\text{ ft}}{\sin 30} = 30\text{ ft}$$

$$\text{Length of curved surface, } L = \frac{15\text{ ft}}{\sin\beta} = \frac{15\text{ ft}}{\sin 50} = 19.6\text{ ft}$$

$$\frac{h}{L} = \sin(\beta - \alpha_S) = \sin(50 - 30)$$

So, $h = 6.7$ ft

$$W_M = \left(\frac{1}{2}L_S h\right)\gamma = \left(\frac{1}{2}(30\text{ ft})(6.7\text{ ft})\right)(120\text{ pcf}) = 12{,}060\ \frac{\text{lb}}{\text{ft}} \approx 12.1\ \frac{\text{kips}}{\text{ft}}$$

Sometimes, you may need to round off while selecting the answer. For example, in this problem, 12.1 kip/ft instead of 12.06 kip/ft. This kind of rounding has been done in many places in this book.

"FE exam results are based on the total number of correct answers that you selected. There are no deductions for wrong answers. The score is then converted to a scaled score, which adjusts for any minor differences in difficulty across the different exam forms. This scaled score represents an examinee's ability level and is compared to the minimum ability level for that exam, which has been determined by subject-matter experts through psychometric statistical methods. NCEES does not publish the passing score. NCEES scores each exam with no predetermined percentage of examinees that should pass or fail. All exams are scored the same way. First-time takers and repeat takers are graded to the same standard." Visit https://ncees.org for more information.

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13 STRUCTURAL ANALYSIS

Fundamentals of Engineering (FE) CIVIL Exam Specification mentions, 6-9 questions may appear in the exam based on Structural Analysis as follows:

A. Analysis of forces in statically determinant beams, trusses, and frames
B. Deflection of statically determinant beams, trusses, and frames
C. Structural determinacy and stability analysis of beams, trusses, and frames
D. Loads and load paths (e.g., dead, live, lateral, influence lines and moving loads, tributary areas)
E. Elementary statically indeterminate structures

Influence Lines for Beams and Trusses

An influence line shows the variation of an effect (reaction, shear and moment in beams, bar force in a truss) caused by moving a unit load across the structure. An influence line is used to determine the position of a moveable set of loads that causes the maximum value of the effect. Influence line is drawn by following the following steps:

Step 1. Apply a unit load at different key points of the member
Step 2. Determine the concern force/moment/shear at the concern point. This determined force is the ordinate for point where unit is applied.
Step 3. Then using the unit load application point and the determined force as (x, y), draw the influence line.

Now practice the following problems for further clarification.

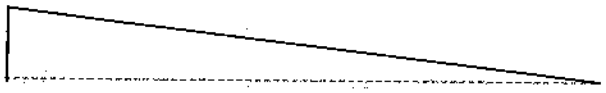
Problem 13.1

The qualitative influence line of the reaction at the left support, A of the following beam is most nearly:

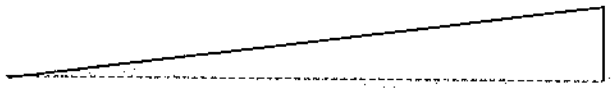


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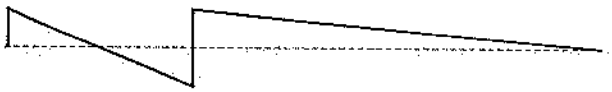
A.



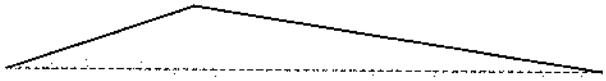
B.



C.



D.



Solution: A

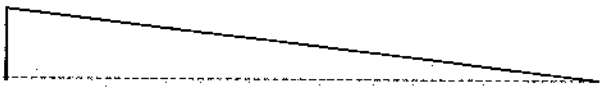
When unit load is applied at $x = 0$ (left support), $A_y = +1$
When unit load is applied at $x = L$ (right support), $A_y = 0$
Then, using $(x, y) = (0, 1)$ and $(L, 0)$, draw the line.

Problem 13.2

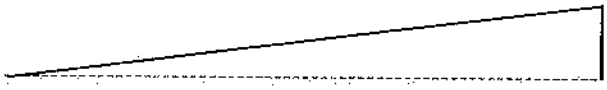
The qualitative influence line of the reaction at the right support, B of the following beam is most nearly:



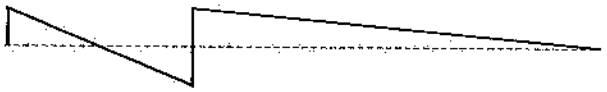
A.



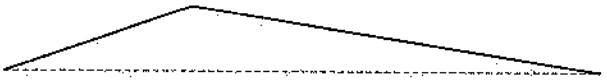
B.



C.



D.



Solution: B

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Problem 13.3

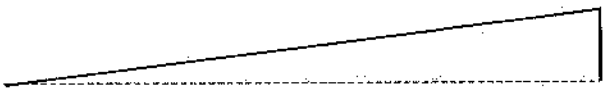
The qualitative influence line of the shear force at C of the following beam is most nearly:



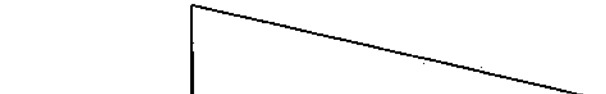
A.



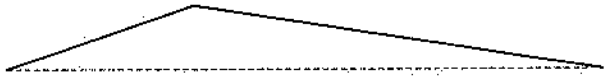
B.



C.



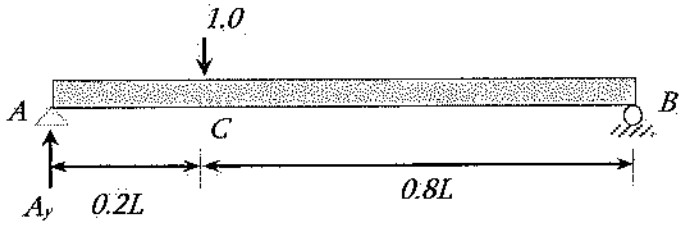
D.



Solution: C

When a unit load is applied at A, $V_c = 0$; because support will resist the load. Similarly, when a unit load is applied at B, $V_c = 0$.

Consider C is located at 0.2L from the left. Apply a unit load just left of C. Then –



$\sum M$ at $B = 0$

$A_y (L) - 1.0\ (0.8L) = 0$

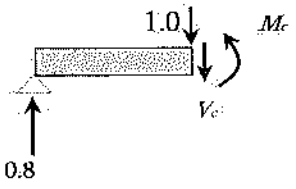
$A_y = 0.8$

Now, cut at C as follows, draw free-body diagram and apply $\sum F_y = 0$

$\sum F_y = 0$

$0.8 - 1.0 - V_c = 0$

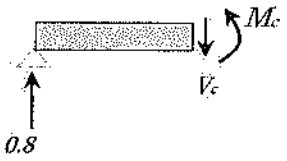
$V_c = -0.2$



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Now, if you place unit load just right of C, then $V_c = 0.8$ as follows:

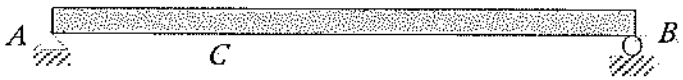
$$\sum F_y = 0$$
$$0.8 - V_c = 0$$
$$V_c = 0.8$$



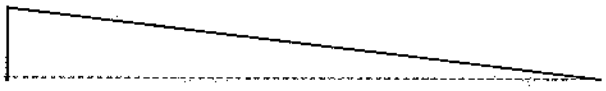
Now, you can draw the diagram using these four points: (0, 0), (Left of C, -0.2), (right of C, 0.8), and (0, 0). Note: we assumed C is located $0.2L$ from the left. For other values proceed accordingly. The shape will be the same, stair-type shape.

Problem 13.4

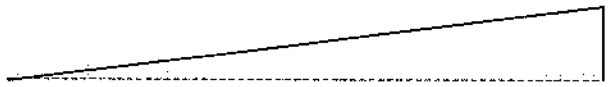
The qualitative influence line of the moment at C of the following beam is most nearly:



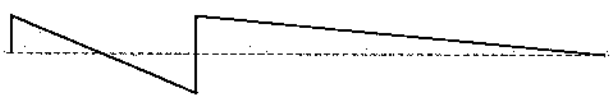
A.



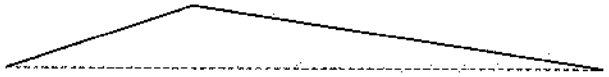
B.



C.



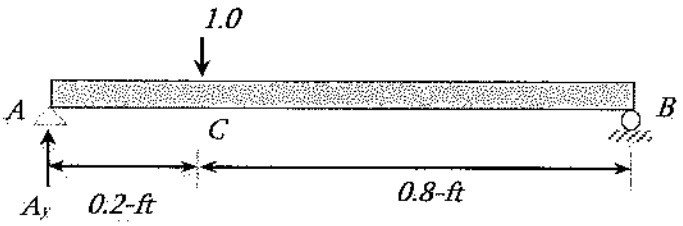
D.



Solution: D

When a unit load is applied at A, $M_c = 0$; because support will resist the load. Similarly, when a unit load is applied at B, $M_c = 0$.

Consider C is located at 0.2-ft from the left of the 1-ft long beam. Apply a unit load just left of C.

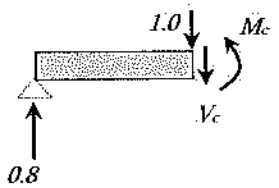


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$\sum M$ at $B = 0$
$A_y\,(1.0\,ft) - 1.0\,(0.8\text{-ft}) = 0$
$A_y = 0.8$

Now, cut at C as follows, draw the free-body diagram and apply $\sum M$ at $C = 0$

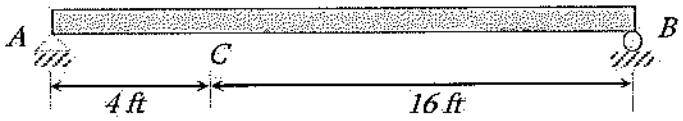
$\sum M$ at $C = 0$
$0.8\,(0.2\text{-ft}) - M_c = 0$
$M_c = 0.16$



Now, you can draw the diagram using these four points: (0, 0), (C, 0.16), and (0, 0). Note: we assumed C is located 0.2-ft from left of 1-ft beam. It doesn't matter whether the unit load is applied at just the left or just the right of C. For other values proceed accordingly. The shape will be the same, two-straight lines meet at a point.

Problem 13.5

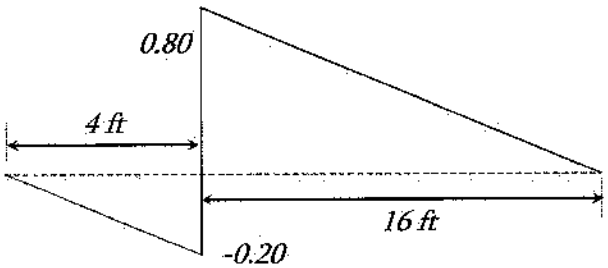
The maximum positive shear force that can be developed at point, C, of the following beam due to a concentrated load moving load of 2 kips and a uniform load of 1 kip/ft is most nearly:



A. 2 kips B. 4.56 kips C. 8.0 kips D. 12.365 kips

Solution: C

Draw Influence line of shear-force first as shown in Problem 13.3. Apply the mentioned loading which gives the maximum shear force.



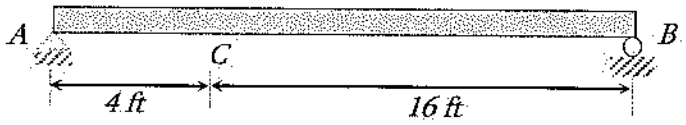
The maximum positive shear force develops when the concentrated load acts at C and the uniform load acts only at the right of C of the beam to the right support.

Therefore, positive V_{max} at $C = 2$ kips $(0.80) + 1$ kip/ft $[\frac{1}{2}(16\text{ ft})(0.80)] = 8.0$ kips

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Problem 13.6

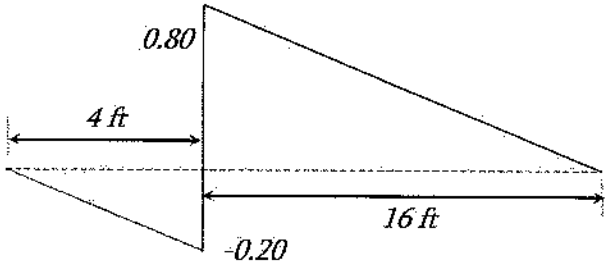
The maximum negative shear force that can be developed at point, *C* of the following beam due to a concentrated load moving load of 2 kips and a uniform load of 1 kip/ft is most nearly:



A. 0.8 kips B. 2.00 kips C. 7.5 kips D. 12.365 kips

Solution: A

Draw Influence line of shear-force first as shown in Problem 13.3. Apply the mentioned loading.

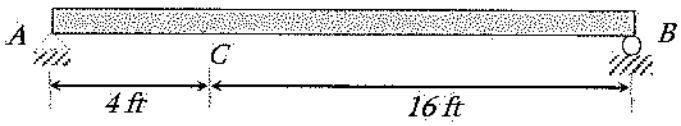


The maximum negative shear force develops when the concentrated load acts at *C* and the uniform load acts only at the left of *C* of the beam to the left support.

Negative V_{max} at C = 2 kips (-0.2) + 1 kip/ft [½(4 ft)(-0.2)] = -0.8 kips

Problem 13.7

The maximum positive moment that can be developed at point, *C* of the following beam due to a concentrated load moving load of 2 kips and a uniform load of 1 kip/ft is most nearly:

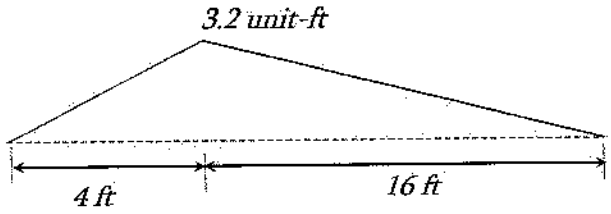


A. 38.4 kips-ft B. 20 kips-ft C. 43 kips-ft D. 12.36 kips-ft

Solution: A

Draw Influence line of moment first as shown in Problem 13.4. Apply the mentioned loading.

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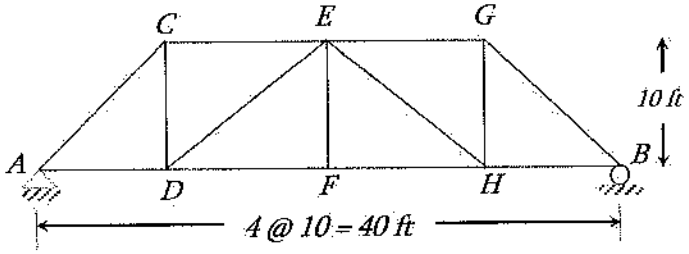


The maximum moment develops when the concentrated load acts at C and the uniform load acts all over the beam.

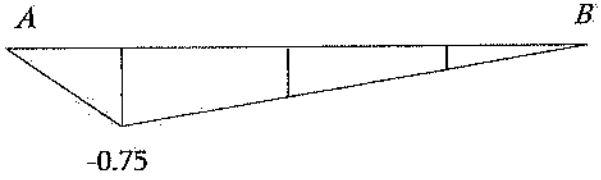
M_{max} at C = 2 kips(3.2) + 1 kip/ft [½(20 ft)(3.2)] = 38.4 kips-ft

Problem 13.8

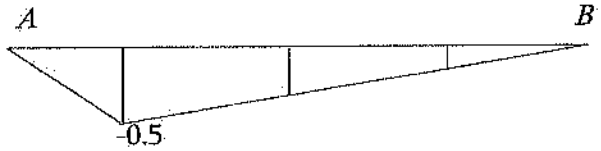
The influence line of member CE of the following truss is most nearly:



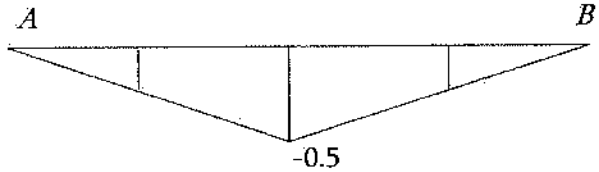
A.



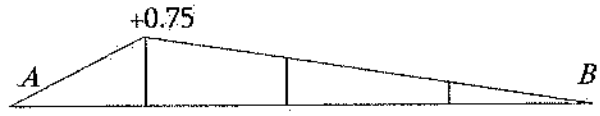
B.



C.



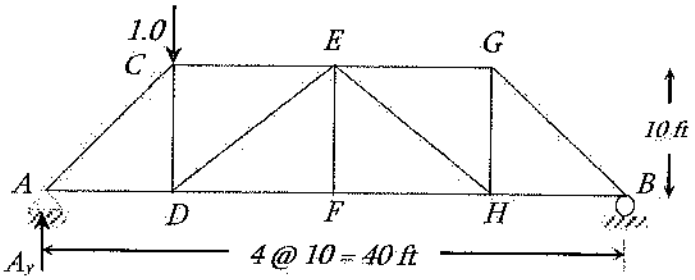
D.



Solution: A

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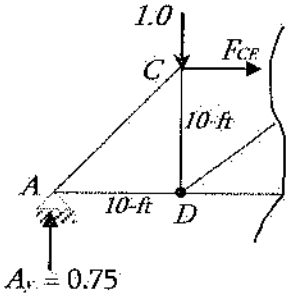
All answer options are different at joint, C, 10-ft from the left support. Therefore, checking a single joint will find out the answer. Let us check when the unit load is placed at C as follows:



$\sum M$ at $B = 0$

$A_y\,(40\,ft) - 1.0\,(30\,ft) = 0$

$A_y = 0.75$



Then, for the cut section, $\sum M$ at $D = 0$

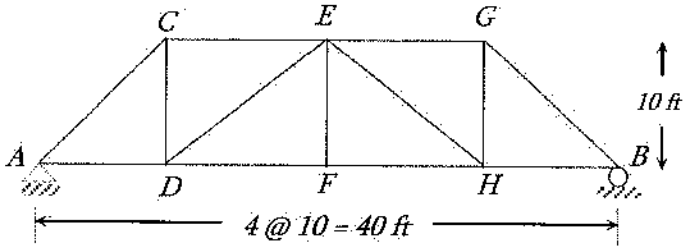
$F_{CE}\,(10\,ft) + A_y\,(10\,ft) = 0$

$F_{CE} = -0.75$

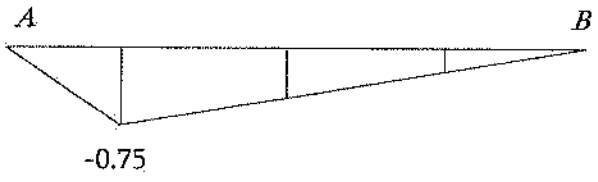
Therefore, Option A is the answer. You can place the unit load at other joints and check further.

Problem 13.9

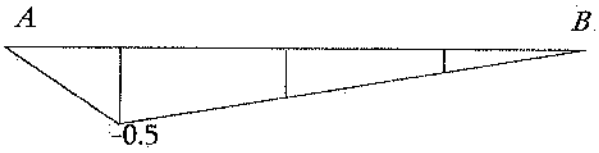
The influence line of member FH of the truss shown below is most nearly:



A.

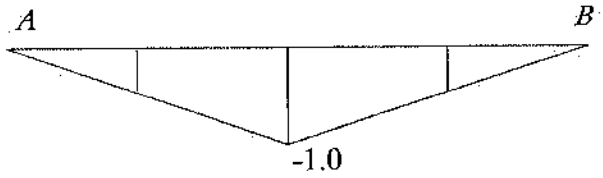


B.

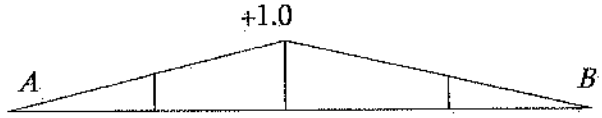


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C.

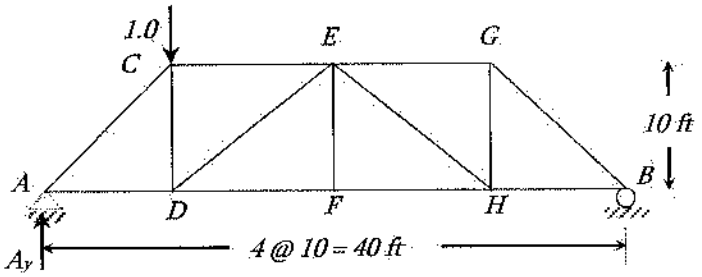


D.



Solution: D

All options are different at joint, *C*, 10-ft from the left support. Therefore, checking a single joint will find out the answer. Let us check when the unit load is placed at *C* as follows:



$\sum M$ at $B = 0$

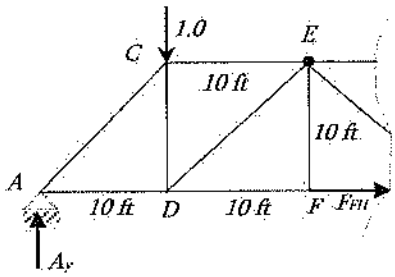
$A_y\,(40\,ft) - 1.0\,(30\,ft) = 0$

$A_y = 0.75$

Then, for the cut section, $\sum M$ at $E = 0$

$-F_{FH}\,(10\,ft) - 1.0\,(10\,ft) + A_y\,(20\,ft) = 0$

$F_{FH} = 0.5$



Therefore, Option D is the answer.

You can place the unit load at other joints and check further.

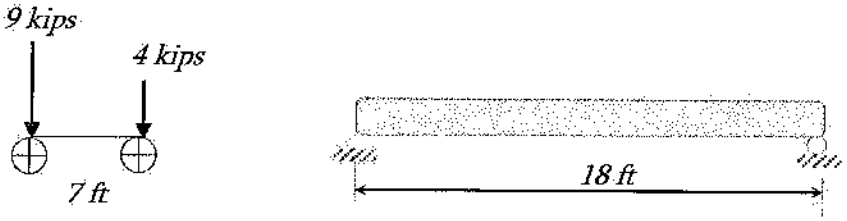
Moving Concentrated Load Sets

When series of loads move over a beam (say, bridge beam), the maximum moment occurs for a specific loading configuration. The absolute maximum moment produced in a beam by a set of "n" moving loads occurs when the resultant "*R*" of the load set and adjacent load are equal distance from the centerline of the beam. In general, two possible load sets positions must be considered, one for each adjacent load. Regarding the maximum shear, it is understandable that the maximum shear occurs when all (or most of the loads) are the closest to the left support or the closest to the right support.

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Problem 13.10

A car has two rows of wheels as shown below. If the car passes over the18 ft bridge beam, what is most nearly the absolute maximum moment produced in the following beam? Assume, the maximum moment occurs under the 9 kips load.

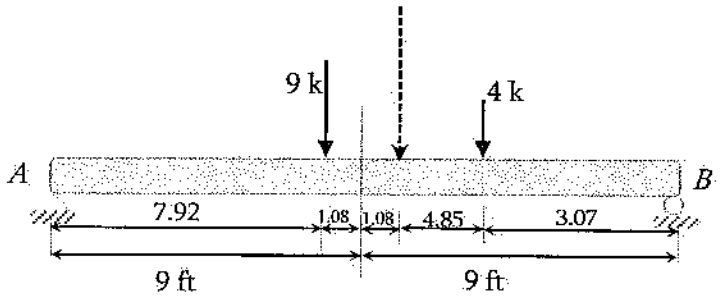


A. 36 kips-ft
B. 45 kips-ft
C. 54 kips-ft
D. 60 kips-ft

Solution: B

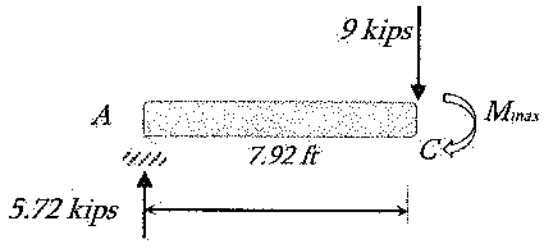
$$\text{Centroid of the loading w.r.t 9 kips load} = \frac{9\,\text{kips}\,(0) + 4\ \text{kips}\,(7\ \text{ft})}{9+4} = 2.15\ \text{ft}$$

Place the 9-kip load and the resultant equal distance from the beam-center as shown below.



$$\sum M_B = 0$$

$$A_y = \frac{4\,\text{k}(3.07\,\text{ft}) + 9\,\text{k}(10.08\,\text{ft})}{18\ \text{ft}} = 5.72\ \text{kips}$$



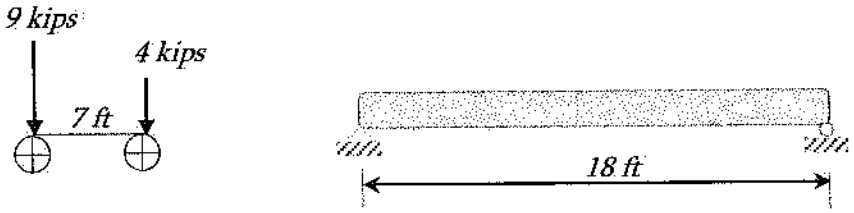
$$\sum M_C = 0$$

M_{max} = 5.72 k (7.92 ft) = 45.32 kips-ft

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Problem 13.11

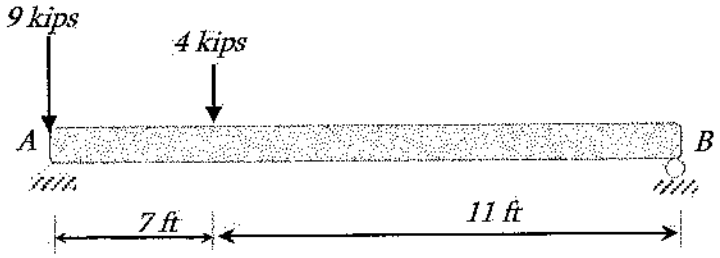
A car has two rows of wheels as shown below. If the car passes over the18 ft beam bridge, the maximum shear force (kips) produced in the beam is most nearly:



A. 7.36
B. 11.44
C. 17.54
D. 19.60

Solution: B

The maximum shear force occurs at the support when the load is on the support. Here, the maximum shear force will develop for the following loading configuration.



$$\sum M_B = 0$$

$$A_y = \frac{4\text{k}(11\text{ft}) + 9\text{k}(18\text{ft})}{18\text{ ft}} = 11.44 \text{ kips}$$

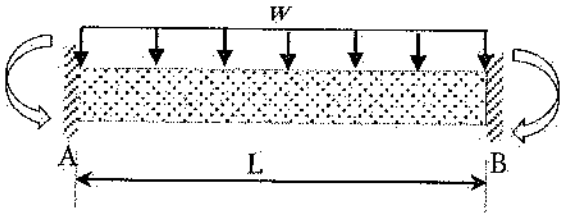
Therefore, $V_{max} = 11.44$ kips

You can try placing the loads close to the right support. But, that will give lower shear force.

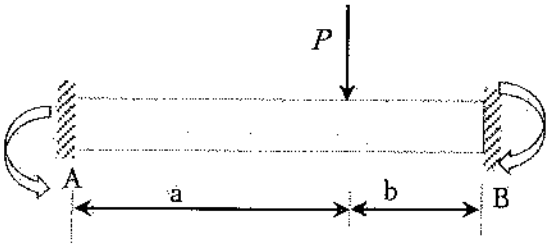
Member Fixed-End Moments (Magnitudes)

Fixed-End Moment (FEM) means the moment developed at end supports of a single span beam considering both ends fixed. This can be determined using the conjugate beam analysis. FEMs for two common cases of beam are listed here as the NCEES FE Ref. Handbook lists these two only. This is used in solving beam using slope-deflection method, moment distribution, etc.

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$$FEM_{AB} = FEM_{BA} = \frac{wL^2}{12}$$



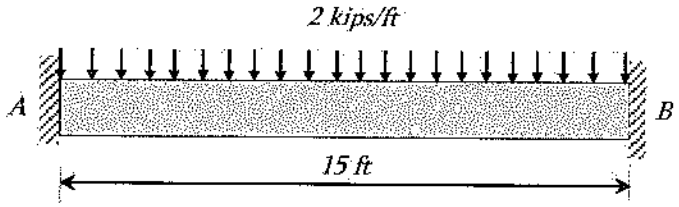
$$FEM_{AB} = \frac{Pab^2}{L^2}$$

$$FEM_{BA} = \frac{Pa^2b}{L^2}$$

Problem 13.12

The fixed end moment for the following beam is _____kip-ft.

[write the answer in one decimal place such as 3.7 or 87.3 etc. without unit]



Solution: 37.5

$$FEM_{AB} = FEM_{BA} = \frac{wL^2}{12} = (2)(15)^2/12 = 37.5 \text{ kip-ft}$$

Stability and Determinacy

A structure is said to be stable if it's support restraints and member organization should be such that it does not collapse due to external loading from any direction and of any kind. Stability can be accessed by mathematical calculation and visual inspection. The following symbols have been used here to discuss this phenomenon.

m = number of members
r = number of external reactions
j = number of joints
c = number of conditions based on known internal moments or forces, such as internal moment of zero at a hinge

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Determinacy is determined by comparing the unknown forces in a structure and the known conditions available.

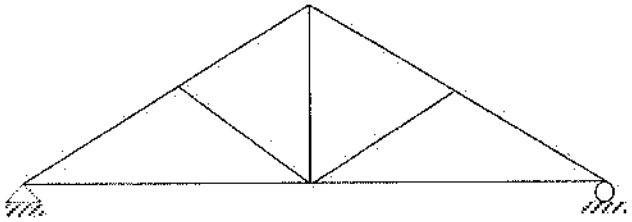
Plane Truss

For plane truss (2D truss), the following formulations can be used to justify the stability (external) and determinacy of a plane truss. To justify the internal stability of truss, it must be accessed visually.

Static Analysis	Classification
$m + r < 2j$	Unstable
$m + r = 2j$	Stable and statically determinate
$m + r > 2j$	Stable and statically indeterminate

Problem 13.13

The following planar truss is most nearly:



A. unstable
B. stable and statically determinate
C. stable and statically indeterminate
D. unstable and statically indeterminate

Solution: B

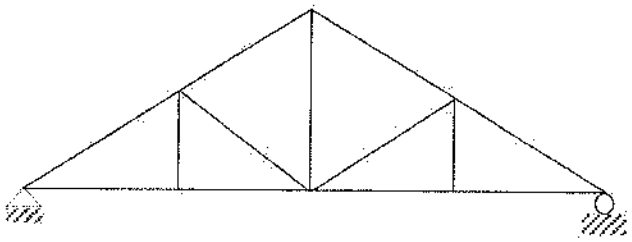
Number of member, $m = 9$
Number of reaction, $r = 3$
Number of joint, $j = 6$

$m + r = 12$
$2j = 12$
$m + r = 2j$
So, determinate and stable

Problem 13.14

The following planar truss is most nearly:

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A. Unstable
B. Stable and statically determinate
C. Stable and statically indeterminate
D. Unstable and statically indeterminate

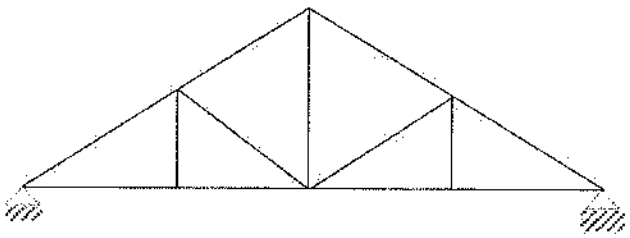
Solution: B

Number of member, $m = 13$
Number of reaction, $r = 3$
Number of joint, $j = 8$

$m + r = 16$
$2j = 16$
$m + r = 2j$
So, determinate and stable

Problem 13.15

The following planar truss is most nearly:



A. Unstable
B. Stable and statically determinate
C. Stable and statically indeterminate
D. Unstable and statically indeterminate

Solution: C

Number of member, $m = 13$
Number of reaction, $r = 4$
Number of joint, $j = 8$

$m + r = 17$
$2j = 16$
$m + r > 2j$
So, indeterminate and stable

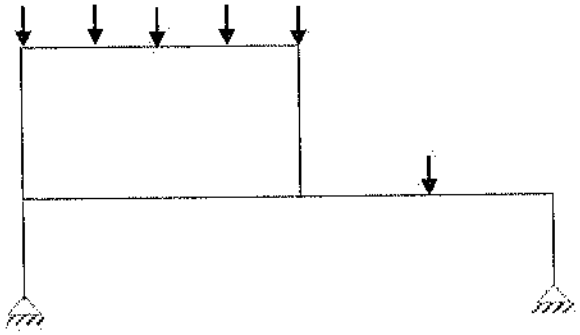
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Plane Frame

Static Analysis	Classification
$3m + r < 3j + c$	Unstable
$3m + r = 3j + c$	Stable and statically determinate
$3m + r > 3j + c$	Stable and statically indeterminate

Problem 13.16

The following planar frame is most nearly:



A. Unstable
B. Stable and statically determinate
C. Stable and statically indeterminate
D. Unstable and statically indeterminate

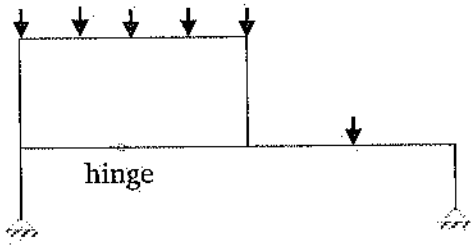
Solution: C

Number of member, $m = 7$
Number of reaction, $r = 4$
Number of joint, $j = 7$
Number of condition, $c = 0$

$3m + r = 25$
$3j + c = 21$
$3m + r > 3j + c$
So, indeterminate and stable

Problem 13.17

The following planar frame is most nearly:



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A. Unstable
B. Stable and statically determinate
C. Stable and statically indeterminate
D. Unstable and statically indeterminate

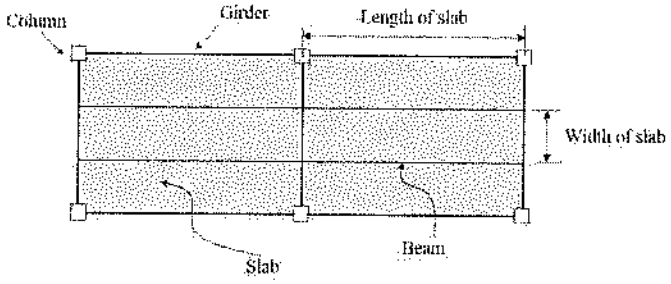
Solution: C

Number of member, $m = 8$	$3m + r = 28$
Number of reaction, $r = 4$	$3j + c = 25$
Number of joint, $j = 8$	$3m + r > 3j + c$
Number of condition, $c = 1$	So, indeterminate and Stable

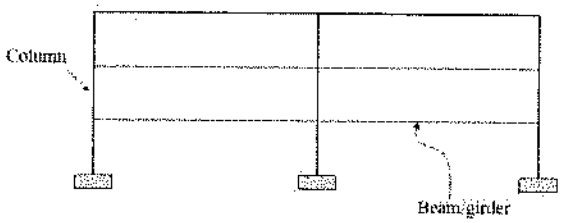
Load Tracing

Before designing a structure, it is necessary to determine the possible load at different members of a structure. To do it, it is essential to know, how load transfers from our footstep through different members of a structure to the foundation. Let me explain how load transfers from structural member to member. At the very beginning load is applied on the floor or slab of a building in the form of human being, furniture, etc. The slab transfers this load to beam as shown in the Figure below. It can be seen in the plan view that the beam is connected to girder; that means the beam load is transferred to girder. Girders then transfer the load to column. The column load then goes to foundation soil. Therefore, load calculation starts from roof slab and then gradually comes downward.

Now let us see, how slab load is transferred. There are two different ways, slab load is transferred to beam: one-way system and two-way system. These are explained using the following Figure. Let us consider the following slab with beams all around.



Plan view of a building structure



Profile of a building structure

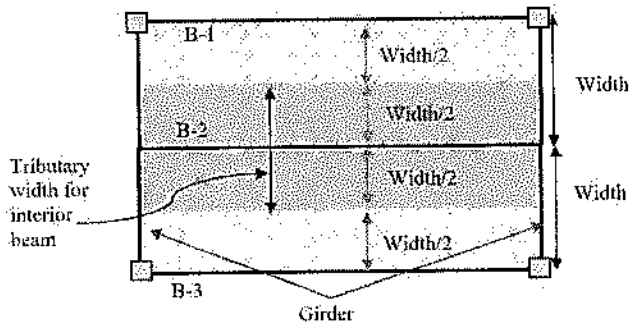
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If, $\frac{\text{Length}}{\text{Width}} > 2.0$, then the load distribution is one-way; the slab load is to be carried by both longer beams equally.

If, $\frac{\text{Length}}{\text{Width}} \leq 2.0$, then the load distribution is two-way; beams all around carry the load.

One-way Load System

For one-way load system $\left(\frac{\text{Length}}{\text{Width}} > 2.0\right)$, the slab load is distributed as shown in the following figure. Beam, *B*-2 carries load 50% of each slab as shown by gray color. Beams, B-1 and B-3 carry load 50% of the slab as shown by the dotted region. The girder, in the left and in the right does not carry any load directly from slab. However, the girders have its self-weight and the concentrated load at its mid span from the interior beam. The exterior beam transfers the load directly to the column. The column load at the lower story is the cumulative of the load from its story and all the stories above.



Let us see an example of one-way load transfer system based on the above figure.

Problem 13.18

Let us assume the following:

Width of slab = 10 ft
Length of beam = 22 ft
Slab load = 2 ksf (including the self-weight)

What is most nearly the load on the interior beam, B-2 ?

Solution:

Length-to-width ratio of the slab = 22/10 = 2.2 >2; so one-way load distribution.
Tributary width of the beam = (10/2)+(10/2) = 10 ft.
Load on interior beam = 2 ksf (10 ft) = 20 kips/ft. The load distribution can be drawn as follows:

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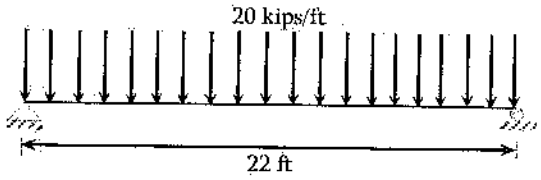
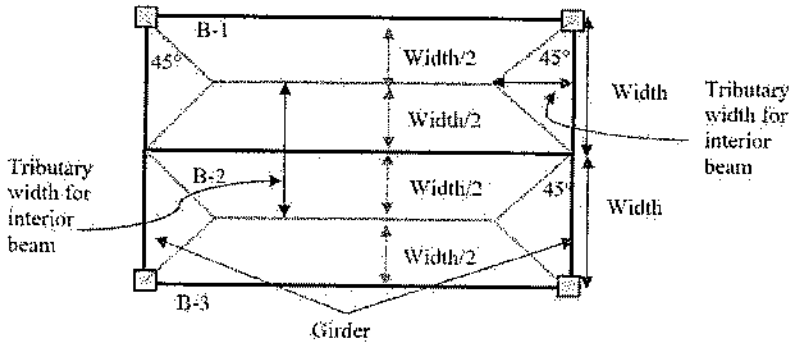


Fig. Load on interior Beam

Note: This problem may be appeared as Multiple-Choice Question, or drag and drop, etc.

Two-way Load System

For two-way load system $\left(\frac{\text{Length}}{\text{Width}} \le 2.0\right)$, the slab load is distributed is shown in the following figure. Beam, *B*-2 carries load 50% of each slab not up to full length. The girder, in the left and in the right, carries some load directly from slab. The area of this load determined by drawing 45° lines from each corner. The exterior beam transfers the load directly to the column. The column load at the lower story is the cumulative of the load from its story and all of the story above.



Let us see an example of two-way load transfer system based on the above figure.

Problem 13.19

Let us assume the following:

Width of slab = 10 ft
Length of beam = 15 ft
Slab load = 2 ksf (including the self-weight)

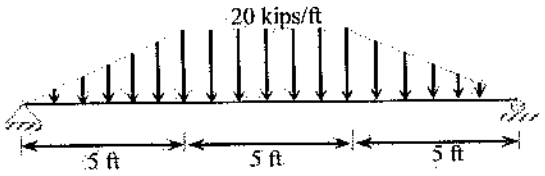
What is most nearly the load and draw it for the interior beam, *B*-2 ?

Solution:

Length-to-width ratio of the slab = 15/10 = 1.5 < 2; so two-way load distribution.
Tributary width of the beam = (10/2) + (10/2) = 10 ft.

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Load on interior beam = 2 ksf (10 ft) = 20 kips/ft up to middle (15 – 2 x 5) = 5 ft, uniformly decreases to zero at the ends. Therefore, the load distribution can be drawn as follows:



Load on Interior beam

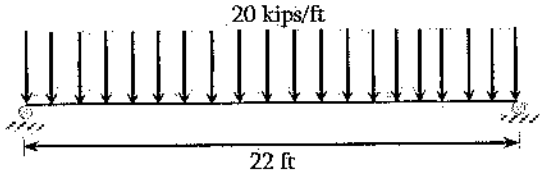
Note: This problem may be appeared as Multiple-Choice Question, or drag and drop, etc.

Lateral Stability of Load Tracing

The sense of lateral stability is more like a common sense. While designing and constructing, it must be borne in mind that there must have adequate restraints on structure to resist any kind of load from any side. For example, wind force may come left to right or right to left, and so on. The following example will clarify it.

Problem 13.20

Let us check whether the following structure is safe or not.



Solution:

The above structure has vertical load; the structure has two vertical supports to resist that load. However, both supports are roller support which resists load in one direction, in this case, the vertical direction. If there any load comes horizontally, by chance, there is not restraint to resist that. Therefore, the structure will move if any lateral load exhibits. So, this structure is not safe. It will be safe if the roller is replaced by pin or fixed support.

Truss Deflection by Unit Load Method

The displacement of a truss joint caused by external effects (truss loads, member temperature change, member misfit) is found by applying a unit load at the point that corresponds to the desired displacement.

$$\Delta_{joint} = \sum_{i=1}^{members} f_i (\Delta L)_i$$

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Δ_{joint} = joint displacement at point of application of unit load (+ in direction of unit load)
f_i = force in member "i" caused by unit load (+ tension)
$(\Delta L)_i$ = change in length caused by external effect (+ for increase in member length)

$= \left(\frac{FL}{AE}\right)_i$ for bar force F caused by external load

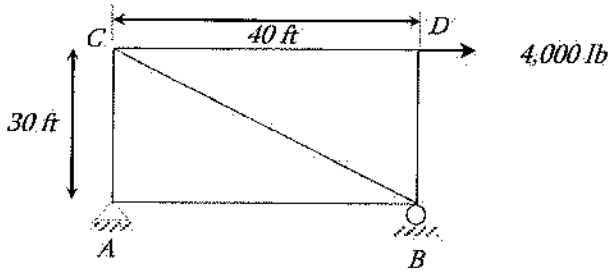
$= \alpha L_i(\Delta T)_i$ for temperature change in member (α = coefficient of thermal expansion)

= member misfit

L, A = member length and cross-sectional area respectively
E = member elastic modulus

Problem 13.21

A horizontal load of 4,000 lb is applied at joint D of the following truss. The corresponding member force due to this 4,000 lb load for each member of the truss is also listed in the table. The cross-sectional area of all members is equal to 5 in², and the modulus of elasticity is 29,000 ksi. The horizontal displacement of the joint D of the following truss is most nearly:



Member	Force (F), lb	Length (L), in	$\frac{FL}{AE}$ (lb.in/in².psi)
AB	4,000	480	0.01324
CD	4,000	480	0.01324
AC	3,000	360	0.00745
BD	0	360	0
BC	-5,000	600	-0.0207

A. 0.26 in
B. 0.06 in
C. 0.058 in
D. 1.11 in

Solution: C

$$\Delta_{joint} = \sum_{i=1}^{members} f_i(\Delta L)_i$$

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To determine f_i, a unit force is to be applied at joint D horizontally to the right. As the member force at each member is known due to the 4,000 lb load, the member force for unit load can be determined by dividing the member forces by 4,000 lb.

Member	Force (F), lb	Length (L), in	f	$f\left(\frac{FL}{AE}\right)$
AB	4,000	480	1	0.01324
CD	4,000	480	1	0.01324
AC	3,000	360	0.75	0.00558
BD	0	360	0	0
BC	-5,000	600	-1.25	0.0259
			$\Sigma =$	0.058

$$\Delta_{joint} = \sum_{i=1}^{members} f_i(\Delta L)_i = \sum_{i=1}^{members} f_i\left(\frac{FL}{AE}\right)_i = 0.058 \text{ inch}$$

Frame Deflection by Unit Load Method

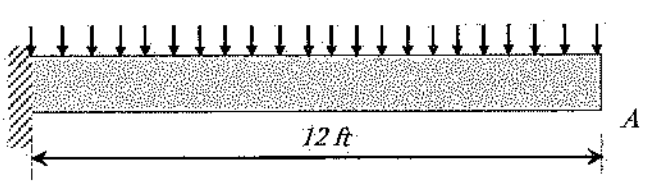
The displacement of any point on a frame caused by external loads is found by applying a unit load at that point that corresponds to the desired displacement:

$$\Delta_{joint} = \sum_{i=1}^{members} \int_{x=0}^{x=l_i} \frac{m_i M_i}{EI_i} dx$$

Δ = displacement at point of application of unit load (+ in direction of unit load)
m_i = moment equation in member "i" caused by the unit load
M_i = moment equation in member "i" caused by loads applied to frame
L_i = length of member "i"
I_i = moment of inertia of member "i"

If either the real loads or the unit load causes no moment in a member, that member can be omitted from the summation.

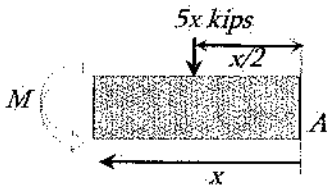
Problem 13.22
The deflection at A of the following beam is most nearly:
Assume, E = 29,000 ksi, and I = 1,500 in^4.



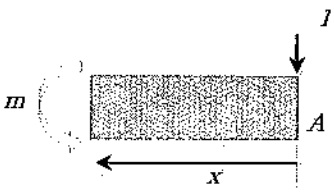
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A. 0.042 in
B. 0.51 in
C. 0.89 in
D. 1.23 in

Solution: B



$M = -5x\,(x/2) = -2.5x^2$ kips-ft



$m = -1x$ kips-ft

$$\Delta_A = \int_0^{L} \frac{mM}{EI}dx = \int_0^{12} \frac{(-1x)(-2.5x^2)}{EI}dx = \frac{2.5}{EI}\int_0^{12} x^3 dx = \frac{2.5}{4EI}\left[x^4\right]_0^{12} =$$

$$= \frac{2.5}{4(29{,}000 \text{ x } 144 \text{ ksf})\left(\frac{1{,}500}{12^4}\text{ ft}^4\right)}(12)^4 = 0.043\text{ ft} = 0.51\text{ in}$$

Positive sign means the direction of loading. So, the deflection is vertically downward.

Alternative Solution: From the NCEES FE Ref. Handbook, Mechanics of Materials

$$v_{max} = -\frac{wL^4}{8EI} = -\frac{5/12 \text{ kip/in } (12\text{x}12\text{in})^4}{8(29{,}000 \text{ ksi})(1{,}500 \text{ in}^4)} = -0.51 \text{ inch [-ve means downward]}$$

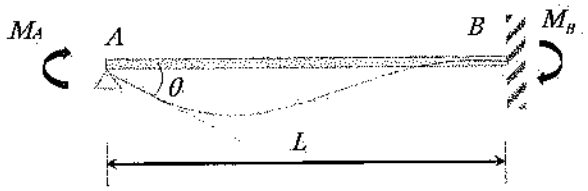
Note that you can use any method you like; both must give the same answer. The unit load method can be used for irregular loading for frame structure. However, the listed formulas in the Mechanics of Materials Section of the NCEES FE Ref. Handbook is for few simple beams with regular loading.

Beam Stiffness and Moment Carryover

Consider the following beam with one end pin and one end fixed. If a moment is applied at the end *A*, the beam is to rotate an angle of θ. Then, using mechanics the following relationship can be developed:

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$M_A = \left(\frac{4EI}{L}\right)\theta = k_{AB}\theta$, where k_{AB} is called the stiffness factor at A.

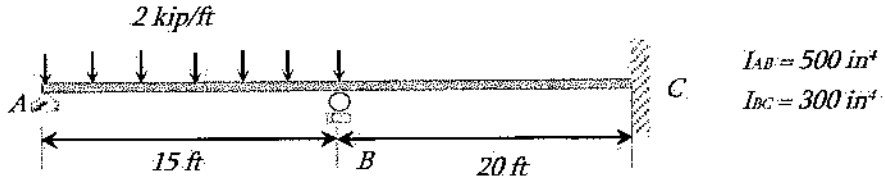


The resulting moment at B can be wrtten as: $M_B = \left(\frac{2EI}{L}\right)\theta = \frac{k_{AB}}{2}\theta$

Therefore, $M_B = \frac{M_A}{2}$, that means the moment at pin, A induces 50% of the moment to the far fixed end. In other words, the carry over factor of this moment is ½. If the end span is pin/roller supported then stiffness factor can be assumed as $k = \frac{3EI}{L}$, then no carry over will occur from the end pin/roller joint.

Problem 13.23

A two-span beam is having a uniformly distributed load of 2 kips/ft in one span and no load at another span as shown below. The lengths and moment of inertias are also given. What is most nearly the moment at C? Assume the stiffness factor of A and C are infinity.



A. -22.5 kip-ft B. -10.5 kip-ft C. -8.72 kip-ft D. -51.56 kip-ft

Solution: B

$$k_{BA} = \frac{3EI_{AB}}{L_{AB}} = \frac{3E(500\ \text{in}^4)}{15(12)\ \text{in}} = 8.33E \quad ; \quad k_{BC} = \frac{4EI_{BC}}{L_{BC}} = \frac{4E(300\ \text{in}^4)}{20(12)\ \text{in}} = 5E$$

$k_{CB} = \infty$.

Distribution factor:

$DF_{CB} = 0$ (end fixed support)

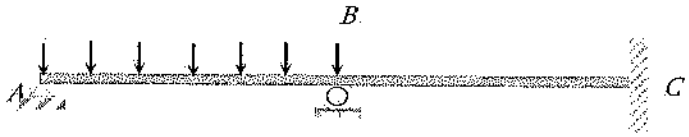
$DF_{AB} = 1$ (end pin support)

$$DF_{BA} = \frac{8.33E}{8.33E + 5E} = 0.625 \quad ; \quad DF_{BC} = \frac{5E}{8.33E + 5E} = 0.375$$

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$$FEM_{AB} = -\frac{wL^2}{12} = -\frac{2(15)^2}{12} = -37.5 \text{ kip.ft}$$

$$FEM_{BC} = \frac{wL^2}{12} = \frac{2(15)^2}{12} = 37.5 \text{ kips.ft}$$



Member	*AB*	*BA*	*BC*	*CB*
DF	1	0.625	0.375	0
FEM	-37.5 +37.5 →	37.5 18.75 -35.2	 -21.1 →	 -10.5
ΣM	0	+21.1	-21.1	-10.5

Approximate Analysis of Indeterminate Structures

Indeterminate structure can analytically be analyzed using the methods discussed above such as unit force method, moment distribution, etc. Logical assumptions can be made to solve indeterminate structure where too much precision is not essential, and time is an issue. Approximate methods of analyzing indeterminate truss are discussed here. Approximate methods of solving indeterminate frame structures are available such as portal method, cantilever method, assuming point of zero moment at 0.1L from end for vertical loading frame, etc. However, this is not listed in the NCEES FE Ref. Handbook, and therefore, the frame structure is omitted.

Truss. For indeterminate truss two scenarios can happen:

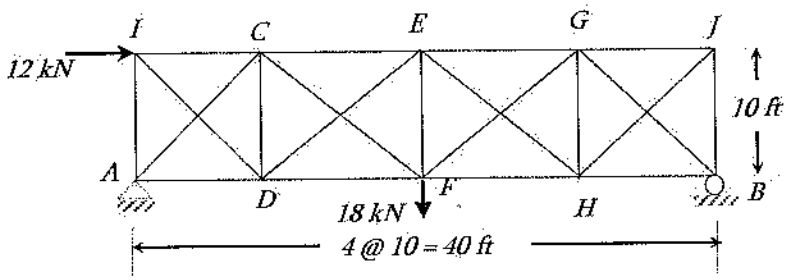
> Case I. If the diagonals are long and slender, it can be assumed that the diagonals can carry only tension force; compressive members can be assumed zero force member.
> Case II. If the diagonals are short and non-slender, it can be assumed that the compression and tension diagonals carry equal amount of force.

Practice the next two problems for better understanding.

Problem 13.24

What is most nearly the member force at *IC* member of the following truss? Assume the diagonals are too slender to carry compressive load.

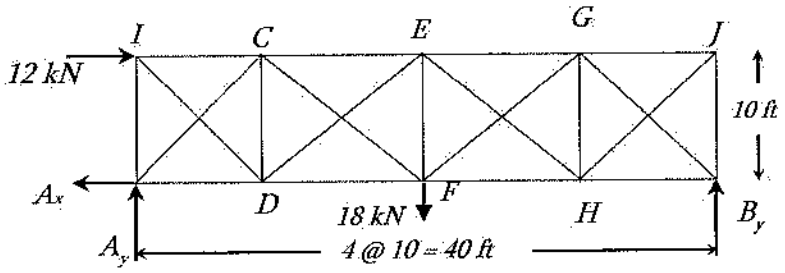
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A. 12 kN B. 8.48 kN C. -8.92 kN D. -14.26 kN

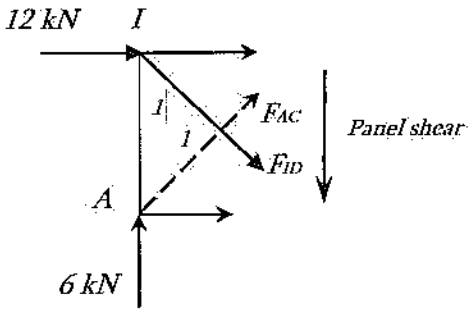
Solution: B

$b + r = 21 + 3 = 24$
$2j = 2(10) = 20 < 24$, indeterminate by 4 degrees.



ΣM at $B = 0$
12 kN (10 ft) – 18 kN (20 ft) + A_y (40 ft) = 0
$A_y = 6.0$ kN

Apply $\Sigma F_y = 0$ in the following cut section,
6 kN – Panel shear = 0
Panel shear = + 6 kN (downward)



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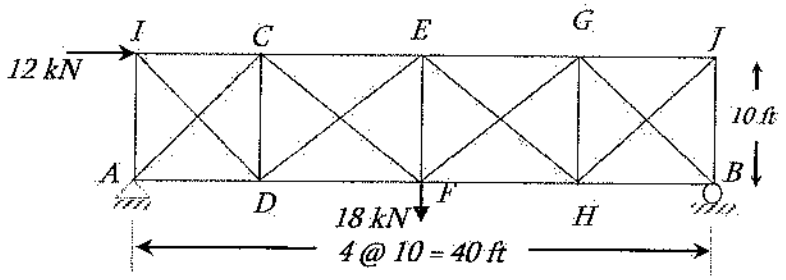
As panel shear is downward, diagonal AC is in compression and cannot take compression as the problem says diagonals cannot take compression. That means ID member will take the panel shear and can be written as:

$$F_{ID}\left(\frac{1}{\sqrt{2}}\right)=6$$

F_{ID} = 8.48 kN

Problem 13.25

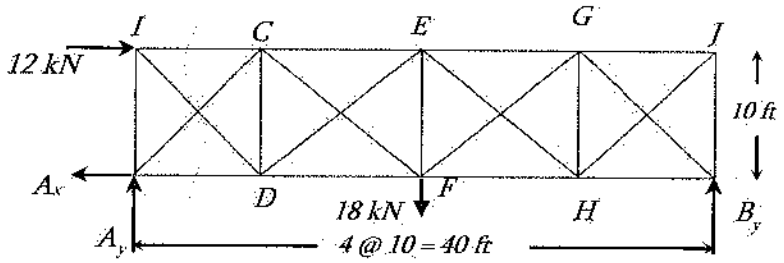
What is most nearly the member force at IC member of the following truss? Assume the diagonals carry equal tension and compression loads.



A. 12 kN B. -12 kN C. -15 kN D. -14.26 kN

Solution: C

$b + r = 21 + 3 = 24 > 2j = 2\,(10) = 20$, indeterminate by 4 degree



As diagonals can carry compressive load, it may be assumed that tension and compression, diagonals carry equal amount forces. Anyway, the reactions at the support A are needed to proceed. Let us find it.

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$\Sigma M_B = 0$

12 kN (10 ft) – 18 kN (20 ft) + A_y (40 ft) = 0

A_y = 6.0 kN

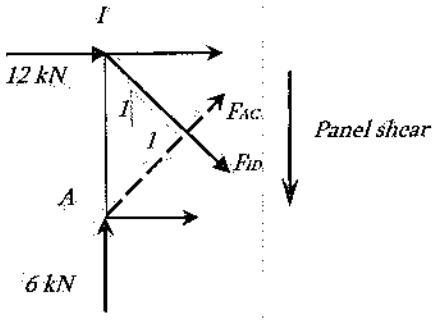
$\Sigma F_x = 0$

A_x = 12 kN

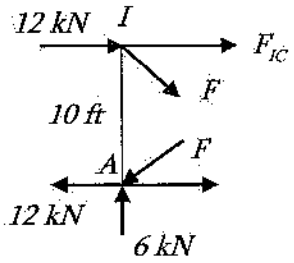
Apply $\Sigma F_y = 0$ in the following cut section,

6 kN – (Panel shear) = 0

Panel shear = + 6 kN (downward)



As panel shear is downward, diagonal AC is in compression and ID is in tension.



Let the force in each diagonal is F.

$\Sigma F_y = 0 \uparrow$ (+ve)

$\Rightarrow -F\frac{1}{\sqrt{2}} - F\frac{1}{\sqrt{2}} + 6 = 0$

$\Rightarrow F = 4.24$ kN

[Slope of diagonal is 1:1]

$\Sigma M_A = 0$

$\Rightarrow F_{IC}$ (10 ft) + 12 kN (10 ft) + $F(1/\sqrt{2})$ (10 ft) = 0

$\Rightarrow F_{IC}$ = -15 kN

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If you pass the NCEES FE exam, you will receive the congratulation letter only. If you fail (hopefully, you will not), you will receive a Diagnostic Report; a sample report is given below:

FE Civil

	Knowledge Area	Number of Items	Your Performance (on a scale of 0 - 15)	Your Performance Compared to the Average Performance of Passing Examinees Average of Passing Examinees ▮ Your Performance ▮
1	Mathematics	7	6.3	
2	Probability and Statistics	4	0.0	
3	Computational Tools	4	5.4	
4	Ethics and Professional Practice	4	8.0	
5	Engineering Economics	4	6.5	
6	Statics	7	6.9	
7	Dynamics	4	9.5	
8	Mechanics of Materials	7	10.4	
9	Materials	4	9.3	
10	Fluid Mechanics	4	9.3	
11	Hydraulics and Hydrologic Systems	8	10.1	
12	Structural Analysis	6	10.0	
13	Structural Design	6	11.2	
14	Geotechnical Engineering	9	8.1	
15	Transportation Engineering	8	8.0	
16	Environmental Engineering	6	6.8	
17	Construction	4	14.9	
18	Surveying	4	10.0	

- *If you cannot solve a problem anyhow, please make the best guess. There is no negative point for wrong answer.*
- *The author believes that the probability of the option 'C' be the correct answer is the highest. However, the author does not have any evidence for it.*
- *Sometimes, the longest option may be answer; the reason behind this is that if this is not the answer, then, the question maker may not invest his time to write a longer option.*

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14 STRUCTURAL DESIGN

Fundamentals of Engineering (FE) CIVIL Exam Specification mentions, 6 - 9 questions may appear in the exam based on Structural Design as follows:

A. Design of steel components (e.g., codes and design philosophies, beams, columns, beam-columns, tension members, connections)
B. Design of reinforced concrete components (e.g., codes and design philosophies, beams, slabs, columns, walls, footings)

Live Load Reduction

The effect on a building member of nominal occupancy live loads may often be reduced based on the loaded floor area supported by the member. A typical model used for computing reduced live load (as found in ASCE 7 and many building codes) is:

$$L_{\text{reduced}} = L_{\text{nominal}}\left(0.25 + \frac{15}{\sqrt{K_{LL}A_T}}\right) \geq 0.4L_{\text{nominal}}$$

$L_{nominal}$ = nominal live load given in the ASCE 7 or a building code
A_T = the cumulative floor tributary area supported by the member
$K_{LL}A_T$ = area of influence supported by the member
K_{LL} = ratio of area of influence to the tributary area supported by the member
= 4 (typical columns)
= 2 (typical beams and girders)

Load Combinations using Strength Design (LRFD or USD)

Nominal loads used in following combinations:

1. $1.4D$
2. $1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R)$
3. $1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (L \text{ or } 0.8W)$
4. $1.2D + 1.6W + 0.5(L_r \text{ or } S \text{ or } R)$

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5. $1.2D + E + L + 0.2S$
6. $0.9D + 1.6W$
7. $0.9D + E$

D = dead loads
E = earthquake loads
L = live loads (floor)
L_r = live loads (roof)
R = rain load
S = snow load
W = wind load

Problem 14.1

The nominal live load given in the ASCE 7 for a beam is 100 psf. The tributary area of loading for the beam is 300 ft². The reduced live load (psf) for that beam is most nearly:

A. 40
B. 56
C. 86
D. 96

Solution: C

$K_{LL} = 2$ for beams

$$L_{reduced} = L_{nominal}\left(0.25 + \frac{15}{\sqrt{K_{LL}A_T}}\right) \geq 0.4L_{nominal}$$

$$L_{reduced} = 100\left(0.25 + \frac{15}{\sqrt{2(300)}}\right) \geq 0.4(100) = 100(0.86) \geq 0.4(100)$$

$$L_{reduced} = 86 \geq 40$$

The reduced load is 86 psf.

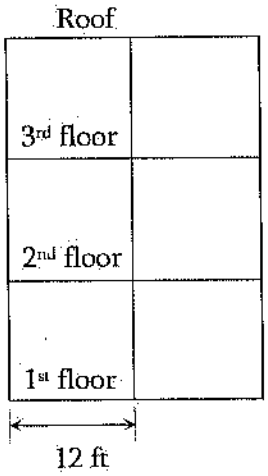
Problem 14.2

A three-story building has columns spaced at 12 ft. in both orthogonal directions and is subjected to the roof and floor loads shown below. The live load reduction factor on a typical interior column at the 1st floor is most nearly:

A. 0.76 B. 0.69 C. 0.57 D. 0.52

Solution: B

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$K_{LL} = 4$

$A_T = 288\ \text{ft}^2$ [2nd and 3rd stories]

$$\text{Factor} = (0.25 + \frac{15}{\sqrt{K_{LL} A_T}})$$

$$= (0.25 + \frac{15}{\sqrt{(4)(288)}})$$

$= 0.69$

Problem 14.3

A roof has the calculated snow load of 30 psf, the rain load of 25 psf, the roof live load of 40 psf, and a dead load of 50 psf. The factored load (psf) on the roof is most nearly:

A. 142 B. 222 C. 124 D. 202

Solution: C

$1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (L \text{ or } 0.8W) = (1.2)\ (50) + 1.6(40) + 0 = 124$ psf

Other load combinations do not win here.

Problem 14.4

A beam has a dead load of 150 lb/ft and a live load of 200 lb/ft. The factored load (lb/ft) on the beam is most nearly:

A. 300 B. 350 C. 450 D. 500

Solution: D

$w_u = 1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R) = 1.2(150\ \text{lb/ft}) + 1.6(200\ \text{lb/ft}) + 0 = 500$ lb/ft

Other load combinations do not win here.

Design of Reinforced Concrete Components

Definitions

a = depth of equivalent rectangular stress block, in

A_g = gross area of column, in^2

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A_s = area of reinforcement, in^2

A_{st} = total area of longitudinal reinforcement, in^2

A_v = area of shear reinforcement within a distance s, in.

b = width of compression face to member, in.

β_1 = ratio of depth rectangular stress block, a, to depth to neutral axis, c

$$= 0.85 \geq 0.85 - 0.05\left(\frac{f'_c - 4{,}000}{1{,}000}\right) \geq 0.65$$

c = distance from extreme compression fiber to neutral axis, in.

d = distance from extreme compression fiber to centroid of tension reinforcement

d_t = distance from extreme compression fiber to extreme tension steel, in.

E_c = modulus of elasticity $= 33 w_c^{1.5}\sqrt{f'_c}$

ε_t = net tensile strain in extreme tension steel at nominal strength.

f'_c = compressive strength of concrete, psi

f_y = yield strength of steel reinforcement, psi

M_n = nominal moment strength, in-lb

φM_n = design moment strength, in-lb

M_u = factored moment, in-lb

P_n = nominal axial load strength at given eccentricity, lb

φP_n = design axial load strength at given eccentricity, lb

P_u = factored axial force, lb

ρ_g = ratio of total reinforcement area to cross-sectional area of column $= \dfrac{A_{st}}{A_g}$

s = spacing of shear ties measured along longitudinal axis of member, in.

V_c = nominal shear strength provided by concrete, lb

V_n = nominal shear strength, lb

φV_n = design shear strength, lb

V_s = nominal shear strength provided by reinforcement, lb

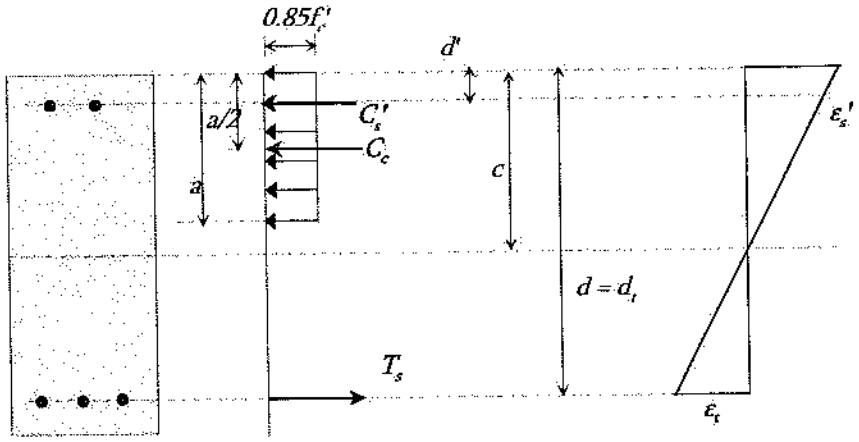
V_u =factored shear force, lb

Beams-Flexure Design Provisions

Internal Forces and Strains

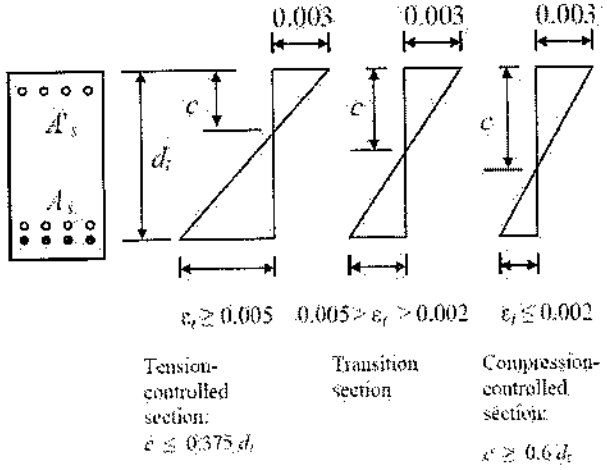
The stress-strain diagrams due to applied moment in a doubly reinforced beam section is shown below. The section has both tensile and compressive reinforcements. Reinforced concrete section is assumed cracked in tension zone. So, the stress in concrete in tension zone is assumed zero. The compressive stress in concrete is assumed rectangular with uniform stress of $0.85 f'_c$. Note that, the stress distribution in concrete is not linear or uniform. The depth of the stress block is a, and the depth of the neutral axis is c. However, the strain distribution is linear as shown.

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Strain Conditions:

The failure behavior (tension failure, or compression failure) can be determined using the strain value in the tensile steel. This is described below:



Resistance Factors, φ

The capacity of material is reduced by applying the strength reduction factor to adopt a safety against the material strength. The factors (ϕ) for different loading conditions are different. The factors are listed below:

Tension-controlled sections $(\varepsilon_t \geq 0.005): \phi = 0.9$

Compression-controlled sections: $(\varepsilon_t \leq 0.005): \phi = 0.65$ [Members with tied reinforcement]

Transition sections: $(0.002 < \varepsilon_t < 0.005): \phi = 0.48 + 83\varepsilon_t$ [Members with tied reinforcement]

Shear and torsion: $\phi = 0.75$

Bearing on concrete: $\phi = 0.65$

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Design Principle

Design Moment Capacity ≥ Factored Moment, $\phi M_n \geq M_u$

For All Beams

Net tensile strain: $\varepsilon_t = \dfrac{(0.003)(d_t - c)}{c}$ where, $a = \beta_1 c$

Design moment strength: $\phi M_n \geq M_u$ [ϕ-values are mentioned in the previous page]

Singly-Reinforced Beams

For a singly reinforced beam, equating the tensile force in tensile steel and the compressive force in concrete, the depth of the rectangular stress block can be determined as:

$$a = \frac{A_s f_y}{0.85 f_c' b}$$

Then, the design moment can be calculated as: $\phi M_n = \phi 0.85 f_c' ab\left(d - \dfrac{a}{2}\right) = \phi A_s f_y\left(d - \dfrac{a}{2}\right)$

Problem 14.5

What is the value of β_1 for a concrete with f_c' = 5 ksi?

A. 0.75 B. 0.80 C. 0.85 D. 0.90

Solution: B

$$\beta_1 = 0.85 \geq 0.85 - 0.05\left(\frac{f_c' - 4{,}000}{1{,}000}\right) \geq 0.65 = 0.85 - 0.05\left(\frac{5{,}000\,\text{psi} - 4{,}000}{1{,}000}\right) = 0.80$$

Problem 14.6

The elastic modulus (ksi) of a concrete with f_c' = 4 ksi and a unit weight = 150 pcf is most nearly:

A. 3,200
B. 3,400
C. 3,600
D. 3,800

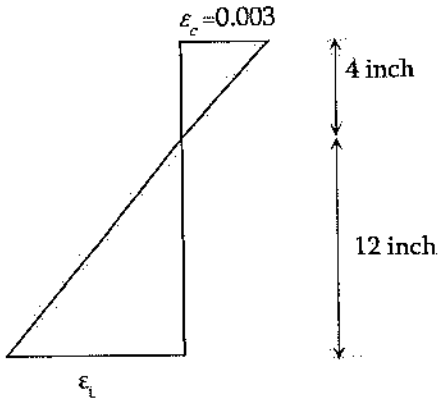
Solution: D

$$E_c = 33 w_c^{1.5} \sqrt{f_c'} = 33(150)^{1.5}\sqrt{4{,}000} = 3{,}834{,}253\,\text{psi} = 3{,}834\,\text{ksi}$$

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Problem 14.7

The value of the tensile strain (ε_t) of a reinforced concrete section whose strain diagram is shown below is most nearly:



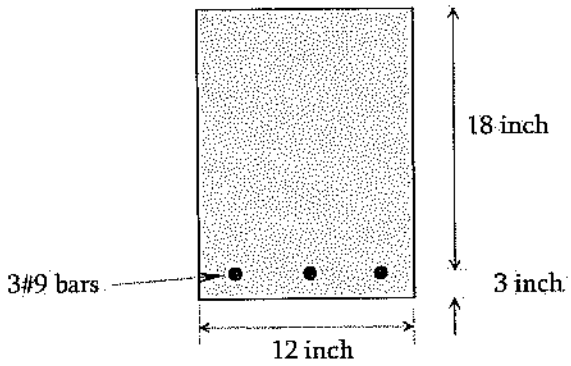
A. 0.9 B. 0.09 C. 0.009 D. 0.0009

Solution: C

$$\frac{\varepsilon_t}{12"}=\frac{0.003}{4"} \qquad \left| \varepsilon_t=\left(\frac{0.003}{4"}\right)(12")=0.009 \right.$$

Problem 14.8

The moment capacity, ϕM_n of the following reinforced concrete beam section is most nearly:
[ϕ= 0.9, f_c'= 4 ksi and f_y = 60 ksi]



A. 2,260 kip.in B. 2,360 kip.in C. 2,460 kip.in D. 2,560 kip.in

Solution: D

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$$A_s = (3)(1.0) = 3 \text{ in}^2$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{3 \text{ in}^2 (60{,}000 \text{ psi})}{0.85(4{,}000 \text{ psi})(12 \text{ in})} = 4.41 \text{ in.}$$

$$\varphi M_n = \varphi A_s f_y \left(d - \frac{a}{2} \right) = (0.9)(3 \text{ in}^2)(60 \text{ ksi}) \left(18 \text{ in} - \frac{4.41 \text{ in}}{2} \right) = 2{,}559 \text{ kip.in}$$

Beams-Shear Design Provisions

The shear design principle is: $\phi V_n \geq V_u$

The design shear force capacity of the section must be equal or greater than the external shear force of the section. In reinforced concrete, the shear is resisted by plain concrete and steel reinforcement. The nominal shear strength of the section can be determined as: $V_n = V_C + V_S$.
The nominal shear strengths of the concrete and steel can be determined as:

$$V_c = 2\sqrt{f_c'}(b_w d) \text{ and } V_S = \frac{A_V f_y d}{s} \quad \left(\text{may not exceed } 8 b_w d \sqrt{f_C'} \right)$$

Required and the maximum-permitted stirrup spacing, s:

No shear reinforcement is required if, $V_u \leq \frac{\phi V_c}{2}$

Use the following table if, $V_u > \frac{\phi V_c}{2}$

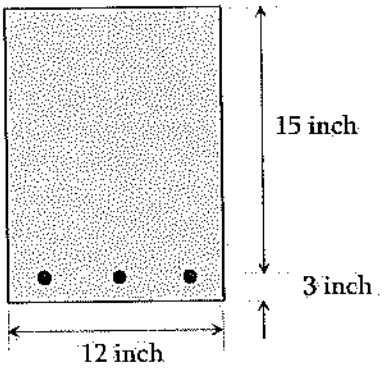
Table 14.1 Summary of Shear Reinforcement Design

	$\frac{\phi V_c}{2} < V_u < \phi V_c$	$V_u > \phi V_c$
Required spacing	Smaller of : $s = \frac{A_v f_y}{50 b_w}$, $s = \frac{A_v f_y}{(0.75)(b_w)\sqrt{f_c'}}$	$s = \frac{A_v f_y d}{V_s}$, $V_s = \frac{V_u}{\phi} - V_c$
Maximum permitted spacing	s_{max} smaller of: $\frac{d}{2}$ and 24 in	If $V_s \leq 4 b_w d \sqrt{f_c'}$ s_{max} smaller of: $\frac{d}{2}$ and 24 in
		If $V_s > 4 b_w d \sqrt{f_c'}$ s_{max} smaller of: $\frac{d}{4}$ and 12 in

Problem 14.9

The design shear strength (lbs) of the plain concrete in the following reinforced concrete section with f_c'=4,000 psi is most nearly:

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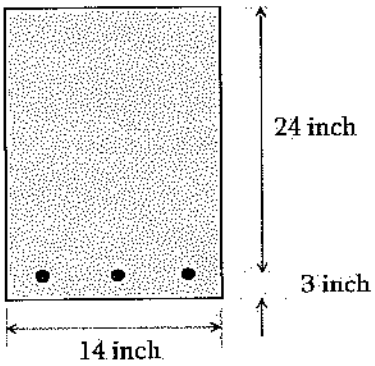
A. 22,768 B. 17,076 C. 20,865 D. 24,122

Solution: B

$$\varphi V_c = \varphi 2\sqrt{f_c'}(b_w d) = (0.75)(2)\sqrt{4,000}(12)(15) = 17,076 \text{ lbs}$$

Problem 14.10

The following reinforced concrete beam section has a factored shear force of 13 kips. Does it require web reinforcement? $f_c' = 3{,}000$ psi?



A. Yes
B. No
C. Cannot be determined
D. Shear reinforcement is always required

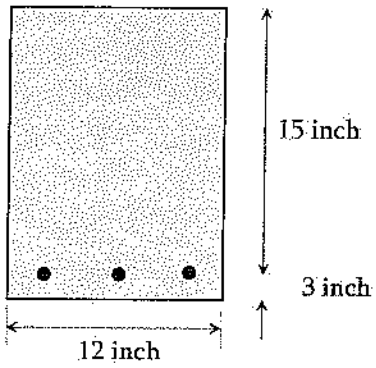
Solution: B

$$\frac{\varphi V_c}{2} = \frac{\varphi 2\sqrt{f_c'}b_w d}{2} = 0.75\sqrt{3,000}(14)(24) = 13,803 \text{ lbs} > 13 \text{ kips}, \text{ No stirrup needed}$$

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Problem 14.11

For the following reinforced concrete beam section, what is most nearly the maximum amount of shear force (lbs) that the web reinforcement can resist? $f_c' = 4{,}000$ psi $\phi = 0.75$.



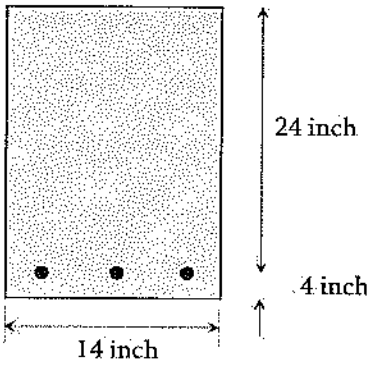
A. 48,300 B. 58,300 C. 68,300 D. 78,300

Solution: C

$$\varphi V_{s(\max)} = \varphi 8 b_w d \sqrt{f_c'} = (0.75)(8)(12)(15)\sqrt{4{,}000} = 68{,}305 \text{ lbs}$$

Problem 14.12

The following reinforced concrete beam section uses f_y of 60 ksi and f_c' of 4,000 psi. The maximum factored shear force experienced by the beam is 30,000 lbs. If a #3 U stirrup is used, the required spacing (inch) of the web reinforcement is most nearly:



A. 16 B. 17 C. 18 D. 19

Solution: C

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$$\frac{\varphi V_c}{2} = \frac{\varphi 2\sqrt{f_c'}\,b_w d}{2} = (0.75)(\sqrt{4,000})(14)(24) = 15,938 \text{ lbs}$$

$$\phi V_c = 2\frac{\phi V_c}{2} = 2 \text{ x } 15,938 \text{ lbs} = 31,876 \text{ lbs},$$

Therefore, $\frac{\phi V_c}{2} < V_u < \phi V_c$

Required spacing:

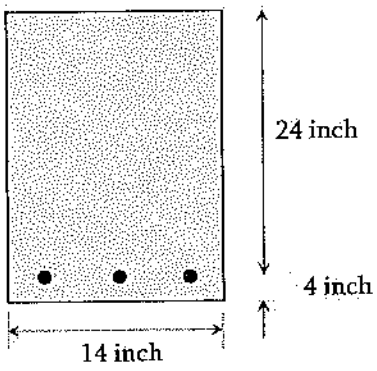
$A_v = 2(0.11 \text{ in}^2) = 0.22 \text{ in}^2$ as U-shaped #3 is used.

$$s = \frac{A_v f_y}{50 b_w} = \frac{2(0.11)(60,000)}{50(14)} = 18.9 \text{ in} \quad \text{(Controls)}$$

$$s = \frac{A_v f_y}{(0.75)(b_w)\sqrt{f_c'}} = \frac{2(0.11)(60,000)}{(0.75)(14)\sqrt{4,000}} = 19.9 \text{ in}$$

Problem 14.13

The following reinforced concrete beam section uses f_y of 60 ksi and f_c' of 4,000 psi. The maximum factored shear force experienced by the beam is 30,000 lbs. If a #3 U stirrup is used, the maximum spacing (in) permitted is most nearly:



A. 8 B. 12 C. 18 D. 24

Solution: B

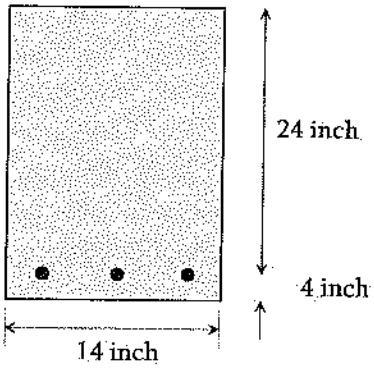
$$\frac{\varphi V_c}{2} = \frac{\varphi 2\sqrt{f_c'}\,b_w d}{2} = (0.75)(\sqrt{4,000})(14)(24) = 15,938 \text{ lbs}, \quad \phi V_c = 2\frac{\phi V_c}{2} = 31,876 \text{ lbs}$$

As, $\frac{\phi V_c}{2} < V_u < \phi V_c$, $s_{max} = \frac{d}{2} = \frac{24}{2} = 12$ in or $s_{max} = 24$ in (smaller one controls)

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Problem 14.14

The following reinforced concrete beam section uses 60-ksi steel and 4,000-psi concrete. The factored shear force experienced by the beam is 40,000 lbs. If a #3 U stirrup is used, the calculated required spacing (in) of the web reinforcement is most nearly:



A. 12 B. 18 C. 24 D. 29

Solution: D

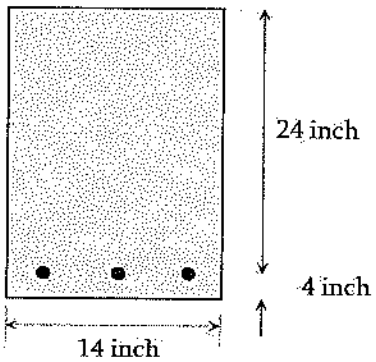
$$\varphi V_c = \varphi 2\sqrt{f_c'}\, b_w d = (0.75)(2)\sqrt{4,000}(14)(24) = 31,876 \text{ lbs}$$

$$\phi V_c = V_u > \phi V_c$$

$$s = \frac{A_v f_y d}{V_s} = \frac{A_v f_y d}{\frac{V_u}{\phi} - V_c} = \frac{2(0.11)(60,000)(24)}{\frac{40,000}{0.75} - \frac{31,876}{0.75}} = 29.2 \text{ in}$$

Problem 14.15

The following reinforced concrete beam section uses f_y of 60 ksi and f_c' of 4,000 psi. The factored shear force experienced by the beam is 100,000 lbs. If a #3 U stirrup is used, the required spacing (in) of the web reinforcement is most nearly:



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A. 2
B. 3.5
C. 12
D. 24

Solution: B

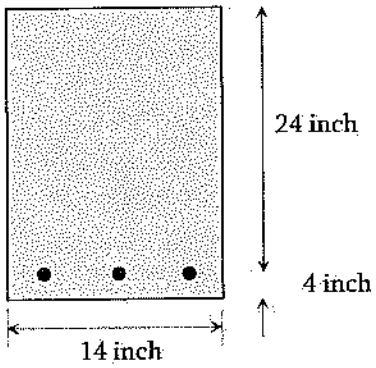
$$\varphi V_c = \varphi 2\sqrt{f_c'}\,b_w d = (0.75)(2)\sqrt{4,000}(14)(24) = 31,876 \text{ lbs}$$

$$V_u = 100,000 \text{ lbs}$$

As, $V_u > \phi V_c$, the required spacing, $s = \dfrac{A_v f_y d}{V_s} = \dfrac{2(0.11)(60,000)(24)}{90,832} = 3.49$ in

Problem 14.16

The following reinforced concrete beam section uses f_y of 60 ksi and f_c' of 4,000 psi. The factored shear force experienced by the beam is 100,000 lbs. If a #3 U stirrup is used, the maximum spacing (in) permitted is most nearly:



A. 18
B. 6
C. 12
D. 24

Solution: B

$$\varphi V_c = \varphi 2\sqrt{f_c'}\,b_w d = (0.75)(2)\sqrt{4,000}(14)(24) = 31,876 \text{ lbs}$$

$$V_s = \frac{V_u}{\varphi} - V_c = \frac{100,000}{0.75} - \frac{31,876}{0.75} = 90,832 \text{ lbs}$$

$4b_w d\sqrt{f_c'} = 4(14)(24)\sqrt{4,000}$ = 85,000 lbs < V_s (okay)

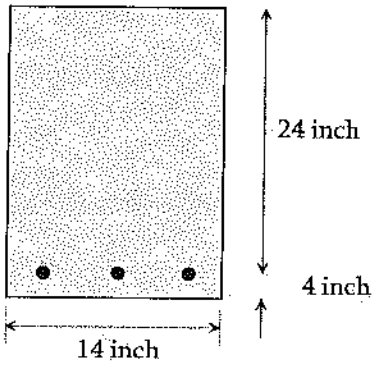
$s_{max} = 12$ inches from the Table

or $s_{max} = \dfrac{d}{4} = \dfrac{24}{4} = 6$ inches (answer)

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Problem 14.17

The following reinforced concrete beam section uses f_y of 60 ksi and f_c' of 4,000 psi. The maximum factored shear force experienced by the beam is 180,000 lbs. If a #3 U stirrup is used, the required spacing (in) of the web reinforcement is most nearly:



A. 4 B. 8 C. 12 D. None of the above

Solution: D

$$\varphi V_c = \varphi 2\sqrt{f_c'}\, b_w d = (0.75)(2)\sqrt{4,000}(14)(24) = 31,876 \text{ lbs}$$

$$V_s = \frac{V_u}{\varphi} - V_c = \frac{180,000}{0.75} - \frac{31,876}{0.75} = 197,500 \text{ lbs}$$

$$8 b_w d\sqrt{f_c'} = (8)(14)(24)\sqrt{4,000} = 170,000 \text{ lbs} < V_s$$

This section is not adequate.

Design of Short Columns

Limits for Main Reinforcements

Column is designed considering both concrete and steel resisting the load on it. The limits of steel ratio are mentioned below:

Steel Ratio limit: $1\% < \rho_g < 8\%$

where: Steel Ratio, $\rho_g = \dfrac{A_{st}}{A_g}$

Design Column Strength, Tied Columns

The design equation for tied column is shown below:

$$\phi P_n = \phi(0.8)\left[0.85 f_c'(A_g - A_{st}) + A_{st} f_y\right]$$

$$\phi = 0.65$$

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Problem 14.18

A tied reinforced concrete column is to support a dead load of 180 kips and a live load of 220 kips. For 2% steel, what would be the required gross area? Assume, $f_c' = 4$ ksi and $f_y = 60$ ksi?

A. 47.5 in^2 B. 57.5 in^2 C. 242 in^2 D. 75.7 in^2

Solution: C

$$P_u = 1.2D + 1.6L$$
$$P_u = 1.2(180\text{ k}) + 1.6(220\text{ k}) = 568\text{ k}$$
$$\varphi P_n = P_u = \varphi(0.8)[0.85 f_c'(A_g - A_{st}) + A_{st} f_y \qquad [A_{st} = \rho_g A_g]$$
$$568\text{ k} = 0.65(0.8)[0.85(4\text{ ksi})(A_g - 0.02A_g) + 0.02A_g(60\text{ ksi})]$$
$$A_g = 242\text{ in}^2$$

Problem 14.19

An axial factored load of 340 kips is applied on a reinforced concrete tied column (18 in x 18 in) with 8-inch eccentricity about one axis. What is the required steel amount for this column? [$f_c' = 4$ ksi, $\gamma = 0.80$, $f_y = 60$ ksi, $\phi = 0.65$]

A. 1.2 in^2 B. 1.8 in^2 C. 4.5 in^2 D. 2.8 in^2

Solution: C

$$A_g = 18x18 = 324\text{ in}^2;\ e = 8\text{ inch};\ h = 18\text{ inch}$$
$$R_n = \frac{P_u e}{\phi f_c' A_g h} = \frac{340\text{k}(8\text{ in})}{0.65(4\text{ksi})(324\text{ in}^2)(18\text{ in})} = 0.18$$
$$K_n = \frac{P_u}{\phi f_c' A_g} = \frac{340\text{k}}{0.65(4\text{ksi})(324\text{ in}^2)} = 0.40$$

From the Graph A.11, $\rho \approx 0.014$ [Try your best interpolation]

$$A_{st} = \rho bh = (0.014)(18\text{ in})(18\text{ in}) = 4.5\text{ in}^2$$

Problem 14.20

The ACI recommended minimum center to center reinforced bar spacing in common reinforced concrete structural member is most nearly:

A. 1.5 in B. 3.0 in C. 1 in D. 1.5 times the bar diameter

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Solution: C

The minimum center to center reinforced bar spacing is 1 inch or the bar diameter.

Problem 14.21

Strength reduction factor (ϕ) is used while designing reinforced concrete structural member following the ultimate strength design method to –

A. account uncertainties in material strength
B. account uncertainties in member dimensions
C. account uncertainties in construction errors
D. All of the above

Solution: D

Strength reduction factor (ϕ) is used to account uncertainties in material strength, dimensions, construction errors, and so on.

Problem 14.22

The minimum flexural reinforcement is recommended in reinforced concrete member in the ultimate strength design method to ensure the ultimate resisting moment of steel –

A. is not less than concrete compressive strength
B. is less than the concrete cracking moment
C. is not less than the concrete cracking moment
D. All of the above

Solution: C

The minimum flexural reinforcement is provided to ensure the ultimate resisting moment of the steel is not less than the concrete cracking moment. Otherwise, the section may fail immediately when a crack in concrete occurs.

Problem 14.23

The minimum stirrup diameter recommended for common reinforced concrete structural member is most nearly:

A. 3/4 inch
B. 3/8 inch
C. 5/8 inch
D. 0.5 times of the bar diameter

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Solution: B

The minimum stirrup diameter is 3/8 inch (#3) for main reinforcement up to #10. For main reinforcement above #10, at least 0.5 inch (#4) is recommended.

Problem 14.24

The minimum clear cover recommended for common interior reinforced concrete beam is most nearly:

A. 3 inch
B. 1.5 inch
C. 5/8 inch
D. 0.5 times of the bar diameter

Solution: B

The minimum clear cover for beam in normal environment is 1.5 in, inside water/soil is 3 in, and for slab it is 0.75 in.

Problem 14.25

The minimum steel ratio recommended for common reinforced concrete column is most nearly:

A. 1% of the gross column area
B. 2% of the gross column area
C. 3% of the gross column area
D. 8% of the gross column area

Solution: A

The steel ratio in column should have the minimum of 1% and the maximum of 8% of the gross column area.

Problem 14.26

The minimum tie size recommended for common reinforced concrete column is most nearly:

A. #3 B. #5 C. #7 D. #6

Solution: A

For tied column, the ties should not be less than #3 for longitudinal bar #10 or smaller. The minimum tie size is #4 for longitudinal bar greater than #10 and bundled bars.

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Problem 14.27

The minimum number of reinforcing bar recommended for rectangular reinforced concrete column is most nearly:

A. 3 B. 4 C. 5 D. 6

Solution: B

The minimum number of bars in column is 4 for rectangular section, 3 for triangular section, 6 for spirally enclosed section in reinforced concrete column.

Problem 14.28

The center-to-center spacing of tie in reinforced concrete column should not be more than-

A. 16 times the diameter of the longitudinal bars
B. 48 times of the tie diameter
C. The least lateral dimension of the column
D. All of the above

Solution: D

The center-to-center spacing of tie should not be more than 16 times the diameter of the longitudinal bars, 48 times of the tie diameter, or the least lateral dimension of the column.

Problems 14.29

Which of the following reinforced concrete members has lower strength reduction factor (ϕ)?

A. Tied column
B. Spiral column
C. Beam
D. All should have equal strength reduction factor

Solution: A

The strength reduction factor (ϕ) for tied column is 0.65, for spiral column is 0.7, and for beam is 0.9. Factor, α is used to account accidental eccentricity in column loading, which is 0.8 for tied column and 0.85 for spiral column.

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Design of Steel Components

Beams

For doubly symmetric compact I-shaped members bent about their major axis, the design flexural strength, $\phi_b M_n$ is determined with $\phi_b = 0.90$ as follows:

Yielding Condition: $M_n = M_p = F_y Z_x$

where:

F_y = specified minimum yield stress
Z_x = plastic section modulus about the x-axis

Lateral-Torsional Buckling

The design moment capacity of a beam depends on bracing where L_b is the length between points that are either braced against lateral displacement of the compression flange or braced against twist of the cross section with respect to the length limits, L_p and L_r. The values of L_p and L_r for different sections are listed in the NCEES FE Ref. Handbook.

When $L_b \le L_p$, the limit state of lateral-torsional buckling does not apply.

When $L_p \le L_b \le L_r$,

$$M_n = C_b\left[M_p - \left(M_p - 0.7F_y S_x\right)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p$$

$$M_n = C_b\left[M_p - BF\left(L_b - L_p\right)\right] \le M_p$$

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C}$$

M_{max} = absolute value of maximum moment in the unbraced segment
M_A = absolute value of maximum moment at quarter point of the unbraced segment
M_B = absolute value of maximum moment at centerline of the unbraced segment
M_C = absolute value of maximum moment at three-quarter of the unbraced segment

$$BF = \frac{M_p - 0.7F_y S_x}{L_r - L_p}$$

Values of BF for different sections can be found in the NCEES FE Ref. Handbook.
C_b can be assumed 1.0 if not given.

The NCEES FE Ref. Handbook does not include the design for $L_p > L_r$.

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Shear

The design shear strength, $\phi_n V_n$ is determined with $\phi_n = 1.0$ for webs of rolled I-shaped members and is determined as follows: $V_n = 0.6F_y(dt_w)$

Problem 14.30

A steel wide flange section, W14 x 90 stands for –

A. A nominal depth of 14 in.
B. A self-weight of 90 lb./ft.
C. Both of the above
D. None of the above

Solution: C

Wide flange section, W14 x 90 stands for nominal depth of 14 in and self-weight of 90 lb./ft.

Problem 14.31

An angle section, L4 x 3 x 1/4 implies

A. A member with a long leg length of 3 in.,
B. A short leg length of 4 in.,
C. Self-weight of ¼ lb per feet
D. A thickness of ¼ inch

Solution: D

Problem 14.32

A HSS 6 x 0.375 implies a round hollow structural steel with an outside diameter of 6 in. and –

A. A uniform wall thickness of 0.375 in.
B. A uniform wall thickness of 0.375 ft
C. Self-weight of 0.375 lb per feet
D. Self-weight of 0.375 lb per inch

Solution: A

Problem 14.33

The strength reduction factor for steel beam design considering moment is most nearly:

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A. 1.0 B. 0.9 C. 0.75 D. 0.65

Solution: B

From the NCEES Ref. Handbook, ϕ_b=0.9.

Problem 14.34

The strength reduction factor for steel beam design considering shear is most nearly:

A. 1.0 B. 0.9 C. 0.75 D. 0.65

Solution: A

Problem 14.35

The shear strength capacity (kips) of W 18 x 55 steel section with yield stress of 50 ksi is most nearly:

A. 122 B. 221 C. 212 D. 197

Solution: C

From the NCEES Ref. Handbook, ϕV_{nx}=212 kips.

Problem 14.36

A steel W 18 x 40 section with 50 ksi yield stress has a laterally unsupported length of 5.0 ft, the moment capacity (kip.ft) of this beam is most nearly:

A. 294 B. 180 C. 810 D. 924

Solution: A

From FE Ref, Handbook, for a W 18 x 40, L_p=5.55ft, L_r=15.9 ft
As $L_b < L_p$, capacity, $\phi_b M_{px}$= 294 kip ft

Problem 14.37

The limit state of lateral torsional buckling not apply, if –

A. $L_b < L_r$
B. $L_b \leq L_p$
C. $L_b > L_p$

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D. $L_b < L_p < L_r$

Solution: B

The limit state of lateral torsional buckling does not apply for $L_b \leq L_p$.

Problem 14.38

A steel W 21x50 section with 50 ksi yield stress has laterally unsupported length of 4 ft. The nominal moment capacity (kip.ft) of the section is most nearly:

A. 413 B. 248 C. 459 D. 549

Solution: C

For W 21 x 50 section, $L_p = 4.59$ft, $L_r = 13.6$ ft.
As $L_b < L_p$, Design/capacity, $\phi_b M_{px} = 413$ kip-ft
So, $M_{px} = 413/\phi_b = 459$ kip-ft

Problem 14.39

A steel W section has a depth of 12 in and web-thickness of 0.25 in. If A 36 steel is used, the nominal shear strength (kips) of the material is most nearly:

A. 65
B. 72.5
C. 56
D. 58

Solution: A

$V_n = 0.6F_y(dt_w) = (0.6)(36 \text{ ksi})(12 \text{ in})(0.25 \text{ in}) = 64.8$ kips

Problem 14.40

A steel W- section has a depth of 18 in and web thickness of 0.33 in. A steel with 42 ksi yield strength is used. The design shear strength (kips) of the material is most nearly:

A. 150 B. 160 C. 170 D. 144

Solution: A

From the NCEES Ref. Handbook, AISC T 1-1, $d = 18$ in, and $t_w = 0.33$ inch. AISC T 3-2 cannot be used as $F_y = 42$ ksi. So, $\phi_v V_n = \phi_v(0.6)F_y(dt_w) = (1.0)(0.6)(42)(18)(0.33) = 150$ kips

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Problem 14.41

A steel beam section used 50 ksi steel. The client wants to use W 21 x 48 section. For this condition, the maximum unbraced length (ft) possible for that beam if the design moment is less than 300 kip.ft is most nearly:

A. 12 B. 13 C. 14 D. 19.5

Solution: B

From the NCEES FE Ref. Handbook, AISC Table 3.10, W 21 x 48 section can have a maximum unbraced length of 13.0 ft.

Problem 14.42

A steel beam section used 50 ksi steel. The client wants to use W 21 x 44 section. For this condition, what is most nearly the maximum unbraced length possible for that beam if the design moment is just less than 315 kip.ft? $F_y = 50$ ksi.

A. 8 ft B. 10 ft C. 12 ft D. 14 ft.

Solution: A

W 21 x 44 section can have the maximum unbraced length of 8.0 ft (AISC Table 3.10). The maximum length 8.0 ft is possible if the design moment is just less than 315 kip.ft.

Problem 14.43

A simply supported beam has 3 concentrated loads (equally spaced), load at quarter points. If the beam is fully laterally unsupported, the value of C_b is most nearly:

A. 1.67 B. 1.32 C. 1.30 D. 1.14

Solution: D

See the Table listed in the Steel Subsection.

Problem 14.44

For a steel W 21 x 55 section, the polar radius of gyration (in) is most nearly:

A. 8.4 B. 8.6 C. 1.73 D. 2.48

Solution: B

For W 21 x 55, $A = 16.2$ in^2, $I_x = 1140$ in^4, $I_y = 48.4$ in^4

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$$\text{Polar radius of gyration, } r_j = \sqrt{\frac{I_x + I_y}{A}} = \sqrt{\frac{1140 + 48.4}{16.2}} = 8.56 \text{ in} \approx 8.6 \text{ in}$$

Problem 14.45

For a steel section, W 14 x 74, the available area (in²) to resist shear is most nearly:

A. 6.01 B. 6.4 C. 2.8 D. 2.8

Solution: B

From the NCEES FE Ref. Handbook, for W 14 x 74, $d = 14.2$ in and $t_w = 0.45$ in
Shear area = dt_w = (14.2 in) (0.45 in) = 6.39 in².

Columns

The design compressive strength, $\phi_c P_n$ is determined with $\phi_c = 0.9$ for flexural buckling of members without slender elements and is determined as follows:

$$\phi P_n = \phi F_{cr} A_g$$

where the critical stress, F_{cr} is determined as follows:

a) When, $\frac{KL}{r} \le 4.71\sqrt{\frac{E}{F_y}}$, $F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right] F_y$

b) When, $\frac{KL}{r} > 4.71\sqrt{\frac{E}{F_y}}$, $F_{cr} = 0.877 F_e$

KL/r is the effective slenderness ratio. KL is determined from AISC Table C-A-7.1 or AISC figures C-A-7.1 and C-A-7.2 of the NCEES Ref. Handbook.

The elastic buckling stress, $F_e = \frac{\pi^2 E}{\left(\frac{KL}{r_{min}}\right)^2}$

Problem 14.46

According to the LRFD design approach, what is the axial load capacity of W 14 x 48 section with respect to minimum radius of gyration if KL = 15 ft? F_y = 50 ksi.

A. 232 kips B. 298 kips C. 332 kips D. 356 kips

Solution: C

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From the AISC Table 4-1 of the NCEES Ref. Handbook, $\phi P_n = 332$ kips.

Problem 14.47

Per the LRFD design approach, what is the axial load capacity of W 10 x 60 section with respect to minimum radius of gyration if $KL = 15$ ft? $F_y = 50$ ksi.

A. 332 kips B. 498 kips C. 555 kips D. 656 kips

Solution: C

From the AISC Table 4-1 of the NCEES Ref. Handbook, $\phi P_n = 555$ kips.

Problem 14.48

Is W14 x 53 section with $KL = 34$ with respect to least radius of gyration a slender section? $F_y = 50$ ksi.

A. Yes B. No C. Cannot be determined

Solution: A

From the AISC Table 4-1 of the NCEES Ref. Handbook, yes. $KL = 34$ is below the heavy line. Heavy line indicates $\frac{KL}{r} > 200$.

Problem 14.49

A steel section has the following properties:

-Area = 20 in^2
-Both ends pinned
-Unsupported length = 20 ft
-Least radius of gyration =1.52 inch
-F_y =50 ksi.

What is the compression load capacity (kips) of that column?

A. 78 B. 521 C. 600 D. 163

Solution: D

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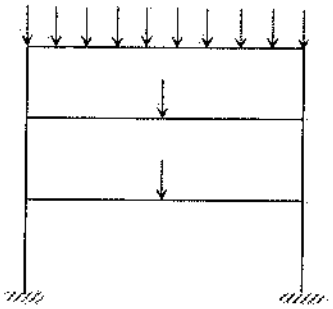
$$\frac{KL}{r} = \frac{1(20\text{x}12)\text{ in}}{1.52\text{ in}} = 158$$

From the AISC Table 4-22 the NCEES Ref. Handbook, ϕF_{cr} =9.05 ksi

Therefore, $\phi_c P_n = \phi_c A_g F_{cr}$ = (0.9) (20 in^2) (9.05 ksi) = 162.9 kips ≈ 163 kips

Problem 14.50

What is the recommended effective length factor of the following base column in the following structure? The foundation is fixed and the upper joints pinned, and side-sway is not prohibited in this structure.



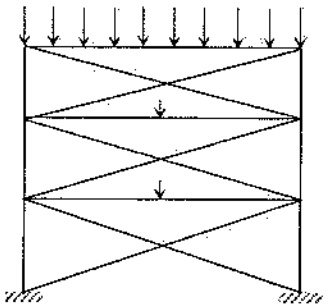
A. 1.0 B. 0.5 C. 2.1 D. 0.8

Solution: C

From Table C-A-7.1 of the NCEES Ref. Handbook, 2.1 is most appropriate.

Problem 14.51

The recommended effective length factor of the following base column in the following braced-frame structure is most nearly:



A. 1.0 B. 0.5 C. 2.1 D. 0.8

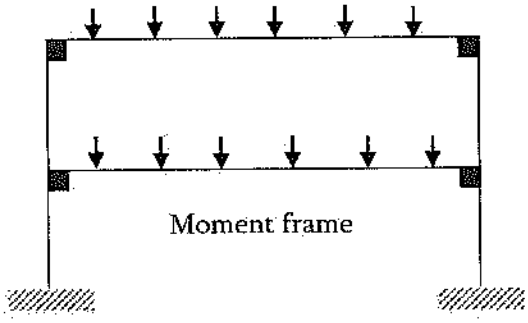
Solution: D

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From Table C-A-7.1 (in Civil Engineering section of the NCEES Ref. Handbook), 0.8 (fixed-pinned): the lowest end is fixed, and the upper ends are pinned with side-sway inhibited.

Problem 14.52

The recommended effective length factor for the following base column is most nearly:



A. 1.0 B. 0.5 C. 2.1 D. 1.2

Solution: D

From Table C-A-7.1 (in Civil Engineering section of the NCEES Ref. Handbook), 1.2 (moment frame, translation free). The lowest end is fixed, and the upper ends are pinned with side-sway is not inhibited.

Problem 14.53

The flexural critical buckling stress of a steel W16 x 67 section is 10 ksi. What is the design compressive strength of that column? $F_y = 50$ ksi.

A. 98 kips B. 122 kips C. 156 kips D. 177 kips

Solution: D

From the NCEES Ref. Handbook, Civil Engineering section, $A_g = 19.7$ in^2

$\phi_c P_n = \phi_c F_{cr} A_g = (0.9)\ (10\ \text{ksi})\ (19.7) = 177.3$ kips.

Problem 14.54

What is the critical buckling stress of a W 10 x 39 steel section with 10 ft length? Assume, K=1.0.

A. 66 kips B. 78 kips C. 88 kips D. 96 kips

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Solution: B

From the NCEES Ref. Handbook, for W10 x 39, $r_{min} = 1.98$ in.

Buckling stress, $F_e = \dfrac{\pi^2 E}{\left(\dfrac{KL}{r_{min}}\right)^2} = \dfrac{\pi^2 (29{,}000 \text{ ksi})}{\left(\dfrac{(1.0)(10)(12) \text{ in}}{1.98 \text{ in}}\right)^2} = 77.9 \text{ ksi}$

Problem 14.55

A both ends pinned column section has a length of 10 ft. and the minimum radius of gyration of 1.5 in. If A36 steel is used, the critical stress (F_{cr}) of that column is most nearly:

A. 44.75 ksi B. 25.7 ksi C. 36 ksi D. None of the above

Solution: B

It is not possible to use the NCEES Ref. Handbook as F_y is not 50 ksi. Therefore, hand calculation required.

$$\frac{KL}{r} = \frac{(1.0)(10)(12) \text{ inch}}{1.5 \text{ inch}} = 80$$

As, $4.71\sqrt{\dfrac{E}{F_y}} > \dfrac{KL}{r}$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000 \text{ ksi}}{36 \text{ ksi}}} = 134$$

$$F_e = \frac{\pi^2 E}{\left(\dfrac{KL}{r}\right)^2} = \frac{\pi^2 (29{,}000)}{80^2} = 44.72 \text{ ksi}$$

$$F_{cr} = [0.658^{\frac{F_y}{F_e}}]F_y = [0.658^{\frac{36}{44.72}}]36 \text{ ksi} = 25.7 \text{ ksi}$$

Problem 14.56

A both ends pinned column section has a length of 10 ft. and the minimum radius of gyration of 1.5 in. If 50 ksi steel is used the critical stress (F_{cr}) of that column is most nearly:

A. 28.2 ksi
B. 31.3 ksi
C. 33.1 ksi
D. 22.8 ksi

Solution: B

It is possible to use the NCEES Ref. Handbook as F_y is 50 ksi.

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$\frac{KL}{r} = \frac{(1.0)(10)(12)\text{ inch}}{1.5\text{ inch}} = 80$. From the NCEES Ref. Handbook, Table 4-22, $\phi_c F_{cr} = 28.2$ ksi.

Therefore, $F_{cr} = \frac{28.2}{0.9} = 31.3$ ksi .

Problem 14.57

A both ends fixed column has a length of 10 ft and uses a W12 x 79 section. If 50 ksi steel is used, the available design critical flexural buckling stress, ϕF_{cr} is most nearly:

A. 42.8 ksi
B. 52.8 ksi
C. 38.8 ksi
D. 56.7 ksi

Solution: A

It is possible to use the NCEES Ref. Handbook as F_y is 50 ksi. $\frac{KL}{r} = \frac{(0.65)(10)(12)\text{ in}}{3.05\text{ in}} = 26$

($r_{min} = r_y = 3.05$ in. from the NCEES Ref. Handbook, Table 1-1). From the NCEES Ref. Handbook, Table 4-22, $\phi_c F_{cr} = 42.8$ ksi.

Problem 14.58

For a sideway uninhibited (moment frame), the value of K if $G_A = 4.0$, and $G_B = 4.0$ is most nearly:

A. 1.8 B. 2.0 C. 2.5 D. 3.0

Solution: B

From Table C-A-7.1 of the NCEES Ref. Handbook, $K = 2.05$

Problem 14.59

For a sideway inhabited (braced frame), the value of the K if $G_A = 2.0$, and $G_B = 0.5$ is most nearly:

A. 0.26 B. 0.65 C. 0.75 D. 0.85

Solution: C

From Table C-A-7.1 of the NCEES Ref. Handbook, $K = 0.76$

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Problem 14.60

A column is expected have a factored load of 100 kips. If the strength reduction factor is 0.9, the minimum nominal capacity (kips) of the column is most nearly:

A. 81 B. 91 C. 101 D. 111

Solution: D

$$\phi P_n = P_u \qquad \text{Then,} \quad P_n = \frac{P_u}{\phi} = \frac{100}{0.9} = 111 \text{ kips}$$

Tension Members

Definitions

Bolt diameter: d_b

Nominal hole diameter: $d_h = d_b + \frac{1}{16}\text{inch}$

Gross width of member: b_g; Member thickness: t

Connection eccentricity: $\bar{x}$

Gross area: $A_g = b_g t$ (use tabulated areas for angles)

Net area (parallel holes): $A_n = \left[b_g - \sum \left(d_b + \frac{1}{16}\text{inch} \right) \right] t$

Net area (staggered holes): $A_n = \left[b_g - \sum \left(d_b + \frac{1}{16}\text{inch} \right) + \sum \frac{s^2}{4g} \right] t$

NCEES used d_b instead of d_h in the above 2 equations which is wrong as $d_h = d_b + \frac{1}{16}$

s = longitudinal spacing of consecutive holes

g = transverse spacing between lines of holes

Effective area (bolted members): $A_e = UA_n$ $\begin{cases} \text{Flat bars} : U = 1.0 \\ \text{Angles} : U = 1 - \frac{\bar{x}}{L} \end{cases}$

Effective area (welded members):

$$A_e = UA_n \begin{cases} \text{Flat bars or angles with transverse welds} : U = 1.0 \\ \text{Flat bars of width } w, \textit{longitudinal} \text{ welds of length } L : U = 1.0 \left(L \geq 2w \right) \\ \qquad\qquad U = 0.87 \left(2w > L \geq 1.5w \right) \\ \qquad\qquad U = 0.75 \left(1.5w > L > w \right) \\ \text{Angles with } \textit{longitudinal} \text{ welds only, } U = 1 - \frac{\bar{x}}{L} \end{cases}$$

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Limit States and Available Strengths

Yielding: $\phi_y = 0.90$ and $\phi_y T_n = \phi_y F_y A_g$

Fracture: $\phi_f = 0.75$ and $\phi_f T_n = \phi_f F_u A_e$

Block shear: $\phi = 0.75$

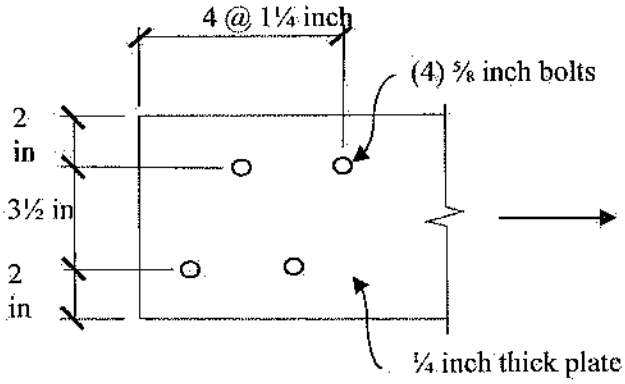
$U_{bs} = 1.0$ (flat bars and angles); A_{gv} = gross area for shear

A_{nv} = net area for shear; A_{nt} = net area for tension

$$\phi P_n = \text{smaller of} \begin{cases} \phi\left[(0.60)(F_u)(A_{nv}) + (U_{bs})(F_u)(A_{nt})\right] \\ \phi\left[(0.60)(F_y)(A_{gv}) + (U_{bs})(F_u)(A_{nt})\right] \end{cases}$$

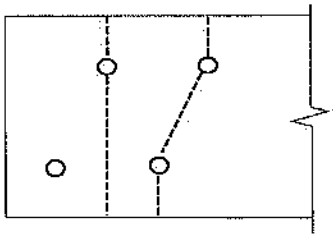
Problem 14.61

The net area (in²) of the following steel section is most nearly:



A. 1.73
B. 1.64
C. 1.94
D. 1.70

Solution: A



Linear Failure Option

$$A_{n1} = A_g - \Sigma d_h t$$

$$A_{n1} = \left(7\tfrac{1}{2} \cdot \tfrac{1}{4}\right) - 1\left(\tfrac{5}{8} + \tfrac{1}{16}\right)\left(\tfrac{1}{4}\right) = 1.70 \text{ in}^2$$

NCEES used (d_h +1/16) which is wrong as $d_h = d_b + \dfrac{1}{16}$

Non-Linear Failure Option

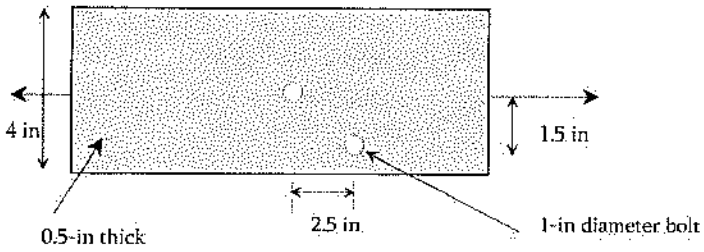
$$A_{n2} = A_g - \Sigma d_h t + \Sigma \frac{s^2}{4g} t$$

$$A_{n2} = \left(7\tfrac{1}{2} \cdot \tfrac{1}{4}\right) - 2\left(\tfrac{5}{8} + \tfrac{1}{16}\right)\left(\tfrac{1}{4}\right) + \frac{\left(1\frac{1}{4}\right)^2}{\left(4 \cdot 3\frac{1}{2}\right)}(0.25) = 1.728 \text{ in}^2$$

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Problem 14.62

The net area (in²) of the following steel section is most nearly:



A. 1.46 B. 0.94 C. 1.22 D. 0.76

Solution: A

Failure through linear line: $A_n = A_g - A_{holes}$

$A_g = (0.5")(4") = 2.0 \text{ in}^2$

$A_{holes} = n\left(d_h + \frac{1}{16}\right)t = 2\left(1" + \frac{1}{16}\right)(0.5") = 1.0625 \text{ in}^2$ [NCEES used (d_h +1/16) which is wrong as $d_h = d_b + \frac{1}{16}$]

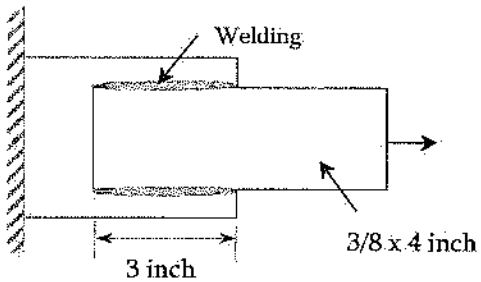
$A_n = 2" - 1.0625 = 0.9375 \text{ in}^2$ (controls)

Failure through a non-linear line:

$$A_n = \left(w_g - \sum d_h + \sum \frac{s^2}{4g}\right)t = 2 - \left(2\left(1 + \frac{1}{16}\right)(0.5)\right) + \left(\frac{2.5^2}{4(1.5)}\right)(0.5) = 1.46 \text{ in}^2$$

Problem 14.63

What is the design tensile strength of the 3/8 x 4 inch plate shown below, assuming the steel has yield and ultimate has of 36 ksi and 58 ksi respectively? Neglect block shear criteria. $F_y = 36$ ksi, $F_u = 58$ ksi.



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A. 48.6 kips
B. 65.3 kips
C. 39.62 kips
D. 72.12 kips

Solution: A

$w = 4\,\text{in}$
$\ell = 3\,\text{in}$
$t = \frac{3}{8}\,\text{in}$
$F_y = 36\ \text{ksi}$
$F_u = 58\ \text{ksi}$

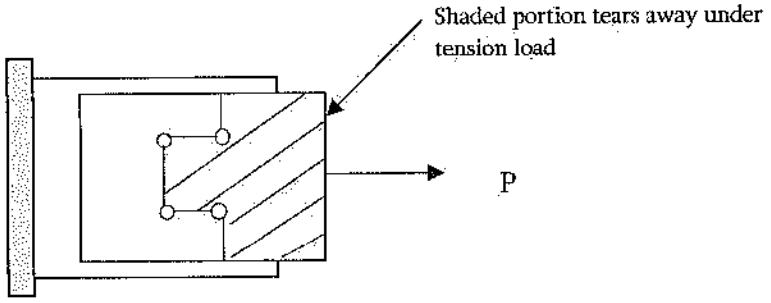
$A_g = (4)\left(\frac{3}{8}\right) = 1.5\,\text{in}^2$
$A_g = A_n$
$U = 1.0$
$A_e = UA_n$
$A_e = 1.0(1.5\,\text{in}^2) = 1.5\ \text{in}^2$

Yielding Condition:
$\phi P_n = \phi F_y A_g$
$= (0.9)(36\ \text{ksi})(1.5\ \text{in}^2)$
$= 48.6\,\text{kips}$ (controls)

Fracture Condition:
$\phi P_n = \phi F_u A_g$
$= (0.75)(58\ \text{ksi})(1.5\ \text{in}^2)$
$= 65.25\,\text{kips}$

Problem 14.64

What is the block shear capacity of the following plate for the shaded region? The bolt diameter is 7/8 inch with center to center spacing of 2.5 in. The tension plate size is 5 x 0.375 inch. $F_y = 36$ ksi, $F_u = 58$ ksi.



A. 56 kips
B. 66 kips
C. 81 kips
D. 96 kips

Solution: C

$$\varphi P_n = \varphi[(0.60)(F_u)(A_{nv}) + (U_{bs})(F_u)(A_{nt})] \le \varphi[(0.60)(F_y)(A_{gv}) + (U_{bs})(F_u)(A_{nt})]$$
$$A_{nv} = A_{gv} - A_{holes}$$

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$$A_{gt} = A_{gt} - A_{holes}$$

$$F_u = 58 \text{ ksi}$$

$$F_y = 36 \text{ ksi}, \quad \phi = 0.75, \quad U_{bs} = 1.0$$

$$A_{gv} = (2)(2.5)(0.375) = 1.875 \text{ in}^2$$

$$A_{holes}(\text{shear}) = (2)\left(\frac{7}{8} + \frac{1}{16}\right)(0.375) = 0.703 \text{ in}^2$$

$$A_{nv} = 1.875 - 0.703 = 1.1718 \text{ in}^2$$

$$A_{nt} = \left[2.5 - \left(\frac{7}{8} + \frac{1}{16}\right)\right](0.375) + 2\left[1.25 - 0.5\left(\frac{7}{8} + \frac{1}{16}\right)\right](0.375) = 1.1718 \text{ in}^2$$

$$\phi P_n = (0.75)[(0.60(58)(1.1718) + (1)(58)(1.1718)] \le (0.75)[(0.60)(36)(1.875) + (1)(58)(1.1718)]$$
$$= 81.56 \le 81.35$$

So, block shear capacity is 81.35 kips

Problem 14.65

Which of the following failures is/are not considered for steel beam design?

A. Torsional Buckling
B. Lateral Buckling
C. Web yielding
D. Longitudinal Buckling
E. I don't know it

Solution: D

Problem 14.66

Which of the following is/are examples of tension member(s)?

A. Hanger
B. Sag rod
C. Cable
D. All of the above

Solution: D

NCEES offers computer-based practice exams. These practice exams contain questions that have been used on past exams and questions written just for study materials to give you extra practice. For more information, visit http://ncees.org/engineering/fe/

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15 HYDRAULICS AND HYDROLOGIC SYSTEMS

Fundamentals of Engineering (FE) CIVIL Exam Specification mentions, 8-12 questions may appear in the exam based on Hydraulics and Hydrologic Systems as follows:

A. Basic hydrology (infiltration, rainfall, runoff, detention, flood flows, watersheds)
B. Basic hydraulics (Manning equation, Bernoulli theorem, open-channel flow, pipe flow)
C. Pumping systems (water and wastewater)
D. Water distribution systems
E. Reservoirs (e.g., dams, routing, spillways)
F. Groundwater (e.g., flow, wells, drawdown)
G. Storm sewer collection systems

Runoff Calculation

Many different approaches are available to calculate the runoff from a catchment. Two methods are mostly popular: NRCS (SCS) method and Rational method.

NRCS (SCS) Rainfall-Runoff

The following equation is used to calculate the runoff of a basin:

$$Q = \frac{(P - 0.2S)^2}{P + 0.8S}; \qquad S = \frac{1{,}000}{CN} - 10$$

P = precipitation (inches); S = maximum basin retention (inches)
Q = runoff (inches); CN = curve number

Rational Formula

The following equation is used to calculate the runoff of a basin: $Q = CIA$

A = watershed area (acres); C = runoff coefficient
I = rainfall intensity (in/hr); Q = peak discharge (cfs)

Note: Be careful about the units. $Q = CIA$ is an empirical equation and units are not consistent.

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Problem 15.1

A 150 acres watershed has 11 distinct land types whose combined curve number is 80. The 30-year storm has a gross rainfall of 2 inches. Using the NRCS method, the net runoff from this watershed is most nearly:

A. 2,000 m^3/s B. 2,000 in C. 0.56 in D. 1.3 in

Solution: C

The maximum basin retention (inches), $S = \frac{1,000}{CN} - 10 = \frac{1,000}{80} - 10 = 2.5$

Given, P = precipitation = 2 inches

The net runoff, $Q = \frac{(P - 0.2S)^2}{P + 0.8S} = \frac{(2 - 0.2(2.5))^2}{2 + 0.8(2.5)} = 0.56$ inches

Problem 15.2

A 150 acres watershed has 3 distinct land types with runoff coefficients as shown below. The 30-year rainfall intensity is 2 in/hr. Using the Rational method, the net runoff from this watershed is most nearly:

Type	Area (Acres)	Runoff Coefficients
Area-1	40	0.3
Area-2	80	0.1
Area-3	30	0.05

A. 43 m^3/s B. 43 cfs C. 43 in D. 430 cfs

Solution: B

Combined Coefficient, $C = \frac{C_1A_1 + C_2A_2 + C_3A_3}{A_1 + A_2 + A_3} = \frac{0.3(40) + 0.1(80) + 0.05(30)}{40 + 80 + 30} = 0.143$

$Q = CIA = 0.143$ (2 in/hr)(150 acres) = 43 cfs

Darcy's Law

Darcy's lay describes the flow of a fluid through a porous medium. It can be expressed as follows:

$$Q = KA\frac{dh}{dx} = KAi$$

$$q = K\frac{dh}{dx} = Ki; \qquad v = \frac{q}{n} = \frac{K}{n}\frac{dh}{dx}$$

Q = discharge rate (ft^3/sec or m^3/s); K = hydraulic conductivity (ft/sec or m/s)

h = hydraulic head (ft or m); A = cross-sectional area of flow (ft^2 or m^2)

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q = specific discharge (also called Darcy velocity or superficial velocity)
v = average seepage velocity; n = effective porosity

Unit hydrograph: The direct runoff that would result from one unit of runoff occurring uniformly in space and time over a specified period.
Transmissivity, T: The product of hydraulic conductivity and thickness, b, of the aquifier (L^2T^{-1}). $T = Kb$.
Storativity or storage coefficient of an aquifer, S: The volume of water taken into or released from storage per unit surface area per unit change in potentiometric (piezometric) head.

Problem 15.3

Water is flowing through a soil whose length is 2 ft, and cross-sectional area of 0.1 ft². If the elevation difference between the inlet and outlet is 0.2 m, the hydraulic gradient is most nearly:

A. 0.1 B. 0.33 C. 10 D. 3.3

Solution: B

$h = 0.2\text{ m} = 0.2\,(3.28)\text{ ft} = 0.656\text{ ft};$ $\quad i = \dfrac{h}{L} = \dfrac{0.656\text{ ft}}{2\text{ ft}} = 0.328 \approx 0.33$

Problem 15.4

Water flows through a soil with effective porosity of 0.4 and cross-sectional area of 0.15 m². The hydraulic gradient is 0.12 and the coefficient of permeability is 5×10^{-4} m/s. The average seepage velocity (m/s) is most nearly:

A. 1.5×10^{-4} B. 9×10^{-4} C. 5×10^{-9} D. None of the above

Solution: A

$$v = \frac{q}{n} = \frac{K}{n}\frac{dh}{dx} = \frac{K}{n}i = (5\times10^{-4}\ \text{m/s})(0.12)/0.4 = 1.5\times10^{-4}\ \text{m/s}$$

Problem 15.5

Water flows through a soil with effective porosity of 0.4 in a confined aquifer of 50 m thickness. The hydraulic gradient is 0.12 and the coefficient of permeability is 5×10^{-4} m/s. The transmissivity of the soil (m²/s) is most nearly:

A. 2.5×10^{-2} B. 5.2×10^{-2} C. 2.5×10^{-4} D. 2.5×10^{-3}

Solution: A

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Transmissivity, $T = Kb = 5\times10^{-4}$ m/s (50 m) $= 2.5\times10^{-2}$ m^2/s

Well Drawdown

Well drawdown is used to determine the permeability of aquifer soil. Two types of conditions may be available: unconfined aquifer and confined aquifer. They are discussed here:

Dupuit's Formula for Unconfined Aquifer

In unconfined aquifer, the flow rate and permeability are related as $K = \dfrac{Q\ln\left(\dfrac{r_2}{r_1}\right)}{\pi\left(h_2^{\,2} - h_1^{\,2}\right)}$

Q = flow rate of water drawn from well (cfs)
K = coefficient of permeability of soil = hydraulic conductivity (ft/sec)
h_1 = height of water surface above bottom of aquifer at perimeter of well (ft)
h_2 = height of water surface above bottom of aquifer at r_2 from well centerline (ft)
r_1 = radius to water surface at perimeter of well, i.e., radius of well (ft)
r_2 = radius of water surface whose height is h_2 above bottom of aquifer (ft)
Q/D_w = specific capacity, where D_w = well drawdown (ft)

Theim Equation for Confined Aquifer

In confined aquifer, the flow rate and permeability are related as follows:

$$Q = \frac{2\pi T\left(h_2 - h_1\right)}{\ln\left(\dfrac{r_2}{r_1}\right)} = \frac{2\pi Kb\left(h_2 - h_1\right)}{\ln\left(\dfrac{r_2}{r_1}\right)}$$

$$\text{Or, } K = \frac{Q\ln\left(\dfrac{r_2}{r_1}\right)}{2\pi b\left(h_2 - h_1\right)}$$

$T = Kb$ = transmissivity (ft^2/sec);
b = thickness of confined aquifer (ft)
h_1, and h_2 = heights of piezometric surface above bottom of aquifer (ft) at r_1, and r_2 radii from pumping well (ft), respectively

Please read Hydrology/Water Resources in Civil Engineering section of the NCEES FE Ref. Handbook for more information.

Problem 15.6

A pumping test was conducted in a well penetrating a 25 ft thick confined aquifer to determine the permeability of the aquifer soil. The equilibrium was reached when the discharge was 5 gal/sec. The water levels at 40 ft and 150 ft distance from the well are 15 ft and 23 ft respectively. The hydraulic conductivity (ft/s) of the aquifer soil is most nearly:

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A. $3.64x10^{-6}$ B. 0.039 C. 0.000039 D. 0.0007

Solution: D

$$K = \frac{Q \ln\left(\frac{r_2}{r_1}\right)}{2\pi b\left(h_2 - h_1\right)} = \frac{5\ \frac{\text{gal}}{\text{sec}}\left(0.134\ \frac{\text{ft}^3}{\text{gal}}\right)\ln\left(\frac{150}{40}\right)}{2\pi(25\,\text{ft})(23\,\text{ft} - 15\ \text{ft})} = 0.0007\ \text{ft/s}$$

Problem 15.7

A pumping test was conducted in a well penetrating an unconfined aquifer to determine the permeability of the aquifer soil. The equilibrium was reached when the discharge was 50 gal/sec. The water levels at 40 ft and 150 ft distance from the well are 15 ft and 23 ft respectively. The hydraulic conductivity (ft/s) of the aquifer soil is most nearly:

A. 0.0925
B. 0.039
C. 0.39
D. 0.0093

Solution: D

$$K = \frac{Q \ln\left(\frac{r_2}{r_1}\right)}{\pi\left(h_2^{\,2} - h_1^{\,2}\right)} = \frac{50\ \frac{\text{gal}}{\text{sec}}\left(0.134\ \frac{\text{ft}^3}{\text{gal}}\right)\ln\left(\frac{150}{40}\right)}{\pi\left(23^2 - 15^2\right)} = 0.0093\ \text{ft/s}$$

Problem 15.8

A pumping test was conducted in a well penetrating a 35 ft thick confined aquifer to determine the permeability of the aquifer soil. The equilibrium was reached when the discharge was 2,000 gal/min. The water levels at 30 ft and 100 ft distance from the well are 7 ft and 18 ft respectively. The hydraulic conductivity (ft/s) of the aquifer soil is most nearly:

A. 0.00525
B. 0.0022
C. 0.39
D. 0.052

Solution: B

$$K = \frac{Q \ln\left(\frac{r_2}{r_1}\right)}{2\pi b\left(h_2 - h_1\right)} = \frac{2{,}000\ \frac{\text{gal}}{\text{min}}\left(0.134\ \frac{\text{ft}^3}{\text{gal}}\right)\left(\frac{1\ \text{min}}{60\ \text{sec}}\right)\ln\left(\frac{100}{30}\right)}{2\pi(35\,\text{ft})(18\,\text{ft} - 7\ \text{ft})} = 0.0022\ \text{ft/s}$$

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Problem 15.9

A pumping test was conducted in a well penetrating an unconfined aquifer to determine the permeability of the aquifer soil. The equilibrium was reached when the discharge was 2,000 gal/min. The water levels at 30 ft and 100 ft distance from the well are 7 ft and 18 ft respectively. The hydraulic conductivity (ft/sec) of the aquifer soil is most nearly:

A. 0.00025 B. 0.029 C. 0.39 D. 0.0062

Solution: D

$$K = \frac{Q \ln\left(\frac{r_2}{r_1}\right)}{\pi\left(h_2^2 - h_1^2\right)} = \frac{2{,}000 \ \frac{\text{gal}}{\text{min}}\left(0.134 \ \frac{\text{ft}^3}{\text{gal}}\right)\left(\frac{1 \ \text{min}}{60 \ \text{sec}}\right)\ln\left(\frac{100}{30}\right)}{\pi\left(18^2 - 7^2\right)} = 0.0062 \ \text{ft/s}$$

Open-Channel Flow

Energy conservation is an important concept when analyzing open channel flow. Energy is conserved for a fluid in an open channel flow if head loss due to friction is neglected. The energy calculated at one location in the flow is equal to the energy calculated at any other location in the same flow. The specific energy can be determined as follows:

$$\text{Specific Energy, } E = \alpha\frac{v^2}{2g} + y = \frac{\alpha Q^2}{2gA^2} + y$$

α = kinetic energy correction factor, usually 1.0
E = specific energy; Q = discharge
v = velocity; y = depth of flow
A = cross-sectional area of flow

Critical Depth is defined as the depth in a channel which has the minimum specific energy. For that depth, the following equation can be deduced:

$$\frac{Q^2}{g} = \frac{A^3}{T}$$

g = acceleration due to gravity; T = width of the water surface

For rectangular channels, the critical depth can be calculate using the following equation:

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

y_c = critical depth; q = unit discharge = Q/B
B = channel width; g = acceleration due to gravity

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Froude Number is used in open channel flow to determine the flow condition (critical or not). It is defined as the ratio of inertial forces to gravity forces as follows:

$$\text{Fr} = \frac{V}{\sqrt{gy_h}} = \sqrt{\frac{Q^2 T}{gA^3}}$$

y_h = hydraulic depth = A/T

Supercritical flow: $F_r > 1$

Subcritical flow: $F_r < 1$

Critical flow: $F_r = 1$

Specific Energy Diagram is used to analyze a channel. The depth of flow vs. $E = \alpha \frac{v^2}{2g} + y$ is plotted to study the channel. The following definitions can be made from the diagram.

Alternate depths: Depths with the same specific energy in open channel flow.
Uniform flow: A flow condition where depth and velocity do not change along a channel.

Problem 15.10

A wide rectangular clean-earth channel has a flow rate of 50 ft²/s. The critical depth (ft) of the flow is most nearly:

A. 4.53 B. 3.87 C. 2.38 D. 4.20

Solution: D

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{50^2}{32.2}\right)^{1/3} = 4.2 \text{ ft}$$

Note: the value of g may not be given. It is listed in the UNITS section of the Ref. Handbook.

Problem 15.11

A wide rectangular channel has a width of 4 m, flow depth of 1.5 m and a flow of 20 m³/s. The specific energy (m) of the flow is most nearly:

A. 1.2 B. 2.1 C. 3.6 D. 4.3

Solution: B

$$V = \frac{Q}{A} = \frac{20 \frac{\text{m}^3}{\text{s}}}{(4\text{ m})(1.5\text{ m})} = 3.33 \text{ m/s}; \qquad E = y + \frac{v^2}{2g} = 1.5\text{m} + \frac{(3.33 \text{ m/s})^2}{2(9.81)} = 2.1 \text{ m}$$

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Problem 15.12

A wide rectangular channel has a width of 4 m, flow depth of 1.5 m and a flow of 20 m^3/s. The flow is most nearly:

A. Critical B. Supercritical C. Subcritical D. None of the above

Solution: C

$$V = \frac{Q}{A} = \frac{20\ \frac{m^3}{s}}{(4\,\text{m})(1.5\,\text{m})} = 3.33\ \text{m/s};$$

$$\text{Froude Number, } Fr = \frac{v}{\sqrt{gy}} = \frac{3.33\ \text{m/s}}{\sqrt{(9.81\ \text{m/s}^2)(1.5\,\text{m})}} = 0.87 < 1.0, \text{Subcritical Flow}$$

Problem 15.13

The Bernoulli equation is an expression of the conservation of -

A. mass B. momentum C. velocity D. energy

Solution: D

Hydraulic Jump

When a liquid of high velocity discharges into a zone of lower velocity, a rather abrupt rise occurs in the liquid surface. This abrupt rise in fluid is called the hydraulic jump. It is frequently observed in open channel flow such as rivers and spillways. The downstream flow depth can be calculated as follows:

$$y_2 = \frac{y_1}{2}\left(-1+\sqrt{1+8Fr_1^2}\right)$$

y_1 = flow depth at upstream supercritical flow location
y_2 = flow depth at downstream subcritical flow location
Fr_1 = Froude number at upstream supercritical flow location

Problem 15.14

A water is flowing in a wide channel at 20 m^2/s and the upstream supercritical flow depth is 1.5 m. If the flow undergoes a hydraulic jump, the downstream flow depth (m) is most nearly:

A. 7.53 B. 3.87 C. 2.38 D. 6.67

Solution: D

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$$\text{Upstream velocity, } V_1 = \frac{q}{y_1} = \frac{20\ \frac{\text{m}^2}{\text{s}}}{1.5\,\text{m}} = 13.33\ \frac{\text{m}}{\text{s}}$$

$$\text{Upstream Froude Number, } Fr_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{13.33\ \text{m/s}}{\sqrt{9.81(1.5)}} = 3.48$$

$$y_2 = \frac{y_1}{2}\left(-1+\sqrt{1+8Fr_1^2}\right) = \frac{1.5\,\text{m}}{2}\left(-1+\sqrt{1+8(3.48)^2}\right) = 6.67\ \text{m}$$

Problem 15.15

A water is flowing in a wide channel at 20 m²/s and the upstream supercritical flow depth is 1.5 m. If the flow undergoes a hydraulic jump, the downstream Froude Number is most nearly:

A. 1.235 B. 0.675 C. 0.375 D. 0.125

Solution: C

$$V_1 = \frac{q}{y_1} = \frac{20\ \frac{\text{m}^2}{\text{s}}}{1.5\,\text{m}} = 13.33\ \frac{\text{m}}{\text{s}}; \qquad Fr_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{13.33\ \text{m/s}}{\sqrt{9.81(1.5)}} = 3.48$$

$$y_2 = \frac{y_1}{2}\left(-1+\sqrt{1+8Fr_1^2}\right) = \frac{1.5\,\text{m}}{2}\left(-1+\sqrt{1+8(3.48)^2}\right) = 6.67\ \text{m}$$

$$\text{Downstream velocity, } V_2 = \frac{q}{y_2} = \frac{20\ \frac{\text{m}^2}{\text{s}}}{6.67\,\text{m}} = 3.0\ \frac{\text{m}}{\text{s}}$$

$$\text{Downstream Froude Number, } Fr_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{3.0\ \text{m/s}}{\sqrt{9.81(6.67)}} = 0.375$$

Manning's Equation

This topic is also discussed in the Fluid Mechanics section. In open-channel flow (gravity driven), Manning's equation is used to determine the velocity or flow rate through a channel whose longitudinal slope, and hydraulic radius are known. The equation is shown below:

$$Q = \frac{K}{n} A R_H^{2/3} S^{1/2}$$

Q = discharge (ft³/sec or m³/s);
A = cross-sectional area of flow (ft² or m²);
P = wetted perimeter (ft or m);
n = Manning's roughness coefficient
K = 1.0 for SI units, 1.486 for USCS units
R_H = hydraulic radius = A/P
S = slope of hydraulic surface

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Weir Formulas

Weirs is used to measure flow rates in open channels and rivers. It consists of an obstruction such as a dam placed across the open channel with a specially shaped opening or notch. Rectangular weirs and triangular or V-notch weirs are often used in water supply, wastewater and sewage systems. They consist of a sharp-edged plate with a rectangular, triangular or V-notch profile for the water flow.

Rectangular Weir

The flow rate over a weir is a function of the head on the weir and can be calculated as:

Free discharge suppressed, $Q = CLH^{3/2}$

Free discharge contracted, $Q = C(L - 0.2H)H^{3/2}$

V-Notch

For V-notch, the flow rate can be determined as follows: $Q = CLH^{5/2}$

Q = discharge (cfs or m^3/s); L = weir length (ft or m)
C = 3.33 for rectangular weir (USCS units); C = 1.84 for rectangular weir (SI units)
C = 2.54 for 90° V-notch weir (USCS units); C = 1.40 for 90° V-notch weir (SI units)
H = head (depth of discharge over weir) ft or m

Hazen-Williams Equation

Hazen-Williams Equation is used to determine the velocity or flow rate through a channel whose longitudinal slope, and hydraulic radius are known. This topic is also discussed in the Fluid Mechanics Section. The equation is shown below:

$$v = k_1 C R_H^{0.63} S^{0.54}$$

where: C = roughness coefficient; k_1 = 0.849 for SI units; k_1 = 1.318 for USCS units; R_H = hydraulic radius (ft or m); S = slope of energy grade line; v = velocity (ft/sec or m/s)

Problem 15.16

A weir in a horizontal channel is 1 m high and 6 m wide. The water depth upstream is 0.8 m. The discharge (m^3/s) of the weir for free discharge suppressed condition is most nearly:

A. 4.3 B. 22.3 C. 7.9 D. 13.65

Solution: C

$$Q = CLH^{3/2} = 1.84(6\,\text{m})(0.8\,\text{m})^{3/2} = 7.9\ \text{m}^3/\text{s}$$

C = 1.84 for rectangular section in SI unit [see Hydraulics subsection in Civil Engineering]

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Problem 15.17

A weir in a horizontal channel is 1 ft high and 6 ft wide. The water depth upstream is 0.8 ft. The discharge (ft^3/s) of the weir for free discharge contracted condition is most nearly:

A. 4.3 B. 22.3 C. 7.9 D. 13.91

Solution: D

$$Q = C(L - 0.2H)H^{3/2} = 3.33(6 - 0.2(0.8))(0.8\,\text{ft})^{3/2} = 13.91\ \text{ft}^3/\text{s}$$

Problem 15.18

The water head in a 90° V-Notch is 1.22 m. The discharge (m^3/s) of the V-Notch is most nearly:

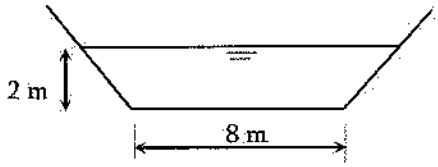
A. 4.3 B. 2.3 C. 7.9 D. 13.65

Solution: B

$$Q = CH^{5/2} = 1.4(1.22\,\text{m})^{5/2} = 2.3\ \text{m}^3/\text{s}$$

Problem 15.19

A trapezoidal brick sewer channel with 1:1 side slopes shown below carries a flow of 100 m^3/s under gravity. The slope of the channel is most nearly:



A. 0.034% B. 0.34% C. 3.4% D. 0.0034%

Solution: B

$$R_H = \frac{\text{Wetted Area}}{\text{Wetted Perimeter}} = \frac{A}{P} = \frac{(8+2)(2\,\text{ft})}{8 + 2\sqrt{2^2 + 2^2}} = \frac{20}{13.66} = 1.46\,\text{m}$$

Area of flow, $A = (8+2)\ \text{m}\ (2\ \text{m}) = 20\ \text{m}^2$

Hazen-Williams coefficient, $C = 100$ for brick sewer (Civil Engineering – Hydrology/Water of the NCEES FE Ref. Handbook). Manning's equation cannot be used as the Manning's coefficient is not given.

$$V = \frac{Q}{A} = \frac{100\ \frac{m^3}{s}}{20\ \text{m}^2} = 5\ \text{m/s}$$

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$$V = 0.849 C R_H^{0.63} S^{0.54}$$
$$\Rightarrow \quad 5\,\text{m/s} = 0.849(100)(1.46)^{0.63} S^{0.54}$$
$$\Rightarrow \quad S = 0.00339 = 0.34\%$$

Problem 15.20

The flow rate (cfs) in a 24-inch diameter storm sewer with the longitudinal slope of 0.003 and n of 0.01 flowing full under the gravity is most nearly:

A. 12 B. 22 C. 16 D. 42

Solution: C

For flowing full, $R_H = D/4 = 24/4 = 6$ in $= 0.5$ ft
From the NCEES FE Ref. Handbook, Fluids Mechanics or Civil Engineering – Hydrology/Water

$$Q = \frac{1.486}{n} A R_H^{2/3} S^{1/2} = \frac{1.486}{0.01}\left(\frac{\pi(2\text{ ft})^2}{4}\right)\left((0.5)^{2/3}\right)\left(0.003^{1/2}\right) = 16\,\text{ft}^3/\text{s}$$

Circular Pipe Head Loss Equation (Head Loss Expressed in Feet)

As discussed in the Fluid Mechanics section, pressure or water head loss occurs in conduit due to friction, conduit bend, connection, and so on. Several head loss equations are discussed in Fluid mechanics. Few more are discussed here. Hazen-Williams equation to determine the head loss is expressed as:

$$h_f = \frac{4.73L}{C^{1.852} D^{4.87}} Q^{1.852}$$

hf = head loss (ft); L = pipe length (ft)
D = pipe diameter (ft); Q = flow (cfs)
C = Hazen-Williams coefficient

Hazen-Williams equation to determine the pressure loss is expressed as:

U.S. Customary Units

$$P = \frac{4.52}{C^{1.85} D^{4.87}} Q^{1.85}$$

P = pressure loss (psi per foot of pipe)
Q = flow (gpm)
D = pipe diameter (inches)

SI Units

$$P = \frac{6.05}{C^{1.85} D^{4.87}} Q^{1.85}$$

P = pressure loss (bars per meter of pipe)
Q = flow (liters/minute)
D = pipe diameter (mm)

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Rated Capacity at 20 psi from Fire Hydrant

The rated capacity at 20 psi from a fire hydrant is calculated as: $Q_R = Q_F \left(\frac{H_R}{H_F} \right)^{0.54}$

Q_R = rated capacity (gpm) at 20 psi
Q_F = total test flow
H_R = static pressure - 20 = P_S – 20 psi
H_F = static pressure - residual pressure = $P_S - P_R$

Fire Hydrant Discharging to Atmosphere

The discharge from a fire hydrant can be determined as: $Q = 29.8 D^2 C_d P^{\frac{1}{2}}$

Q = discharge (gpm)
D = outlet diameter (in.)
P = pressure detected by pilot gage (psi)
C_d = hydrant coefficient based on hydrant outlet geometry

Fire Sprinkler Discharge

The discharge through a fire sprinkler can be computed as: $Q = KP^{1/2}$

Q = flow (gpm); P = pressure (psi)
K = measure of the ease of getting water out, gpm per $\sqrt{\text{psi}}$

The Sprinkler factors (K) for standard (0.5 in), large (17/32 in) and extra-large (5/8 in) orifices are 5.6, 8.0, and 11.2 respectively.

Problem 15.21

A circular new cast iron pipe has a diameter of 12 inch, and length of 52 ft. If the flow of the pipe is 20 cfs, most nearly, the head loss in linear foot is most nearly:

A. 3.23 B. 5.39 C. 7.68 D. 9.65

Solution: C

Hazen-Williams coefficient, C = 130 for new cast iron pipe (Civil Engineering – Hydrology/Water of the NCEES FE Ref. Handbook)

Q = 20 cfs ; D = 1 ft; $h_f = \frac{4.73L}{C^{1.852} D^{4.87}} Q^{1.852} = \frac{4.73(52\text{ ft})}{130^{1.852}(1\text{ ft})^{4.87}} (20\text{ cfs})^{1.852} = 7.68\text{ ft}$

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Problem 15.22

A circular new cast iron pipe has a diameter of 12 inch, and length of 52 ft. If the flow of the pipe is 200 gal/min, most nearly, the pressure loss per foot is most nearly:

A. $5.56\text{x}10^{-5}$ B. $1.56\text{x}10^{-4}$ C. $9.6\text{x}10^{-6}$ D. $1.3\text{x}10^{-2}$

Solution: A

Hazen-Williams coefficient, $C = 130$ for new cast iron pipe (Civil Engineering – Hydrology/Water of the NCEES FE Ref. Handbook)

$Q = 200$ gal/min (gpm); $D = 12$ in,

$$P = \frac{4.52}{C^{1.85}D^{4.87}}Q^{1.85} = \frac{4.52}{130^{1.85}(12\text{ in.})^{4.87}}(200\text{ gpm})^{1.85} = 5.56\text{x}10^{-5}\text{ psi per ft}$$

Note: Be careful about the units. It is an empirical equation and units are not consistent.

Problem 15.23

A standard Fire Sprinkler discharges water at 15 psi. The discharge volume in gallon per minute (gpm) is most nearly:

A. 12.3
B. 31.2
C. 43.4
D. 21.7

Solution: D

$$Q = KP^{1/2} = 5.6(15\text{ psi})^{1/2} = 21.7\text{ gpm}$$

$K = 5.6$ gpm per $\sqrt{\text{psi}}$ is obtained from the Hydrology/Water Resources subsection in Civil Engineering of the NCEES FE Ref. Handbook.

Problem 15.24

Which of the following statements is false?

A. Values of Hazen-Williams Coefficient for plastic pipe is 150
B. Fire hydrant coefficient depends on the hydrant outlet geometry
C. Uniform flow is a flow condition where depth and velocity do not change along a channel
D. None of the above

Solution: D

All three statements are true.

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16 TRANSPORTATION ENGINEERING

Fundamentals of Engineering (FE) CIVIL Exam Specification mentions, 8-12 questions may appear in the exam based on Transportation Engineering as follows:

A. Geometric design of streets and highways
B. Geometric design of intersections
C. Pavement system design (e.g., thickness, subgrade, drainage, rehabilitation)
D. Traffic safety
E. Traffic capacity and traffic flow theory
G. Traffic control devices
H. Transportation planning (e.g., travel forecast modeling)

Vehicle Signal Change Interval

The yellow and red signals in a signalized intersection can be determined using the following equations:

$$\text{Yellow signal period, } y = t + \frac{v}{2a \pm 64.4G}$$

$$\text{Red signal period, } r = \frac{W + l}{v}$$

y = length of yellow interval to nearest 0.1 sec (sec)
r = length of red clearance interval to nearest 0.1 sec (sec)
a = deceleration rate (ft/sec^2)
$\pm G$ = percent grade divided by 100 (uphill grade "+")
W = width of intersection, curb-to-curb (ft)
l = length of vehicle (ft)
v = speed of vehicle in feet per second (fps)
V = speed of vehicle in miles per hour (mph); this speed is used in the next section.

Sight Distance

Stopping Sight Distance (SSD) is one of several types of sight distance used in road design. It is a near worst-case distance a vehicle driver needs to be able to see in order have room to stop before

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colliding with something in the roadway, such as a pedestrian in a crosswalk, a stopped vehicle, or road debris. Insufficient sight distance can adversely affect the safety or operations of a roadway or intersection. SSD is calculated using the following equation:

$$SSD = 1.47Vt + \frac{V^2}{30\left(\frac{a}{32.2} \pm G\right)}$$

t = driver reaction time (sec)
V = vehicle approach speed (mph)

Intersection Sight Distance (ISD) is an important measurement in traffic engineering and comes heavily into play when analyzing access options and modifications for existing and proposed developments. The American Association of State Highway and Transportation Officials (AASHTO) says, specified areas along intersection approach legs and across their included corners should be clear of obstructions that might block a driver's view of potentially conflicting vehicles. It is calculated using the following equation:

$$ISD = 1.47V_{major}t_g$$

Peak Hour Factor (PHF)

The peak hour factor (PHF) is the hourly volume during the maximum-volume hour of the day divided by the peak 15-minute flow rate within the peak hour; a measure of traffic demand fluctuations within the peak hour.

$$PHF = \frac{\text{Hourly Volume}}{\text{Hourly Flow Rate}} = \frac{\text{Hourly Volume}}{4 \text{ x Peak 15-minute Volume}}$$

Now, practice the following problems for better understanding.

Problem 16.1

In a 3% uphill grade the maximum velocity may be expected is 55 mph. What should be most nearly the yellow interval length if an average deceleration rate of 5 ft/sec^2 is available? The driver reaction time is 1 sec.

A. 4.7 sec B. 11 sec C. 7.8 sec D. More information is needed to calculate

Solution: C

$$55\ \frac{\text{mi}}{\text{hr}} = \frac{55\ \text{mi}\left(5{,}280\ \frac{\text{ft}}{\text{mi}}\right)}{3{,}600\ \frac{\text{sec}}{\text{hr}}} = 80.7\ \frac{\text{ft}}{\text{s}}$$

G = 0.03 (+ for uphill)

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Yellow interval, $y = t + \dfrac{v}{2a \pm 64.4G} = 1 + \dfrac{80.7}{2(5) + 64.4(0.03)} = 7.8 \text{ s}$

Note: the conversion factor (mph to fps) may not be given in the problem. Also, if the grade, up or down, not given, check both. If both assumptions give an answer that is in the list, take the higher one to be conservative.

Problem 16.2

The curb–to-curb length of an intersection is 50 ft with an upward slope of 2%. If a 22 ft long truck approaches at 45 mph at intersection, the amount of red clearance interval (seconds) recurred for the truck is most nearly:

A. 0.6
B. 0.8
C. 1.1
D. 0.9

Solution: C

$$45 \frac{\text{mi}}{\text{hr}} = \frac{45 \text{ mi}\left(5{,}280 \dfrac{\text{ft}}{\text{mi}}\right)}{3{,}600 \dfrac{\text{sec}}{\text{hr}}} = 66 \frac{\text{ft}}{\text{s}}$$

$$r = \frac{W + l}{v} = \frac{(50 + 22) \text{ ft}}{66 \dfrac{\text{ft}}{\text{sec}}} = 1.09 \text{ sec} \approx 1.1 \text{ sec}.$$ r is expressed to nearest 0.1 sec.

Note: the conversion factor (mph to fps) may not be given in the problem.

Problem 16.3

The design speed of a highway is 103 mph with an uphill grade of 1%. The driver reaction time is 2.5 sec for that highway. If a car sees an object ahead and decelerates at 30 ft/s², the stopping sight distance in feet for that car is most nearly:

A. 700 B. 750 C. 850 D. 950

Solution: B

$$\text{SSD} = 1.47Vt + \frac{V^2}{30\left(\left(\dfrac{a}{32.2}\right) + G\right)} = 1.47(103)(2.5 \text{ s}) + \frac{103^2}{30\left(\dfrac{30}{32.2} + 0.01\right)} = 754 \text{ ft}$$

Note: Be prepared to do some rounding while selecting the best answer (754 ft vs 750 ft).

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Sight Distance Related to Curve Length

The sight distance is an important concern for vertical and horizontal curves for nighttime conditions. The NCEES FE Ref. Handbook (Civil Engineering - Transportation), listed the equations needed to determine the sight distance for both crest and sag curves for different idealizations.

For horizontal curves, the following equations could be used:

Table 16.1 Some parameters for horizontal curves

Side friction factor (based on super-elevation):	$0.01e+f=\dfrac{V^2}{15R}$
Spiral Transition Length:	$L_S=\dfrac{3.15V^3}{RC}$
Horizontal sight line offset (HSO)(to see around obstruction):	$\text{HSO}=R\left[1-\cos\left(\dfrac{28.65S}{R}\right)\right]$

For vertical curves, the following equations could be used:

Table 16.2 Some parameters for vertical curves

	$S \le L$	$S > L$
Crest Vertical Curve General Equation: Standard Criteria: h_1=3.50 ft and h_2 =2.0 ft	$L=\dfrac{AS^2}{100\left(\sqrt{2h_1}+\sqrt{2h_2}\right)^2}$ $L=\dfrac{AS^2}{2158}$	$L=2S-\dfrac{200\left(\sqrt{h_1}+\sqrt{h_2}\right)^2}{A}$ $L=2S-\dfrac{2158}{A}$
Sag Vertical Curve (standard headlight criteria)	$L=\dfrac{AS^2}{400+3.5S}$	$L=2S-\left(\dfrac{400+3.5S}{A}\right)$
Sag Vertical Curve (based on riding comfort)	$L=\dfrac{AV^2}{46.5}$	
Sag Vertical Curve (based on adequate sight distance under an overhead structure to see an object beyond a sag vertical curve)	$L=\dfrac{AS^2}{800\left(C-\dfrac{h_1+h_2}{2}\right)}$	$L=2S-\dfrac{800}{A}\left(C-\dfrac{h_1+h_2}{2}\right)$
	C= vertical clearance for overhead structure (overpass) located within 200 feet of the midpoint of the curve	

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e = superelevation (%); f = side friction factor
L = length of curve (ft); Ls = spiral transition length (ft)
R = radius of curve (ft); SSD = stopping sight distance (ft)
t = driver reaction time (sec); V = design speed (mph)
A = absolute value of algebraic difference in grades (%)
h_1 = height of driver's eyes above the roadway surface (ft)
h_2 = height of object above the roadway surface (ft)

Now, practice the following problems for better understanding.

Problem 16.4

A highway is to be designed with the required stopping sight distance of 700 ft. The equal-tangent crest vertical curve must be designed to connect grades of 1.5% and -2.5%. The minimum length (ft) of the curve required? Assume, $S \leq L$ and standard criteria of h_1 and h_2 is most nearly:

A. 908 B. 712 C. 855 D. 1,010

Solution: A

$$L = \frac{AS^2}{2158} = \frac{|1.5-(-2.5)|(700)^2}{2158} = 908 \text{ ft}$$

Problem 16.5

A highway is to be designed with the required stopping sight distance of 700 ft. The equal- tangent crest vertical curve must be designed to connect grades of 1.5% and -1.5% with $S > L$. The minimum length in feet of the curve required assuming standard criteria of h_1 and h_2 is most nearly:

A. 680
B. 560
C. 700
D. 922

Solution: A

$$L = 2S - \frac{2{,}158}{A} = 2(700) - \frac{2{,}158}{|1.5-(-1.5)|} = 681 \text{ ft}$$

Problem 16.6

A highway is to be designed with the minimum stopping sight distance of 700 ft. The sag vertical curve is to be designed bared on the standard headlight criteria connecting -1.5% and 1.5% curves.

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Assuming the length of curve is less than the sight distance, the required curve length in feet is most nearly:

A. 350 B. 355 C. 475 D. 450

Solution: D

$$L = 2S - \frac{400 + 3.5S}{A} = 2(700) - \frac{400 + 3.5(700)}{|-1.5 - 1.5|} = 450 \text{ ft}$$

Problem 16.7

A highway is to be designed speed of 88 mi/hr and the super elevation of 6%. If the coefficient of side friction is 0.2, the radius of curvature (ft) of the horizontal curve required is most nearly:

A. 1,000 B. 1,500 C. 2,000 D. 2,500

Solution: C

$$0.01e + f = \frac{V^2}{15R}$$

$$0.01(6) + 0.2 = \frac{88^2}{15R} \qquad \text{Therefore, } R = 1{,}986 \text{ ft} \approx 2{,}000 \text{ ft}$$

Problem 16.8

The radius of a curvature is 2,000 ft in a horizontal curve, and the design speed of 103 mi/hr. The spiral transition length in feet required is most nearly:

A. 1,720 B. 1,920 C. 2,120 D. 1,320

Solution: A

C = 1 ft/sec^3, as not given

$$L_s = \frac{3.15V^3}{RC} = \frac{3.15(103)^3}{(2{,}000)(1)} = 1{,}721 \text{ ft}$$

Problem 16.9

A horizontal curve is designed with the stopping sight distance of 730 ft and the radius of curvature of 2,000 ft. The horizontal sight line offset (HSO) in feet is most nearly:

A. 23.2 B. 30.2 C. 36.2 D. 33.2

Solution: D

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$$\text{HSO} = R\left[1-\cos\left(\frac{28.65S}{R}\right)\right] = 2000\left[1-\cos\left(\frac{28.65(730)}{2{,}000}\right)\right] = 33.2 \text{ ft}$$

Basic Freeway Segment Highway Capacity

A freeway is a divided highway with full access control and two or more lanes in each direction. Base conditions (i.e.: good visibility, no heavy vehicles, at least 12-ft lanes, drivers are regular users, etc.) must be satisfied for a basic freeway segment to operate at its maximum capacity. However, the base conditions are always not possible in real freeways. Thus, the Level of Service (LOS) is very often measured to describe the operating conditions. LOS is a qualitative measure that describes the operating conditions within a given freeway segment using density (pc/mi) as the quantitative variable and an alphabetic grading system, ranging from A to F. Conveys how the density values are perceived by drivers and passengers when traveling in the traffic stream at various demand flow rates, passenger cars per hour per lane (pc/h/ln).

Table 16.3 Variation of density with level of service

LOS	Short Description	Density (pc/mi/ln)
A	Free flow conditions prevail	≤ 11
B	Reasonably free flow conditions still exist	> 11-18
C	Speeds are at or near the free flow speed	> 18 – 26
D	Speeds begin to decline slightly with increasing flows.	> 26 – 35
E	Operations are volatile because there are virtually no gaps	> 35 – 45
F	Breakdown conditions exist, uniform moving impossible	> 45

To determine the LOS, the density of the freeway is to be calculated first from the following equation:

$$D = \frac{v_p}{S}$$

where

D = density(pc/mi/ln)

v_p = demand flow rate (pc/h/ln) under base conditions

S = mean speed of traffic stream under base conditions (mi/h)

Therefore, our target is to determine v_p and S first, to calculate the density (D). Once D is known LOS can be found out from the above table. Let us see how to determine v_p and S. S is basically the Free-Flow Speed (FFS) - the maximum average speed of passenger cars during low to moderate flow when one car does not affect another car. If flow rate, passenger cars per hour per lane (pc/h/ln), increases the FFS decreases as the speed of one car is affected by another car. The flow rate, at which the FFS decreases is called Breakpoint. The value of Breakpoint flow rate is listed in the table below. For example, the Breakpoint flow rate is 1200 pc/h/ln for FFS of 70 mi/h. For demand volumes beyond the breakpoint, flow rate declines until it reaches the capacity of the segment. The reduced FFS is then known as S and can be calculated using the equations listed in the table below.

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Table 16.4 Recommended breakpoint flow rates and speed beyond breakpoint flow rates

Free-Flow Speed (FFS) (mph)	Breakpoint Flow Rate (pc/h/ln)	Max. Flow Rate (pc/h/ln)	Reduced Speed (mph) beyond Breakpoint Flow Rate
75	1,000	2,400	$75-0.00001107\left(v_p-1{,}000\right)^2$
70	1,200	2,400	$70-0.00001160\left(v_p-1{,}200\right)^2$
65	1,400	2,350	$65-0.00001418\left(v_p-1{,}400\right)^2$
60	1,600	2,300	$60-0.00001816\left(v_p-1{,}600\right)^2$
55	1,800	2,250	$55-0.00002469\left(v_p-1{,}800\right)^2$

* This table is available in the NCEES Ref. Handbook in different organizations.

The FFS of a freeway can be estimated using the following equations:

$$FFS = 75.4 - f_{LW} - f_{LC} - 3.22TRD^{0.84}$$

where

FFS = free flow speed of basic freeway segment (mi/h)
f_{LW} = adjustment for lane width, Table 16.5 is shown below to find it out.
f_{LC} = adjustment for right-side lateral clearance, Table 16.6 is shown below to find it out.
TRD = total ramp density (ramps/mi), number of entry and exits in a mile

Table 16.5 Reduction in FFS due to due to lane width

Average Lane Width, ft	Reduction in FFS, f_{LW} (mi/hr)
≥ 12	0.0
≥ 11 – 12	1.9
≥ 10 - 11	6.6

Table 16.6 Reduction in FFS due to due to right-side lateral clearance

Right-Side Lateral Clearance, ft	Reduction in FFS, f_{LC} (mi/hr)			
	Lanes in One Direction			
	2	3	4	≥ 5
≥6	0.0	0.0	0.0	0.0
5	0.6	0.4	0.2	0.1
4	1.2	0.8	0.4	0.2
3	1.8	1.2	0.6	0.3
2	2.4	1.6	0.8	0.4
1	3.0	2.0	1.0	0.5
0	3.6	2.4	1.2	0.6

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Once FFS is calculated, v_p is calculated using the following equation:

$$v_p = \frac{V}{PHF\ N\ f_{HV}\ f_p}$$

where

v_p = demand flow rate under equivalent base conditions (pc/h/ln)
V = demand volume under prevailing conditions (veh/h)
PHF = peak-hour factor
N = number of lanes in analysis direction
f_{HV} = adjustment factor for presence of heavy vehicles in traffic stream
f_p = adjustment factor for unfamiliar driver populations, for regular users it is 1.0.

$$f_{HV} = \frac{1}{1 + P_T\left(E_T - 1\right) + P_R\left(E_R - 1\right)}$$

where

f_{HV} = heavy-vehicle adjustment factor
P_T = proportion of trucks and buses in traffic stream
P_R = proportion of RVs in traffic stream
E_T = passenger-car equivalent (PCE) of one truck or bus in traffic stream
E_R = PCE of one RV in traffic stream

Table 16.6 Passenger-Car Equivalent (PCE) by Terrain

Vehicle	Passenger-Car Equivalent (PCE) by Terrain		
	Level	Rolling	Mountainous
Trucks and buses, E_T	1.5	2.5	4.5
RVs, E_R	1.2	2.0	4.0

If v_p is less than the breakpoint flow rate, then $S = FFS$
If v_p is more than the breakpoint flow rate, then S = reduced speed beyond breakpoint
Also, check v_p with the maximum possible.

Problem 16.10

A freeway is to be designed to provide LOS C for the following conditions:

Trucks = 6%;
No lateral obstructions; rolling terrain;
Total ramp density of 0.5 ramps/km.
Two-way number of lanes = 6

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The f_{hv} (heavy vehicle adjustment factor) is most nearly:

A. 0.82
B. 0.92
C. 0.99
D. None of the above

Solution: B

From Table, E_T (PCE for trucks, rolling), $E_T = 2.5$
Determine E_R (PCE for RVs, rolling), $E_R = 2.0$
P_T = proportion of trucks and buses in traffic stream = 0.06
P_R = proportion of RVs in traffic stream = 0

$$f_{HV} = \frac{1}{1+P_T(E_T-1)+P_R(E_R-1)} = \frac{1}{1+0.06(2.5-1)+0(2.0-1)} = 0.92$$

Problem 16.11

A freeway is to be designed to provide LOS C for the following conditions:
Design hourly volume of 11,200 veh/h;
PHF = 0.92;
Trucks = 6%;
No of lanes in one-way = 3
No lateral obstructions; rolling terrain;
Total ramp density of 0.5 ramps /km.
Assume mainly commuter traffic for driver population factor. The demand flow rate (pc/hr/ln) under equivalent base conditions is most nearly:

A. 3,410
B. 3,810
C. 4,110
D. 4,410

Solution: D

From Table, E_T (PCE for trucks, rolling), $E_T = 2.5$
Determine ER (PCE for RVs, rolling), $E_R = 2.0$
P_T = proportion of trucks and buses in traffic stream = 0.06
P_R = proportion of RVs in traffic stream = 0

$$f_{HV} = \frac{1}{1+P_T(E_T-1)+P_R(E_R-1)} = \frac{1}{1+0.06(2.5-1)+0(2.0-1)} = 0.92$$

V = 11,200 veh/hr

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f_p (driver population factor); assume mainly commuter traffic, $f_p = 1.0$

$$v_p = \frac{V}{PHF\,N\,f_{HV}\,f_p} = \frac{11,200}{0.92(3)(0.92)(1.0)} = 4,410\ \frac{\text{pc}}{\text{hr.ln}}$$

Problem 16.12

A freeway is to be designed for the following conditions:

Heavy vehicle adjustment factor = 0.80
Total ramps density = 0.75 ramps/mi
Two lanes each way with average lane width = 12 ft
Right-Side Lateral Clearance = 4 ft
No of lanes in one-way = 2

The free flow speed (mph) of basic freeway segment is most nearly:

A. 65.6
B. 68.6
C. 71.7
D. 74.4

Solution: C

Reduction in FFS for lane width ≥ 12 ft, $f_{LW} = 0$
Adjustment for right-side lateral clearance, $f_{LC} = 1.2$

$$FFS = 75.4 - f_{LW} - f_{LC} - 3.22\,TRD^{0.84} = 75.4 - 0 - 1.2 - 3.22(0.75)^{0.84} = 71.7\ \frac{\text{mi}}{\text{hr}}$$

Problem 16.13

For a freeway the following information have been obtained.

Free-flow speed = 68.2 mph
Demand flow rate under equivalent base conditions = 1,450 pc/h/ln

The LOS of the freeway is most nearly:

A. LOS A B. LOS B C. LOS C D. LOS D

Solution: C

FFS = 68.2 mph
v_p = 1,450 pc/h/ln
From Table 16.4, the breakpoint for FFS 68.2 mph is about 1,200 < 1,450 pc/h/ln

$$S = 70 - 0.00001160\left(v_p - 1,200\right)^2 = 70 - 0.00001160\left(1,450 - 1,200\right)^2 = 69.3\ \frac{\text{mi}}{\text{h}}$$

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$$D = \frac{v_p}{S} = \frac{1,450}{69.3} = 20.9 \text{ pc/mi/ln}$$

From Table 16.3, the LOS is C for density 18 – 36 pc/mi/ln.

Traffic Flow Relationships

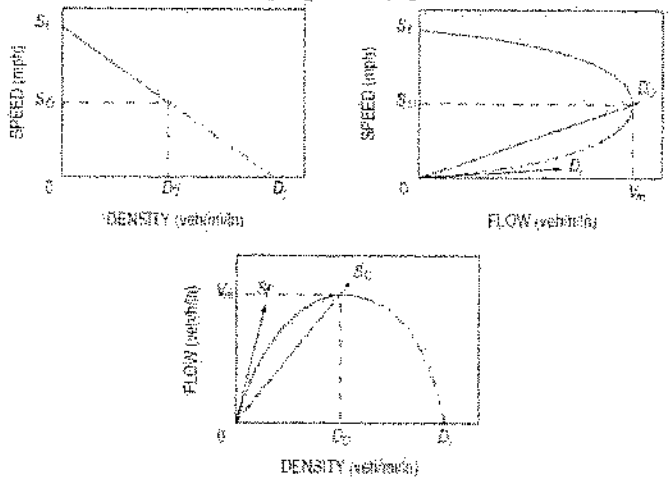
NCEES FE Ref. Handbook, presented the traffic flow relationship. Review it before practicing the problems.

Speed-Density Relationship. The vehicle speed (mph) and traffic density (veh/mile) per lane is inversely proportional with each other. The vehicle speed is the maximum when the density is the minimum and vice-versa. It is sensible that the more vehicles are on a road, the slower the velocity.

Speed-Flow Relationship. Speed-flow diagram is parabolic in nature. For any highway segment, there is an optimum speed-flow condition. If the speed is above the optimum one, speed decreases with the increase in flow (the upper part of the curve). If the speed is below the optimum one, speed decreases with the decrease in flow (the lower part of the curve).

Flow-Density Relationship. The traffic flow (veh/hr) per lane increases with the increase in density (veh/mi) in parabolic nature until the optimum condition (capacity) is reached. Beyond that, the flow decreases with the increase in density.

The above-mentioned relationships can be graphically presented as follows:



The following relationships can be developed from the above relationships.

$$S = S_f - \frac{S_f}{D_j} D \quad \text{[This is also called Greenshields Model]}$$

$$V = SD$$

$$V = S_f D - \frac{S_f}{D_j} D^2$$

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$$V_m = \frac{D_j S_f}{4} \left[D_o = \frac{D_j}{2} \text{ and } S_o = \frac{S_f}{2} \right]$$

D = Density (veh/mile)
S = Speed (mile/hr)
V = Flow (veh/hr)
V_m = Maximum flow
D_o = Optimum density (sometimes called critical density)
Dj = Jam density
S_o = Optimum speed (sometimes called critical speed)
S_f = Theoretical speed selected by the first driver entering a facility (i.e., under zero density and free flow conditions)

Problem 16.14

Data obtained from aerial photography showed six vehicles on a 700 ft-long section of road. Traffic data collected at the same time indicated an average time headway of 3.7 sec. The density (veh/mile) on the highway is most nearly:

A. 0.86 B. 28.3 C. 36.2 D. 45.3

Solution: D

$$D = \frac{\text{Vehicle}}{\text{Mile}} = \frac{6}{700\,\text{ft}\left(\frac{1\ \text{mile}}{5{,}280\ \text{ft}}\right)} = 45.3\,\frac{\text{veh}}{\text{mi}}$$

Problem 16.15

Data obtained from aerial photography showed six vehicles on a 700 ft-long section of road. Traffic data collected at the same time indicated an average time headway of 3.7 sec. The flow on the road in veh/hr is most nearly:

A. 5,837
B. 973
C. 2,863
D. 137

Solution: B

$$V = \frac{\text{Vehicle}}{\text{Hour}} = \frac{1}{3.7\,\text{sec}\left(\frac{1\ \text{hr}}{3{,}600\ \text{sec}}\right)} = 973\,\frac{\text{veh}}{\text{hr}}$$

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Problem 16.16

Data obtained from aerial photography showed six vehicles on a 700 ft-long section of road. Traffic data collected at the same time indicated an average time headway of 3.7 sec. The space mean speed in miles per hour is most nearly:

A. 37.4 B. 34.3 C. 28.5 D. 21.5

Solution: D

$$D = \frac{\text{Vehicle}}{\text{Mile}} = \frac{6}{700\,\text{ft}\left(\frac{1}{5{,}280}\frac{\text{mile}}{\text{ft}}\right)} = 45.3\frac{\text{veh}}{\text{mi}}$$

$$V = \frac{\text{Vehicle}}{\text{Hour}} = \frac{1}{3.7\,\text{sec}\left(\frac{1}{3{,}600}\frac{\text{hr}}{\text{sec}}\right)} = 973\frac{\text{veh}}{\text{hr}}$$

$$S = \frac{V}{D} = \frac{973\frac{\text{veh}}{\text{hr}}}{45.3\frac{\text{veh}}{\text{mi}}} = 21.5\frac{\text{mi}}{\text{hr}}$$

Problem 16.17

Data from a 1.8 m by 1.8 m inductive loop detector collected during a 30-second time period indicate that the mean speed of traffic is 80 km/h among the 16 vehicles counted. Assume an average vehicle length of 5.8 m. The density (veh/km) on the highway is most nearly:

A. 12 B. 18 C. 24 D. 30

Solution: C

$$D = \frac{V}{S} = \frac{\frac{16}{30\,\text{sec}\left(\frac{1}{3{,}600}\frac{\text{hr}}{\text{sec}}\right)}\frac{\text{veh}}{\text{hr}}}{80\frac{\text{km}}{\text{hr}}} = 24\frac{\text{veh}}{\text{km}}$$

Problem 16.18

In a freeway traffic stream, the capacity flow was observed to be 2,200 veh/h/lane, and the jam density at this location had been observed to be 125 veh/ln/km. The free flow speed is most nearly:

A. 70.4 km/hr B. 67.5 km/hr C. 62.1 km/hr D. 60.23 km/hr

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Solution: A

$$V_m = \frac{D_j S_f}{4}$$

$$S_f = \frac{4V_m}{D_j} = \frac{4\left(2{,}200\,\frac{\text{veh}}{\text{hr}}\right)}{125\,\frac{\text{veh}}{\text{km}}} = 70.4\,\frac{\text{km}}{\text{hr}}$$

Problem 16.19

In a freeway traffic stream, the capacity flow was observed to be 2,200 veh/h/lane, and the jam density at this location had been observed to be 125 veh/ln/km. The optimum/critical density in veh/km is most nearly:

A. 62.5
B. 57.5
C. 55
D. 250

Solution: A

$$D_o = \frac{D_j}{2} = \frac{125\,\frac{\text{veh}}{\text{km}}}{2} = 62.5\,\frac{\text{veh}}{\text{km}}$$

Problem 16.20

In a freeway traffic stream, the capacity flow was observed to be 2,200 veh/h/lane, and the jam density at this location had been observed to be 125 veh/lane/km. The optimum/critical speed in km/hr is most nearly:

A. 30.2 B. 32.1 C. 35.2 D. 38.5

Solution: C

$$V_m = \frac{D_j S_f}{4}$$

$$S_f = \frac{4V_m}{D_j} = \frac{4\left(2{,}200\,\frac{\text{veh}}{\text{hr}}\right)}{125\,\frac{\text{veh}}{\text{km}}} = 70.4\,\frac{\text{km}}{\text{hr}}$$

$$S_o = \frac{S_f}{2} = \frac{70.4\,\frac{\text{km}}{\text{hr}}}{2} = 35.2\,\frac{\text{km}}{\text{hr}}$$

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Problem 16.21

The relationship between traffic density (D, veh/mile) and mean vehicle speed (S, mile/hr) can be represented as follows:

$$V = 50 - 0.33D$$

The traffic flow relationship is given by, $V = SD$, the maximum capacity of overall volume (V_{max}) for this roadway in veh/mile is most nearly:

A. 2,000 B. 1,600 C. 1,700 D.1,900

Solution: D

$$V = SD = 50D - 0.33D^2; \qquad \frac{dV}{dD} = 50 - 0.66D$$

$$\frac{dV}{dD} = 0$$

$$50 - 0.66D = 0$$

Therefore, $D = 75.8$ veh/mile

$$V_{max} = 50D - 0.33D^2 = 50(75.8) - 0.33(75.8)^2 = 1{,}894 \approx 1{,}900 \frac{\text{veh}}{\text{hr}}$$

[Be prepared to do some rounding]

Gravity Model

The gravity model assumes that the trips produced at an origin and attracted to a destination are directly proportional to the total trip productions at the origin and the total attractions at the destination. The calibrating term or friction factor, F represents the reluctance or impedance of persons to make trips of various duration or distances. The general friction factor indicates that as travel times increase, travelers are increasingly less likely to make trips of such lengths. Calibration of the gravity model involves adjusting the friction factor. The socioeconomic adjustment factor is an adjustment factor for individual trip interchanges. An important consideration in developing the gravity model is balancing productions and attractions. Balancing means that the total productions and attractions for a study area are equal. Standard form of gravity model –

$$T_{ij} = P_i \frac{A_j F_{ij} K_{ij}}{\sum (A_j F_{ij} K_{ij})}$$

where

T_{ij} = number of trips that are produced in zone i and attracted to zone j
P_i = total number of trips produced in zone i
A_j = number of trips attracted to zone j
F_{ij} = a friction factor that is an inverse function of travel time between zones i and j
K_{ij} = socioeconomic adjustment factor for interchange ij

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Before the gravity model can be used for prediction of future travel demand, it must be calibrated. Calibration is accomplished by adjusting the several factors within the gravity model until the model can duplicate a known base year's trip distribution. For example, if you knew the trip distribution for the current year, you would adjust the gravity model so that it resulted in the same trip distribution as was measured for the current year.

Problem 16.22

An origin-destination survey was conducted in three zones in Long Island, New York. The results are presented below:

Zone	1	2	3	Total
Productions	350	400	250	1,000
Attractions	400	300	300	1,000

The survey's results for the zones' travel time in minutes are mentioned below:

Zone	1	2	3
1	2	6	1
2	6	5	3
3	2	3	4

The following table shows travel time versus friction factor.

Time (min)	1	2	3	4	5	6
Friction Factor	42	37	36	34	33	29

Assume $K_{ij} = 1$, the values of T_{11}, T_{12} and T_{13} (respectively) are most nearly:

A. 143.5, 74.8, 68.0
B. 143.5, 84.3, 80.6
C. 143.5, 48.7, 80.6
D. 143.5, 84.3, 122.2

Solution: D

The mathematical formulation for the gravity model is shown below:

$$T_{ij} = P_i \frac{A_j F_{ij} K_{ij}}{\sum\left(A_j F_{ij} K_{ij}\right)}$$

$$T_{11} = 350 \frac{(400 \times 37)}{(400 \times 37 + 300 \times 29 + 300 \times 42)} = 143.5$$

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$$T_{12} = 350\frac{(300 \text{ x } 29)}{(400 \text{ x } 37 + 300 \text{ x } 29 + 300 \text{ x } 42)} = 84.3$$

$$T_{13} = 350\frac{(300 \text{ x } 42)}{(400 \text{ x } 37 + 300 \text{ x } 29 + 300 \text{ x } 42)} = 122.2$$

Logit Models

The Logit model says, the probability that a certain mode choice will be taken is proportional to exponentiated utility over the sum of the exponentiated utility. The Logit Model also says that if a new mode of transportation is added to a system (or taken away) then the original modes will lose an amount of travels proportional to their share originally.

$$U_x = \sum_{i=1}^{n} a_i X_i$$

where

U_x = utility of mode x
n = number of attributes
X_i = attribute value (time, cost, and so forth)
a_i = coefficient value for attributes i (negative, since the values are disutilities)

The concept of utility assumes that you have a method of combining the various features (called attributes) of all the alternatives including their price, to give one measure of utility which is consistent across all the alternatives within the set of choices (or choice set) open to you. If two modes, auto (A) and transit (T), are being considered, the probability of selecting the auto mode A can be written as:

$$P(A) = \frac{e^{U_A}}{e^{U_A} + e^{U_T}}$$

$$P(x) = \frac{e^{U_x}}{\sum_{x=1}^{n} e^{U_x}}$$

Problem 16.23

The utility of private auto and mass transit is -3.11 and -3.77 respectively. The most nearly proportions of person-trips of the modes are respectively:

A. 66%, 34%
B. 56%, 44%
C. 62%, 38%
D. 52%, 48%

Solution: A

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$$P(auto)=\frac{e^{U_{auto}}}{e^{U_{auto}}+e^{U_{transit}}}=\frac{e^{-3.11}}{e^{-3.11}+e^{-3.17}}=0.66=66\%$$

$$P(transit)=\frac{e^{U_{mass}}}{e^{U_{auto}}+e^{U_{transit}}}=\frac{e^{-3.17}}{e^{-3.11}+e^{-3.17}}=0.34=34\%$$

Problem 16.24

The utility functions of auto and transit of a highway can be presented as follows:

$$U_{Auto}=-0.46-0.35T_1-0.1T_2-0.01C$$
$$U_{Transit}=-0.06-0.05T_1-0.15T_2-0.05C$$

where:

T_1 = total travel time (minutes)
T_2 = Waiting time (minutes)
C = Cost in cents

The surveying data shown the following travel characteristics:

	Auto	Transit
T_1	15	25
T_2	7	5
C	315	125

The proportions of travel in this zone by auto and transit, respectively are most nearly:

A. 22.3%, 77.7%
B. 32.3%, 67.7%
C. 46.5%, 53.5%
D. 56.5%, 43.5%

Solution: A

$$U_{Auto}=-0.46-0.35T_1-0.1T_2-0.01C=-0.46-0.35(15)-0.1(7)-0.01(315)=-9.56$$
$$U_{Transit}=-0.06-0.05T_1-0.15T_2-0.05C=-0.06-0.05(25)-0.15(5)-0.05(125)=-8.31$$

$$P(Auto)=\frac{e^{U_{Auto}}}{e^{U_{Auto}}+e^{U_{Transit}}}=\frac{e^{-9.56}}{e^{-9.56}+e^{-8.31}}=\frac{0.0000705}{0.0000705+0.000246}=22.3\%$$

$$P(Transit)=\frac{e^{-9.56}}{e^{-9.56}+e^{-8.31}}=\frac{0.000246}{0.0000705+0.000246}=77.7\%$$

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Traffic Safety

The crash rates at intersections can be determined using the following equation:

$$RMEV = \frac{1{,}000{,}000\,A}{V}$$

RMEV = crash rate per million entering vehicles
A = Number of crashes, total or by type occurring in a single year at the location
V = Average Daily Traffic (ADT) x 365

The crash rates for roadway segments can be calculated using the following equation:

$$RMVM = \frac{1{,}000{,}000\,A}{VMT}$$

RMVM = crash rate per hundred million vehicles miles
A = Number of crashes, total or by type occurring at the study location in a given period
VMT = Vehicle miles of travel during the given period
= ADT x Number of days in study period x length of road

The crash reduction in an intersection can be computed as:

$$\text{Crashes prevented, } = N(CR)\frac{\text{ADT(after improvement)}}{\text{ADT(before improvement)}}$$

N = expected number of crashes if countermeasure is not implemented and if the traffic volume remains the same
CR = Overall crash reduction factor for multiple mutually exclusive improvements at a single site

Problem 16.25

The minimum perception-reaction time recommended by the AASHTO for calculating stopping sight distance (sec) is most nearly:

A. 3 B. 2.5 C. 4 D. Depends on the city

Solution: B

Problem 16.26

The past 10- year report at an intersection says that 10 vehicle crashes per year at that intersection. The average daily traffic untiring at that intersection is 5,000. The crash rate per million entering vehicles at that intersection is most nearly:

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A. 2,000
B. 5.5
C. 200
D. 0.1

Solution: B

$$\text{RMEV} = \frac{A(1{,}000{,}000)}{V} = \frac{A(1{,}000{,}000)}{ADT(365)} = \frac{10(1{,}000{,}000)}{(5{,}000)(365)} = 5.48$$

Problem 16.27

In a test section of 5 miles, 10 vehicles crash in 6-month period. This test section has 300 traffic each day of the test. The crash rate per million vehicle miles (RMVM) is most nearly:

A. 3,700
B. 7,300
C. 370
D. 730

Solution: A

$$\text{VMT} = 300(6 \text{ months})\left(30\frac{\text{days}}{\text{mo}}\right)(5 \text{ miles}) = 270{,}000 \text{ veh-miles}$$

$$\text{RMVM} = \frac{A(100\text{x}10^6)}{\text{VMT}} = \frac{10(100\text{x}10^6)}{270{,}000} = 3{,}704 \approx 3{,}700$$

Problem 16.28

After a crash preventing measure being adopted the crash reduces to 80% of the original crash rate of 100 crashes per year. Note that the daily traffic is same/equal before and after the measure being adopted. The number of crashes prevented each year by this preventive measure is most nearly:

A. 20 per year
B. 30 per year
C. 25 per year
D. 35 per year

Solution: A

$$\text{Crash reduction factor, } CR = \frac{20\%}{100\%} = 0.2 \quad \text{and} \quad \frac{ADT(\text{after})}{ADT(\text{before})} = 1$$

$$\text{Crashes prevented, } = N(CR)\frac{ADT(\text{after})}{ADT(\text{before})} = (100)(0.2)\,(1) = 20/\text{year}$$

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Pavement Design

AASHTO Structural Number

The pavement thickness design by the AASHTO is conducted using Structural Number (SN). The SN for any pavement site is determined by considering the traffic demand, soil strength, environment, and serviceability condition. The SN is then correlated with the pavement layers' thicknesses and materials types by as follows:

$$SN = a_1D_1 + a_2D_2 + a_3D_3$$

D_i = thickness of different layers

a_i = layer coefficient; stiffer the material higher the number

The optimum combination of layers and materials are chosen for final pavement section considering site condition, local materials, economy, etc. Now, practice the following problems.

Problem 16.29

The layer coefficients of different layers of a pavement system are shown below. The structural number of this pavement system is 6. The thickness (inch) of the asphalt layer is most nearly:

Asphalt Concrete: 0.4
Aggregate Base (6 inch thick): 0.23
Subbase (8 inch thick): 0.11

A. 8.5
B. 9.5
C. 10.5
D. 7.5

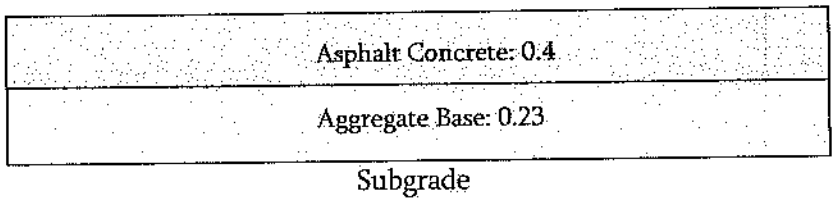
Solution: B

$$6 = a_1D_1 + a_2D_2 + a_3D_3 = 0.4D_1 + 0.23(6) + 0.11(8)$$
$$D_1 = 9.35 \text{ in} \approx 9.5 \text{ in}$$

Problem 16.30

The layer coefficients of different layers of a pavement system are shown below. Considering the top layer only, the structural number has been found to be 2.5. Considering the top two-layer only, the structural number has been found to be 4.0. The thicknesses (inch) of the top layer and base layer respectively is most nearly:

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A. 6.25, 6.5
B. 3.5, 4.5
C. 6.5, 6.25
D. 5.5, 6.5

Solution: A

Considering the top layer only:

$$SN_1 = a_1 D_1$$
$$2.5 = (0.4) D_1$$
$$D_1 = \frac{2.5}{0.4} = 6.25 \text{ in}$$

Considering the top two-layer only:

$$SN_2 = a_1 D_1 + a_2 D_2$$
$$4.0 = (0.4)(6.25) + (0.23) D_2$$
$$D_2 = \frac{4.0 - (0.4)(6.25)}{0.23} = 6.5 \text{ in}$$

Equivalent Single Axle Load (ESAL)

ESAL establishes a damage relationship for comparing the effects of different types of axles with different loads. It converts all axles into a reference axle load which is 18-kip single axle with dual tires. To convert other axles into 18-kip single axle two tires, a factor is multiplied. The factors can be found in the NCEES FE Ref. Handbook. For example, The ESAL factor for 40-kip tandem axle is 2.08. It means this axle produces 2.08 times of damage compared to 18-kip single axle. Now practice following problems.

Problem 16.31

In a construction site, a construction vehicle with a 24 kips single axle and a 44 kips tandem axle travels 30 times per day from the plant to the job site. If the construction period is 6 months (180 days) with no rest days, the total ESAL of that vehicle is most nearly:

A. 108,000 B. 54,000 C. 33,000 D. 5.3×10^6

Solution: C

ESAL of the vehicle, = 3.03+3 = 6.03 per travel
No of travel = 6 months x 30 day/mo x 30 time a day = 5,400 travels
Total ESAL = 5,400 travels x 6.03 ESAL per travel = 32,562 ≈ 33,000

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Problem 16.32

The ADT of a highway is 10,000, the total number of traffic in millions in a year is most nearly:

A. 10 B. 3.65 C. 7.85 D. 22.23

Solution: B

Total traffic in a year = (ADT) (365 days) =10,000(365) = 3.65×10^6 = 3.65 millions

Performance Grade (PG) Binder Grading

Superpave performance grading (PG) is reported using two numbers. The first on the average 7-day maximum pavement temperature (°C), and the second being the minimum pavement design temperature likely to be experienced (°C). Thus, a PG 64-28 is intended for use where the average 7-day maximum pavement temperature is 64 °C and the expected minimum pavement temperature is -28 °C. For different binders, the laboratory testing protocols are different. These are listed in the NCEES FE Ref. Handbook.

Problem 16.33

In a region, average 7-day maximum pavement design temperature is 50 °C, and the minimum pavement design temperature is -12 °C. The recommended asphalt binder for this region is most nearly:

A. PG 52-16 B. PG 50-10 C. PG 52-22 D. All of the above

Solution: A

From FE Ref. Guide, for the average 7-day maximum pavement design temperature less than 52°C, PG 52 is recommended. Then, for the minimum pavement design temperature is -12 °C, the second number is -16 (as -12 > -16). Therefore, the PG 52-16 is recommended.

Problem 16.34

For the asphalt binder, PG 64-28, the recommended design temperature for direct tension test of binder is most nearly:

A. 64 °C B. 28 °C C. -28 °C D. -18 °C

Solution: D

From the NCEES FE Ref. Handbook, the recommended design temperature for PG 64-28 binder for direct tension test is -18 °C.

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Problem 16.35

The average 7-day maximum pavement design temperature of a pavement is 60 °C and the minimum pavement design temperature is -26 °C. The recommend asphalt binder is most nearly:

A. PG 58-28 B. PG 64-28 C. PG 64-22 D. PG 64-34

Solution: B

From the NCEES FE Ref. Handbook, top-right corner of the PG Table, PG 64-28.

Superpave Mixture Design

Superpave mixture design replaces the old mixture designs which were using the volumetric properties of the compacted mixture. Superpave mix design selects asphalt binder and aggregate as a part of mix design and consider the climate, and traffic level as well. The compaction effort is determined considering the climate and traffic level. NCEES FE Ref. Handbook, lists the Superpave mixture design requirements. Now, practice the following problems.

Problem 16.36

A pavement is to be designed in a region with total ESAL of 5 millions. The average design high year temperature is 40 °C. The design number of gyration for the test asphalt cylinder using the super pave mixture design method is most nearly:

A. 8 B. 106 C. 95 D. 169

Solution: B

The N_{des} from the Table is 106 for ESEL < 10 million at 40 °C.

Problem 16.37

The minimum VMA requirement at 4% air void for 19 mm aggregate using the Superpave mix design is most nearly:

A. 15% B. 14% C. 13% D. 12%

Solution: C

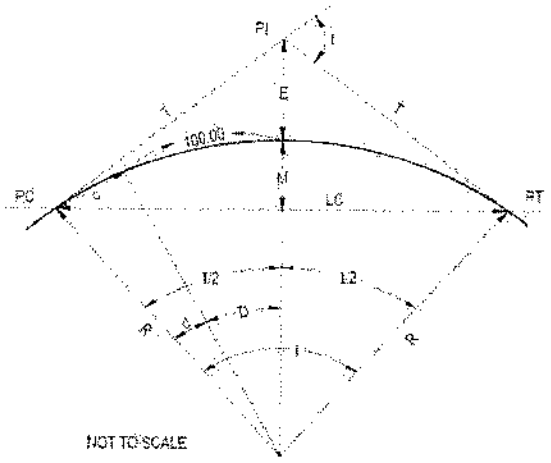
From the NCEES FE Ref. Handbook, the minimum VMA requirement at 4% air void for 19 mm aggregate is 13%.

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Horizontal Curve

Horizontal curves occur at locations where two roadway segments intersect. It helps a vehicle turns smoothly at a gradual rate rather than a sharp turning. This curve is semicircle to provide with a constant turning rate. The intersection point of the two roads is defined as the Point of Intersection (PI). The location of the curve's start point is defined as the Point of Curve (PC), and the location of the curve's end is considered the Point of Tangent (PT). Both the PC and PT are the distance T from the PI, where T is defined as the Tangent Length. The following notations have been used in future formulation:

D = Degree of Curve, Arc Definition
PC = Point of Curve (also called BC)
PT = Point of Tangent (also called EC)
PI = Point of Intersection
I = Intersection Angle (also called Δ)
Angle Between Two Tangents
L = Length of Curve, from PC to PT
T = Tangent Distance
E = External Distance
R = Radius
LC = Length of Long Chord
M = Length of Middle Ordinate
c = Length of Sub-Chord
d = Angle of Sub-Chord
l = Curve Length for Sub-Chord



The following formulas are listed in the NCEES FE Ref. Handbook,

$$R = \frac{5729.58}{D}$$

$$R = \frac{LC}{2\sin\left(\frac{I}{2}\right)}$$

$$T = R\tan\left(\frac{I}{2}\right) = \frac{LC}{2\cos\left(\frac{I}{2}\right)}$$

$$L = RI\left(\frac{\pi}{180}\right) = \frac{I}{D}(100)$$

$$M = R\left[1-\cos\left(\frac{I}{2}\right)\right]$$

$$\frac{R}{E+R} = \cos\left(\frac{I}{2}\right)$$

$$\frac{R-M}{R} = \cos\left(\frac{I}{2}\right)$$

$$c = 2R\sin\left(\frac{d}{2}\right)$$

$$l = Rd\left(\frac{\pi}{180}\right)$$

$$E = R\left[\frac{1}{\cos\left(\frac{I}{2}\right)} - 1\right]$$

-Deflection angle per 100 feet of arc length equals $D/2$.
-The degree of curvature is the central angle, D subtended by a chord of 100 feet.

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Problem 16.38

The intersection angle of a 6°-horizontal curve is 60°50', and the PC is located at 140 + 22.75. The station of the PT is most nearly:

A. 145 + 36.64
B. 130 + 36.64
C. 135 + 36.64
D. 150 + 36.64

Solution: D

$$PT = PC + L = ?$$

$$R = \frac{5729.58}{D} = \frac{5729.58}{6} = 954.98 \text{ ft}$$

$$I = 60^{\circ}50' = \left(60 + \frac{50}{60}\right)^{\circ} = 60.833^{\circ}$$

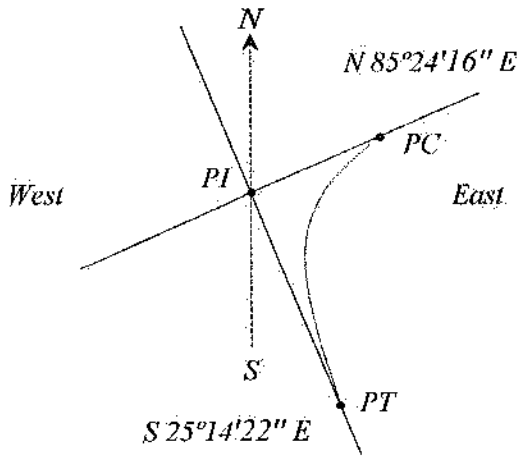
$$L = RI\left(\frac{\pi}{180}\right) = (954.98)(60.833)\left(\frac{\pi}{180}\right) = 1{,}013.89 \text{ ft}$$

$$PT = PC + L = (140 + 22.75) + (10 + 13.89) = 150 + 36.64$$

Problem 16.39

A simple-circular horizontal curve has the following features:

- Degree of curve = 15°
- Bearing on incoming (back) tangent is N 85°24'16" E
- Bearing on outgoing (forward) tangent is S 25°14'22" E
- Station of the *PI* = 178 + 33.87



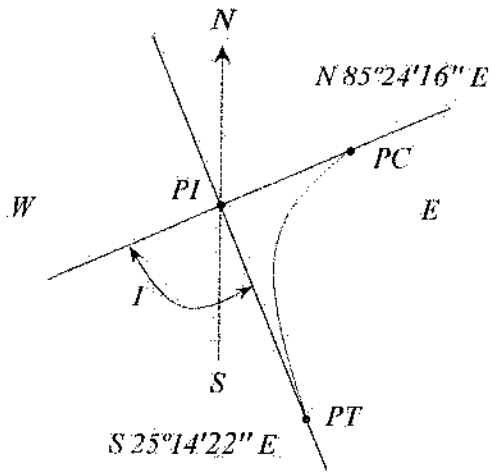
The tangent length (ft) of the curve is most nearly:

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A. 240 B. 265 C. 300 D. 320

Solution: B

$T = R\ tan(I/2) = ?$



$I = 85°24'16'' + 25°14'22'' = 110°38'38'' = 110.411°$

$I = 180° - 110.411° = 69.589°$ [Acute angle of I is considered]

$$R = \frac{5,729.58}{D} = \frac{5,729.58}{15} = 381.97 \text{ ft}$$

$T = R\ tan(I/2) = 381.97 \tan(69.589/2) = 265.4$ ft

Problem 16.40

A horizontal curve has a radius of 1,300 ft and an intersection angle of 22°. The point of intersection (*PI*) is located at 15+33 station. The position (station) of the point of curve (PC) is most nearly:

A. 13+10 B. 13+53 C. 13+20 D. 12+80

Solution: D

$PC = PI - T = ?$

Tangent distance, $T = R \tan\left(\frac{I}{2}\right) = 1,300 \tan\left(\frac{22}{2}\right) = 253$ ft

$PI = 15 + 33$ station $= 1,533$ ft

$PC = PI - T = 1,533 \text{ ft} - 253 \text{ ft} = 1,280 \text{ ft} = 12 + 80$ station

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Problem 16.41

A horizontal curve is designed with a 1,500 ft radius and a tangent length of 300 ft. If the *PI* is located at 15+80 station, the station of *PT* is most nearly:

A. 12+80 B. 17+22 C. 18+72 D. 18+10

Solution: C

$$PT = PC + L$$
$$PC = PI - T = 1{,}580 \text{ ft} - 300 \text{ ft} = 1{,}280 \text{ ft} = 12 + 80 \text{ station}$$
$$L = RI\left(\frac{\pi}{180}\right) \quad (I \text{ is still unknown})$$
$$T = R\tan\left(\frac{I}{2}\right)$$
$$\Rightarrow \quad \tan\left(\frac{I}{2}\right) = \frac{T}{R} = \frac{300}{1{,}500} = 0.2 \qquad \text{So, } I = 22.62°$$
$$L = RI\left(\frac{\pi}{180}\right) = 1{,}500(22.62)\left(\frac{\pi}{180}\right) = 592 \text{ ft}$$
$$PT = PC + 592 \text{ ft} = 1{,}280 + 592 = 1{,}872 \text{ ft} = 18 + 72 \text{ station}$$

Problem 16.42

A horizontal curve is to be designed with 1,500 ft radius and angle at intersection of 30°. The external distance (ft) is most nearly:

A. 53 B. 63 C. 43 D. 66

Solution: A

$$E = R\left[\frac{1}{\cos\left(\frac{I}{2}\right)} - 1\right] = 1{,}500\left[\frac{1}{\cos 15} - 1\right] = 52.9 \text{ ft}$$

Problem 16.43

A horizontal curve is to be designed for a two-lane road in Colorado Spring. The following data are known:

Intersection angle: 40 degrees
Tangent length = 450 ft

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Station of PI: 3300+12.65
$e = 10\%$
$f = 0.15$

The maximum design speed (mph) feasible is most nearly:

A. 48 B. 58 C. 68 D. 78

Solution: C

$T = R \tan(I/2)$

$$R = \frac{T}{\tan\left(\frac{I}{2}\right)} = \frac{450}{\tan\left(\frac{40}{2}\right)} = 1,236 \text{ ft}$$

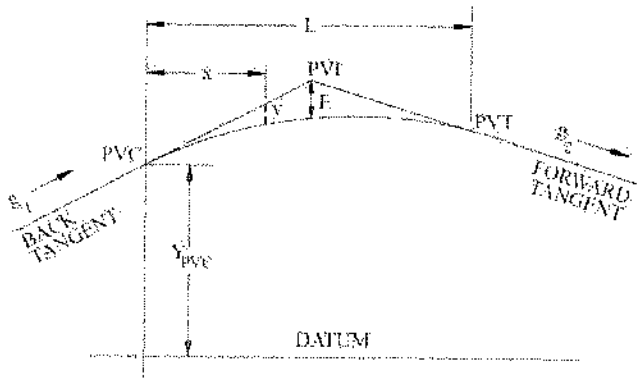
$$R = \frac{V^2}{15(0.01e + f)}$$

$$V = \sqrt{15R(0.01e + f)} = \sqrt{15(1,236)(0.01(10) + 0.15)} = 68 \text{ mph}$$

Vertical Curve

A vertical curve provides a transition between two sloped (upgrade and downgrade) roadways. It helps vehicles pass the elevation rate change smoothly at a gradual rate rather than a sharp change. These curves are parabolic and are assigned stationing based on a horizontal axis.

L = Length of curve
PVC = Point of vertical curvature
PVI = Point of vertical intersection
PVT = Point of vertical tangency
g_1 = Grade of back tangent
x = Horizontal distance from PVC to point on curve
g_2 = Grade of forward tangent
a = Parabola constant
E = Tangent offset at PVI $= \frac{AL}{800}$
r = Rate of change of grade
K = Rate of vertical curvature



The following formulas are listed in the NCEES FE Ref. Handbook:

$$x_m = \text{Horizontal distance to min/max elevation on curve} = -\frac{g_1}{2a} = \frac{g_1 L}{g_1 - g_2}$$

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$$\text{Tangent elevation} = Y_{PVC} + g_1 x \text{ and} = Y_{PVC} + g_2\left(x - \frac{L}{2}\right)$$

$$\text{Curve elevation} = Y_{PVC} + g_1 x + ax^2 = Y_{PVC} + g_1 x + \left[\frac{g_2 - g_1}{2L}\right]x^2$$

$$a = \frac{g_2 - g_1}{2L}; \quad E = a\left(\frac{L}{2}\right)^2; \quad r = \frac{g_2 - g_1}{L}; \quad K = \frac{L}{A}$$

Problem 16.44

A crest vertical curve connects a +3.33% grade and a –5.55% grade. The PVC is at station 121 + 22 at an elevation of 401.77 ft. The design speed is 60 mph, and length of the curve is 1,340.88 ft. The elevation (ft) of the high point is most nearly:

A. 367 B. 420 C. 380 D. 410

Solution: D

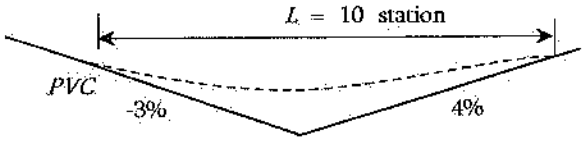
$$x_m = \frac{g_1 L}{g_1 - g_2} = \frac{0.033(1{,}340.88\,\text{ft})}{0.033 - (-0.055)} = 502.83\ \text{ft}$$

$$Y_{high} = Y_{PVC} + g_1 x_m + \left[\frac{g_2 - g_1}{2L}\right]x_{pi}^2 = 401.77 + 0.033(502.83) + \left(\frac{-0.055 - 0.033}{2(1{,}340.88)}\right)(502.83)^2$$

$$= 410.1\,\text{ft}$$

Problem 16.45

In a vertical curve, a back tangent with a -3% grade meets at the station of 1200+50 with a forward tangent of 4%, as shown below:



PVI at station 1200+50
PVI Elev. = 5,500 ft

The tangent offset (ft) at the *PVI* is most nearly:

A. 10.0
B. 0.9
C. 0.8
D. 9.0

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Solution: D

$$E = \frac{AL}{800} = \frac{|-3-4|(10\text{ station})}{800} = 0.088\text{ station} = 8.8\text{ ft} \approx 9\text{ ft}$$

Problem 16.46

An equal-tangent crest vertical curve joining 3% and -2% grade is to be designed. The tangents intersect at station (248 + 00) at an elevation of 2,250 ft, and $K = 300$. The elevation (ft) on the curve at 50-ft from the *PVC* is most nearly:

A. 2,123.64
B. 2,228.99
C. 2,199.62
D. 2,239.38

Solution: B

$$Y = Y_{PVC} + g_1 x + \left[\frac{g_2 - g_1}{2L}\right]x^2 = ?$$

$L = KA = 300(3 - (-2) = 1{,}500$ ft;
Therefore, $L/2 = 750$ ft
Elevation of *PVC* = 2,250 – 3% of 750 ft = 2,227.5 ft

$$Y_{50} = 2{,}227.5 + (0.03)(50) + \left[\frac{-0.02 - 0.03}{2(1{,}500)}\right](50)^2 = 2{,}228.99$$

Problem 16.47

A sag vertical curve joining -3% and 4% grade is to be designed for design speed of 70 mph (K = 181). If the elevation of the PVC is 123.72 ft located at station 122 + 77, the station of the PVT is most nearly:

A. 133 + 44
B. 135 + 44
C. 137 + 44
D. 139 + 44

Solution: B

$PVT = PVC + L = ?$
$L = KA = 181(4 - (-3) = 1{,}267$ ft.
Station of PVC = 122 + 77
Station of PVT = (122 + 77) + (12 + 67) = 135 + 44

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Problem 16.48

The point of vertical curve is located at 3,200 ft elevation. The uphill grade is 2%. The tangent elevation at 500 ft from the PVC in feet is most nearly:

A. 3,200 B. 3,210 C. 3,220 D. 3,250

Solution: B

$$\text{Tangent elevation} = Y_{PVC} + g_1 x = 3,200 + \left(\frac{2}{100}\right)(500) = 3,210 \text{ ft}$$

Earthwork Formulas

Earthwork is essential in transportation engineering to minimize construction cost and increase in drivers' comfort. Calculating the volume is important to determine the amount of earthwork on a given site. For linear highways, volumes can easily be calculated by integrating the areas of the cross sections for the entire length of the segment. More simply, several cross sections can be selected and an average can be taken for the entire length. Two popular volume calculation equations are presented below:

Average End Area Formula, $V = \frac{L}{2}(A_1 + A_2)$

Prismoidal Formula, $V = \frac{L}{6}(A_1 + 4A_m + A_2)$

Volume of Cone, $V = \frac{h}{3}(\text{Base Area})$

where: A_M = area of mid-section and L = distance between A_1 and A_2

Problem 16.49

The length of a highway segment is 100 ft. The cross-sectional area of that highway measured at the ends are 40 ft^2 and 60 ft^2. The design document says the area at the middle of this segment is 45 ft^2. The volume (ft^3) of this highway segment is most nearly:

A. 4,500 B. 4,667 C. 5,233 D. 3,267

Solution: B

$$\text{Volume}, V = \frac{L}{6}(A_1 + 4A_m + A_2) = \frac{100 \text{ ft}}{6}(40 + 4 \times 45 + 60) = 4,667 \text{ ft}^3$$

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Area Formulas

Areas can be calculated using the following three rules, as listed the NCEES FE Ref. Handbook. Each equation is for specific purpose. The problems may not specify which equation to be used. Students must identify which equation to be used. The first one is straight forward for coordinate system. The trapezoidal rule is for any number of segment where the ordinates are connected by straight lines. The Simpson's 1/3 rule is for odd number of ordinates where the ordinates can be connected by curve lines. The following few problems will clarify this.

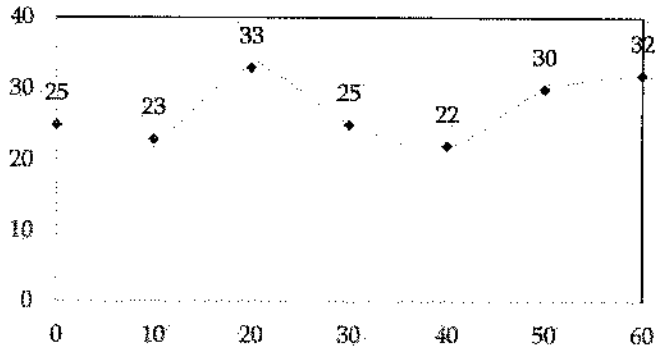
Area bounded by coordinates, $A = \frac{1}{2}[x_A(y_B - y_N) + x_B(y_C - y_A) + ...]$

Area by Trapezoidal Rule, $A = w\left(\frac{h_1 + h_n}{2} + h_2 + h_3 + h_4 + ... + h_{n-1}\right)$

Area by Simpson's 1/3 rule, $A = \frac{w}{3}[h_1 + 2\sum_{k=3,5..}^{n-2} h_k + 4\sum_{k=2,4..}^{n-1} h_k + h_n]$, n = odd

Problem 16.50

The area of the following curve using the Simpson's 1/3 rule is most nearly:



A. 1,622
B. 1,595
C. 1,456
D. 1,326

Solution: B

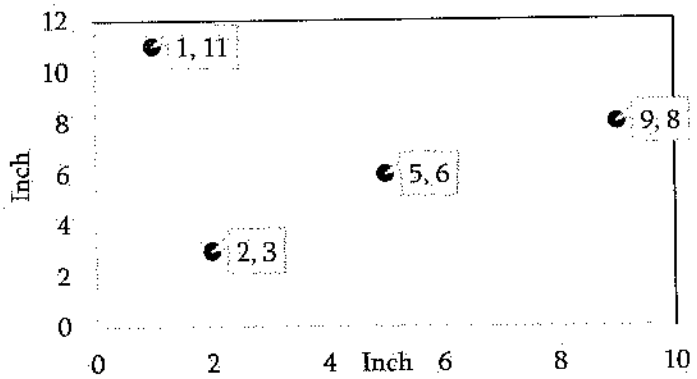
$$\text{Area} = \frac{w}{3}[h_1 + 2\sum_{k=3,5..}^{n-2} h_k + 4\sum_{k=2,4..}^{n-1} h_k + h_n] = \frac{10}{3}[25 + 2(33 + 22) + 4(23 + 25 + 30) + 32]$$

=1,595 sq-unit

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Problem 16.51

The area (in²) bounded by the following four coordinates is most nearly:



A. 32.7
B. 22.7
C. 27.5
D. 31.5

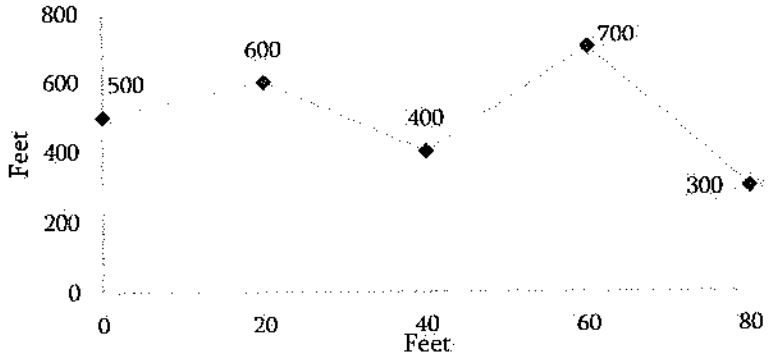
Solution: C

$$A = \frac{1}{2}[x_A(y_B - y_N) + x_B(y_C - y_A) + ...]$$

$$A = \frac{1}{2}[2(6-11) + 5(8-3) + 9(11-6) + 1(3-8)] = 27.5 \text{ in}^2$$

Problem 16.52

The area (ft²) of the following region using the trapezoidal rule is most nearly:



A. 38,000
B. 42,000
C. 45,000
D. 48,000

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Solution: B

$$A = w\left(\frac{h_1 + h_n}{2} + h_2 + h_3 + h_4 + \ldots\ldots + h_{n-1}\right)$$

$$A = 20\left(\frac{500+300}{2} + 600 + 400 + 700\right) = 42{,}000 \text{ ft}^2$$

Problem 16.53

The cross-section areas at two ends of a 100-ft highway segment are 2,000 ft² and 2,300 ft². The volume (ft³) of the segment is most nearly:

A. 215,000
B. 213,000
C. 214,000
D. 212,000

Solution: A

$$V = \frac{L}{2}(A_1 + A_2) = \frac{100 \text{ ft}}{2}(2000 + 2300) \text{ ft}^2 = 215{,}000 \text{ ft}^3$$

NCEES FE Civil exam includes traditional multiple-choice questions as well as alternative item types (AITs). AITs include but are not limited to the following:

- *Multiple correct—allows examinees to select multiple answers*
- *Point and click—requires examinees to click on part of a graphic to answer*
- *Drag and drop—requires examinees to click on and drag items to match, sort, rank, or label*
- *Fill in the blank—provides a space for examinees to enter a response to the question*

The last chapter of this book contains some examples. Visit https://ncees.org for more information.

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17 SURVEYING

Fundamentals of Engineering (FE) CIVIL Exam Specification mentions, 4–6 questions may appear in the exam based on Surveying as follows:

A. Angles, distances, and trigonometry
B. Area computations
C. Earthwork and volume computations
D. Closure
E. Coordinate systems (e.g., state plane, latitude/longitude)
F. Leveling (e.g., differential, elevations, percent grades)

The NCEES FE Ref. Handbook does not clearly mention the topics to be covered in surveying other than areas and volumes calculation which is discussed in Transportation Engineering. Few other topics listed above are discussed here.

Angles, Distances, and Trigonometry

This topic is discussed in the Mathematics chapter based on mathematics point of view. It is further discussed here using few more problems for better understanding.

Problem 17.1

Which of the following devices is a distance measuring device/method?

A. Pacing
B. Odometer
C. Stadia
D. All of the above

Solution: D

Examples of distance measuring devices/methods are Pacing, Scaling, Odometer, EDM (electronic distance measurement), Stadia, Subtense bar, Fiberglass tapes, Steel tapes, etc.

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Problem 17.2

Which of the following devices is a taping accessory?

A. Plumb bob
B. Hand level
C. Clinometer
D. All of the above

Solution: D

Taping accessories are Plumb bob, Hand level, Clinometer, Range pole, Tension handle, etc.

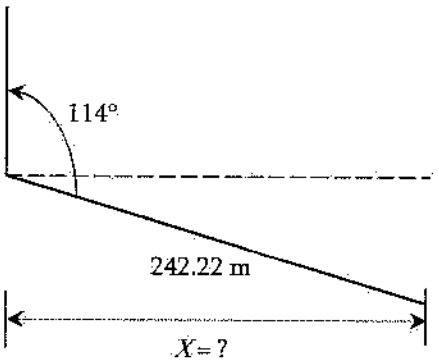
Problem 17.3

The slope distance and the zenith of a land, measured using a total station, are 242.22 m and 114° respectively. The horizontal distance (m) of that land is most nearly –

A. 219.38
B. 223.65
C. 232.36
D. 221.28

Solution: D

$114° - 90° = 24°$



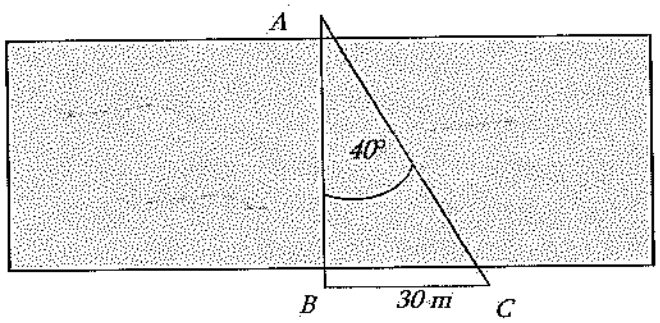
$$\cos 24 = \frac{X}{242.22}$$

$\Rightarrow \quad X = 221.28$ m

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Problem 17.4

A surveyor wants to measure the width of a river. His measurement on three points on the river bank are shown below. The width (m) of the river is most nearly:



A. 22.98
B. 19.28
C. 35.75
D. 25.2

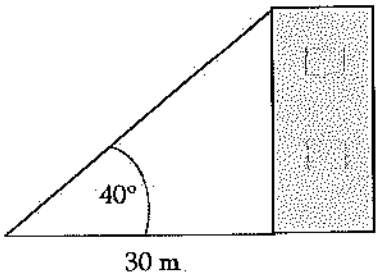
Solution: C

$$\tan 40 = \frac{30\,\text{m}}{\text{Width}}$$

$$\text{Width} = 30 / \tan 40 = 35.75\ \text{m}$$

Problem 17.5

A surveyor wants to measure the height of a building. His measurements are shown below. The height (m) of the building is most nearly:



A. 22.98
B. 19.28
C. 35.75
D. 25.2

Solution: D

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$$\tan 40 = \frac{\text{Height}}{30\,\text{m}}$$

Height = 30 tan 40 = 25.2 m

Problem 17.6

Which of the following statements is false?

A. With 0 to 90 degree azimuth, your bearing = 0 to 90 degree bearing
B. With 90 to 180 degree azimuth, your bearing = 180 – azimuth
C. With 180 to 270 degree azimuth, your bearing = azimuth – 180
D. With 270 to 360 degree azimuth, your bearing = 360 – azimuth
E. None of the above

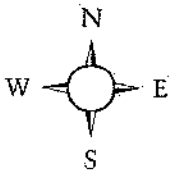
Solution: E

All the four statements are correct. Azimuths are horizontal angles observed clockwise from any reference meridian commonly from north in plane surveying. The bearing of a line is defined as the acute horizontal angle (less than 90°) between a reference meridian and the line. The angle is observed from either the north or south toward the east or west, to give a reading value smaller than 90°.

Problem 17.7

The azimuth of a boundary line is 147°13′26″. Its bearing is most nearly:

A. S 34°46′14″ E
B. S 32°46′14″ W
C. S 32°46′14″ E
D. N 32°46′14″ E



Solution: C

180 - 147°13′46″ = S 32°46′14″ E
(angle is measured clock-wise in surveying)

Problem 17.8

2.341539 radians is most nearly:

A. 134°16′37″
B. 134°37′16″
C. 268°32′37″

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D. $134^{\circ}9'37''$

Solution: D

$$\pi \text{ rad} = 180^{\circ}$$

$$2.341539 = \frac{180}{\pi}(2.341539) = 134.1603 = 134^{\circ}9'37''$$

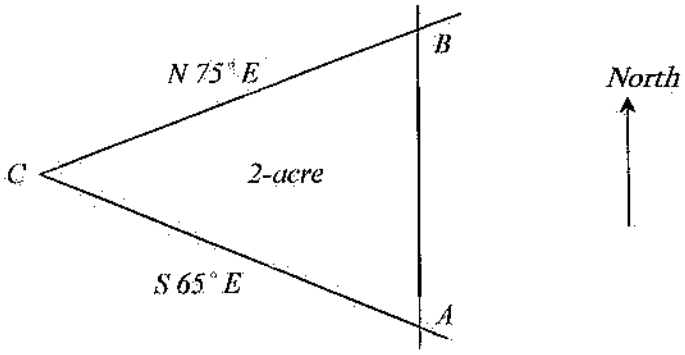
$$[1^{\circ} = 60' \text{ and } 1' = 60'']$$

Area, Earthwork and Volume computations

These topics have been discussed in Transportation Engineering. Some more practice problems:

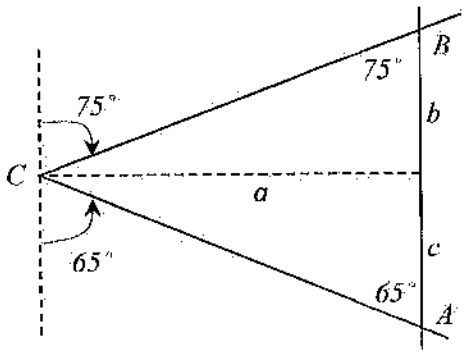
Problem 17.9

A client wants to create a 2-acre parcel by establishing a North-South line, *AB*, as shown in the figure below. The length (ft) of Side *AB* is most nearly:



A. 300
B. 387
C. 427
D. 487

Solution: D



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Area = 2 acres

$$\frac{1}{2}(b+c)a = 2\left(43{,}560\ \text{ft}^2\right)$$

$$ab + ac = 174{,}240$$

Now, $\frac{a}{b} = \tan 75$ or, $b = \frac{a}{\tan 75}$

Similarly, $c = \frac{a}{\tan 65}$

Thus, $a\left(\frac{a}{\tan 75}\right) + a\left(\frac{a}{\tan 65}\right) = 174{,}240$

$$0.7342a^2 = 174{,}240$$

$$a = 487.2\ \text{ft}$$

Problem 17.10

A small rectangular lot measures 100±0.3 ft by 200±0.7 ft. The area (ft²) of the lot is best stated as:

A. 20,000 ± 130.2
B. 20,000 ± 170.0
C. 20,000 ± 21.0
D. 20,000 ± 92.2

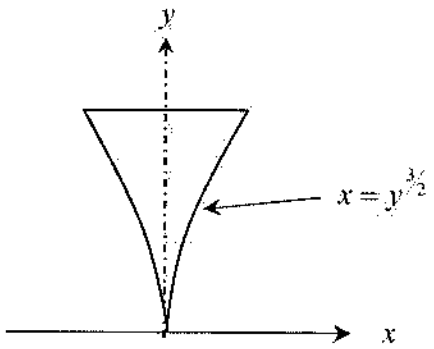
Solution: D

$$\text{Error} = \pm\sqrt{\left(A\,\sigma_B\right)^2 + \left(B\,\sigma_A\right)^2} = \pm\sqrt{\left(100\text{x}0.7\right)^2 + \left(200\text{x}0.3\right)^2} = \pm 92.2$$

Area = 100 x 200 ± 92.2 = 20,000 ± 92.2 ft²

Problem 17.11

A thin-walled tank is constructed as a body of revolution of a parabola, as shown in the figure. The base diameter is 30 ft, and the height of tank is 50 ft. The volume (yd³) of water in the tank when full is most nearly:



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A. 4,908,750
B. 181,805
C. 1,562,500
D. 811,800

Solution: B

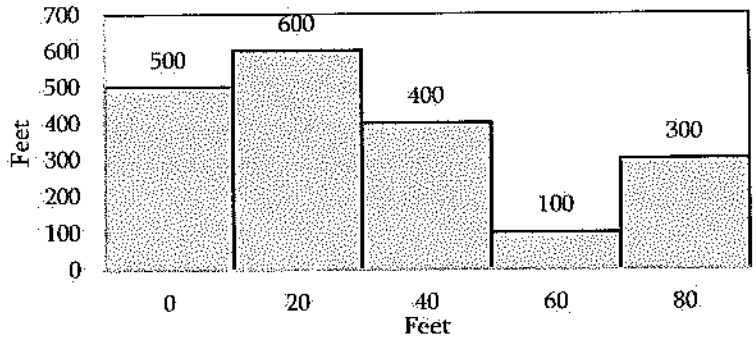
$$V = \int_0^{25} A\,dy = \int_0^{50} \left(\pi x^2\right) dy = \pi \int_0^{50} \left(y^{3/2}\right)^2 dy = \pi \int_0^{50} y^3 dy = \pi \left[\frac{y^4}{4}\right]_0^{50} = 4{,}908{,}750\ \text{ft}^3$$

$$= \frac{4{,}908{,}750\ \text{ft}^3}{27\ \frac{\text{ft}^3}{\text{yd}^3}} = 181{,}805\ \text{yd}^3$$

Problem 17.12

The area of the following region using the trapezoidal rule is _____ ft².

[Write the answer in nearest thousands with comma and without unit such as 7,000 or 87,000 etc.]



Solution: 30,000

$$A = w\left(\frac{h_1 + h_n}{2} + h_2 + h_3 + h_4 + \ldots\ldots + h_{n-1}\right)$$

$$A = 20\left(\frac{500 + 300}{2} + 600 + 400 + 100\right) = 30{,}000\ \text{ft}^2$$

Problem 17.13

The area in acres of triangular lots shown on a plat having the recorded right-angle sides of 335.36 ft and 804.02 ft is most nearly:

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A. 6.18
B. 3.09
C. 1.34
D. 8.18

Solution: B

$$A = \frac{1}{2}bh = \frac{1}{2}(335.36)(804.02) = 134,818\ ft^2 = \frac{134,818}{43,560}acre = 3.09\ acre$$

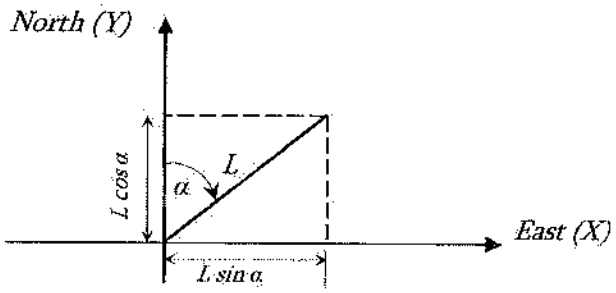
Traversing

Traversing consists of series of lines (open ended or closed) to determine the relative locations of points or object. Closed traverse can be further divided into two categories: polygon and link. Polygon traverse, returns to the starting point, thus forming a closed area. Link traverses ends to a point which have a positional accuracy. Open traverse does not return to the starting point or close upon a point of questionable accuracy. There are different methods of observing traverse such as using interior angles, angles to the right, deflection angles, and azimuths. The length of the traverse line can be determined by taping or any distance measuring technique. The length of traverse is very often called course. Traversing stations can be selected based on accuracy, utility, and efficiency of the survey; however, the reference point should be picked up in a way that it can located even after years so that the whole works can be reproduced if required after years.

The summation of the interior angles of a closed area should be $(n\text{-}2)^*180°$, where n is the number of sides. If the sum of the observed angles is less or more that the calculated one, then it is considered that angular misclosure occurred. Angles of a closed traverse can be adjusted elementary to the correct geometric total by applying one of two methods:

1. Applying an average correction to each angle
2. Making larger corrections to angles where poor observing conditions may be present

After balancing the angles and calculating preliminary azimuths (or bearings), traverse closure is checked by computing the departure and latitude of each line. Therefore, understanding of calculating of preliminary azimuths is important. Departure and latitude are defined as follows:



Departure = $L \sin \alpha$
Latitude = $L \cos \alpha$

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where L is the horizontal length and the azimuth of the course. The linear misclosure is defined as

$$= \sqrt{(\text{Error in Latitudes})^2 + (\text{Error in Departures})^2}$$

Latitude is also called northing or southing. Departures are sometimes called eastings or westings.

Problem 17.14

Which of the following statements is false?

A. The closure of a traverse is checked by computing the latitudes and departures of each sides
B. The latitude of a line is its projection on the north–south meridian
C. The departure of a line is its projection on the east–west line
D. None of the above

Solution: D

Problem 17.15

Which of the following statements is false for enclosure?

A. $\text{Error of Closure} = \sqrt{(\text{Error in Latitudes})^2 + (\text{Error in Departures})^2}$
B. Σ Latitudes = 0
C. Σ Departures = 0
D. None of the above

Solution: D

All the three statements are valid for traversing.

Problem 17.16

While surveying, bad sighting conditions can be caused by -

A. Alternative sun and shadow
B. Sighting into sun
C. Line of sight too close to ground
D. All of the above

Solution: D

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Bad sighting conditions is caused by (a) alternate sun and shadow, (b) visibility of only the rod's top, (c) line of sight passing too close to the ground, (d) lines that are too short, and (e) sighting into the sun.

Problem 17.17

Which of the following is a mistake in traversing?

A. Mistake in note taking
B. Σ Latitudes = 0
C. Σ departures = 0
D. All of the above

Solution: A

Some mistakes in traversing are: 1. Occupying or sighting on the wrong station. 2. Incorrect orientation. 3. Confusing angles to the right and left. 4. Mistakes in note taking. 5. Misidentification of the sighted station.

Problem 17.18

Departures are sometimes called -

A. Northing
B. Southing
C. Easting
D. None of the above

Solution: C

Departures are sometimes called eastings or westings.

Problem 17.19

Latitudes are sometimes called -

A. Northing B. Westing C. Easting D. None of the above

Solution: A

Latitude is also called northing or southing.

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Problem 17.20

Traverse consists of -

A. Closed loop only
B. Series of lines
C. Open lines only
D. None of the above

Solution: B

Transverse consists of series of lines open or closed.

Problem 17.21

Two categories of closed traverses exist: polygon and what?

A. Link
B. Triangle
C. Hexagon
D. Open traverse

Solution: A

Two categories of closed traverses exist: polygon and link.

Problem 17.22

Angles or directions of traverse lines can be observed by-

A. Interior angles
B. Azimuth
C. None of the above
D. Both (A) and (B)

Solution: D

Angles or directions of traverse lines can be observed by using interior angles, angles to the right, deflection angles, and azimuths.

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Problem 17.23

Which of the following statements is false?

A. The closure of a traverse is checked by computing latitudes and departures of each sides
B. The latitude of a line is its projection on the north–south meridian
C. The departure of a line is its projection on the east–west line
D. None of the above

Solution: D

Leveling (e.g., differential, elevations, percent grades)

The determination of elevations is called leveling. Measuring relative elevations changes is a comparatively simply process. It is done by a level which is a high-powered telescope (combined with a system to orient the line of sight in a horizontal plane).

Problem 17.24

Which of the following statements is false for leveling?

A. A sighting with a level back to a point of known elevation is called Backshot (Backsight)
B. A sighting with a level to determine the elevation of a point is called foreshot (Foresight)
C. Elevation of the telescope cross-hairs is called Elevation of Instrument
D. None of the above

Solution: D

Problem 17.25

The elevation difference between two points can be determined using which of the following surveying methods?

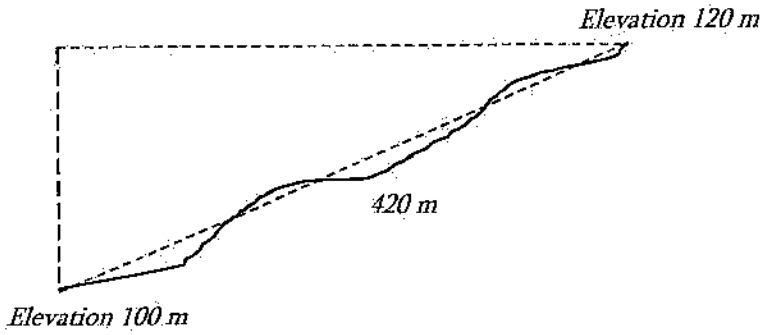
A. Closure
B. Leveling
C. Balancing
D. Pacing
E. I don't know it

Solution: B

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Problem 17.26

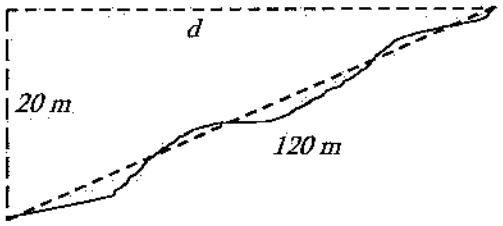
After surveying in a landscape, it was found that the slant distance from one point to another point is 420 m. The elevations of these two points are shown below. The horizontal distance (m) between these points is most nearly:



A. 122
B. 108
C. 112
D. 118

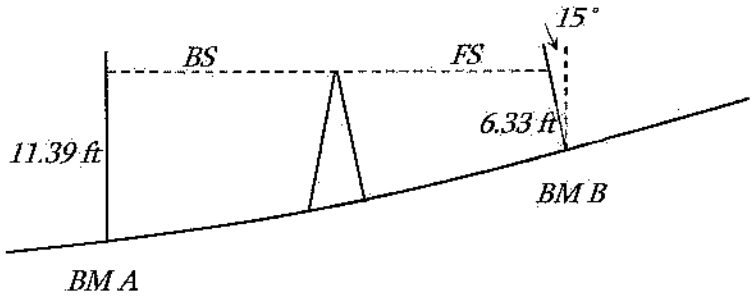
Solution: D

$$d = \sqrt{120^2 - 20^2} = 118 \text{ m}$$



Problem 17.27

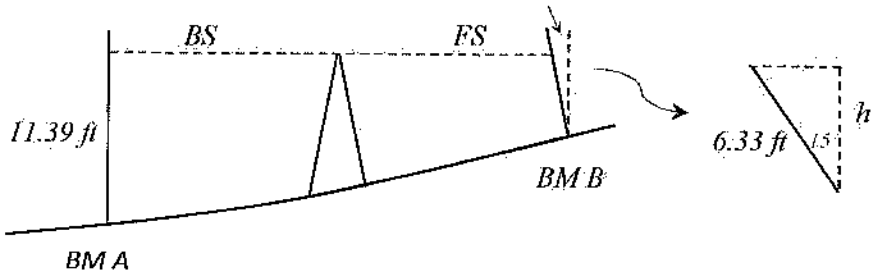
The elevation of BM A is 1,422.00 ft. A level in perfect adjustment is set midway between BM A and BM B. The backsight reading is 11.39 ft, and the foresight reading is 6.33 ft. If the level rod at BM B is held at an angle of 15° to the vertical, then the correct elevation (ft) of BM B is:



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A. 1,427.28 B. 1,427.06 C. 1,416.72 D. 1,439.5

Solution: A



$h = 6.33 \cos 15 = 6.11$ ft
Elevation at $B = 1{,}422 + 11.39 - 6.11 = 1{,}427.28$ ft

Miscellaneous Problems

Problem 17.28

The most effective way to link the proposed infrastructure of a development to the existing adjacent infrastructure is determined by a:

A. Topographic survey
B. Control survey
C. Boundary survey
D. Construction survey

Solution: A

Topographical surveys are essential for showing natural terrain features as well as man-made features such as roads, sanitary, and storm drain system.

Problem 17.29

Your client requests a conference with you to discuss the relationship between boundary locations and fences. You should advise the client that fences have control as to boundary location when the:

A. fences exist along the property lines of adjoins created simultaneously
B. fences are less than 20 years old from the time of the original survey
C. adjacent owners, as arbitrated by the surveyor, decide that the location is controlling
D. fences were built with the original survey and no original or other monuments exist

Solution: D

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Possession representing the location of original survey may be used to prove original survey lines.

Problem 17.30

Your firm has professional liability (errors and omissions) insurance, but the court found your firm not liable for a surveying error. The firm is now free from:

A. any cost arising out of the litigation
B. any cost arising out of the litigation except the deductible
C. any cost arising out of the litigation except the deductible and the legal fees
D. any cost arising out of the litigation except the deductible needed for legal fees

Solution: D

This is a standard practice in the insurance industry.

Problem 17.31

You have been asked to testify as an expert witness in court regarding the officially drawn map of a survey performed by another surveyor. You might legally and ethically testify as to:

A. the accuracy of the survey
B. the competency of the surveyor
C. the accuracy of the map
D. your conclusions regarding the map

Solution: D

This is a standard practice for expert witness testimony.

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About the Author



M. R. Islam is an Assistant Professor at the Colorado State University Pueblo, and previously at the State University of New York, Farmingdale. He is also a registered Professional Engineer (PE), and an ABET Program Evaluator. Dr. Islam received his Ph.D. in Civil Engineering (with distinction in research, and GPA 4.0 in coursework) from the University of New Mexico. He has a Masters degree in Civil Engineering jointly from the University of Minho (Portugal) and Technical University of Catalonia (Barcelona, Spain). He also obtained a B.S. degree in Civil Engineering from Bangladesh. Before pursuing his Ph.D., Dr. Islam served as a tenured full-time instructor of Civil Engineering in Stamford University Bangladesh for two years. Dr. Islam has about 100 publications including text books, scholarly articles in top ranked journals, ASCE book chapters, and conference papers.

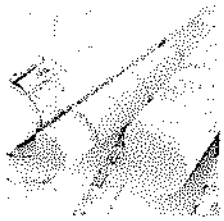
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18 CONSTRUCTION

Fundamentals of Engineering (FE) CIVIL Exam Specification mentions, 4–6 questions may appear in the exam based on Construction as follows:

A. Construction documents
B. Procurement methods (e.g., competitive bid, qualifications-based)
C. Project delivery methods (e.g., design-bid-build, design build, construction management, multiple prime)
D. Construction operations and methods (e.g., lifting, rigging, dewatering and pumping, equipment production, productivity analysis and improvement, temporary erosion control)
E. Project scheduling (e.g., CPM, allocation of resources)
F. Project management (e.g., owner/contractor/client relations)
G. Construction safety
H. Construction estimating

Construction Documents



A construction contract consists of the following documents:

- Agreement
- Conditions of the Contract
- Plans
- Specifications

The agreement describes the work to be performed, completion time, contract sum, provisions for progress payments and final payment, and lists the other documents making up the complete contract. The general conditions contain those contract provisions written by the owner. The special conditions contain any additional contract provisions applicable to the specific project.

Construction plans are drawings that show the location, dimensions, and details of the work to be performed. Taken together with the specifications, they provide a complete description of the facility to be constructed. Types of contract drawings include site drawings and detailed working drawings.

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Specifications provide the detailed requirements for the materials, equipment, and workmanship to be incorporated into the project. Contract drawings and specifications must be used together.

Procurement Methods

Bid Preparation. The selection of a bid for a construction contract, is a mixture of art and science. The bid price actually submitted by a contractor is usually based on an analysis of the expected competition and the state of the construction market in addition to the contractor's estimate of the cost to execute that project.

Bidding Procedure. The principal steps in the bidding procedure for a fixed-price construction contract include solicitation, bid preparation, bid submission, bid opening, selection of the lowest qualified bid, and contract award. Solicitation may range from an invitation sent to selected contractors to public. U.S. governmental agencies are required to solicit bids by public advertising. Contractors indicating an interest in bidding should be supplied with at least one complete set of contract documents. A deposit may be required to ensure the return of project plans and specifications furnished to unsuccessful bidders.

Contract Award. After the bids are opened, they are evaluated by the owner to determine the lowest qualified bid. The qualification of a contractor is the determination that the contractor possesses both the technical and financial ability to perform the work required by the contract. The method of qualification used will depend on the owner. Another method of bidder qualification is called prequalification. Under this procedure only those contractors determined to be capable of performing are invited to submit bids for the project. A more common, although indirect, method of prequalification is to require bonding of the contractor. Bonds used in construction include bid bonds, performance bonds, and payment bonds. A bid bond guarantees that a contractor will provide the required performance and payment bonds if awarded the contract.

Subcontracts. Subcontracts are contracts between a prime contractor and secondary contractors or suppliers. Subcontracts are widely used in building construction for the electrical, plumbing, heating and ventilating systems. Subcontractors are responsible only to the prime contractor (not to the owner) in the performance of their subcontracts.

Project Delivery Methods

A project delivery method is a system used by an agency or owner for organizing and financing design, construction, operations, and maintenance for a structure or facility by entering into legal agreements with one or more entities or parties. Common project delivery methods include:

Design-Bid-Build. An owner develops contract documents with an architect or an engineer consisting of a set of blueprints and specification. Bids are solicited from contractors based on these documents; a contract is then awarded to the lowest responsive and responsible bidder.

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Design-Build. An owner develops a conceptual plan for a project, then solicits bids from joint ventures of architects and/or engineer and builders for the design and construction of the project.

Construction Management. With partially completed contract documents, an owner will hire a construction manager to act as an agent. As substantial portions of the documents are completed, the construction manager will solicit bids from suitable subcontractors. This allows construction to proceed more quickly and allows the owner to share some of the risk inherent in the project with the construction manager.

Multiple Primes. An owner contracts directly with separate trade contractors for specific and designated elements of the work, rather than with a single general or prime contractor.

Construction Operations and Methods

Excavating and Lifting. An excavator is a power-driven digging machine. Major types of excavators used includes hydraulic excavators and the members of the cable-operated crane-shovel family (shovels, draglines, hoes, and clamshells). Dozers, loaders, and scrapers can also serve as excavators.

Loading and Hauling. Loading and hauling equipment includes dozer, tractor, scrapers, trucks, wagons, conveyor belts, and trains. A tractor equipped with a front-mounted earthmoving blade is known as a dozer or bulldozer. A dozer moves earth by lowering the blade and cutting until a full blade load of material is obtained. It then pushes the material across the ground surface to the required location. A tractor equipped with a front-end bucket is called a loader, front-end loader, or bucket loader. Both wheel loaders and track loaders are available. Scrapers are capable of excavating, hauling, and dumping material over medium to long haul distances.

Compacting. Compaction is the process of increasing the soil density by mechanically forcing the soil particles closer together, i.e., expelling air from the void spaces in the soil. Equipment includes tamping foot rollers, grid or mesh rollers, vibratory compactors, smooth steel drum rollers, pneumatic rollers, segmented pad rollers, and tampers or rammers.

Finishing. Finishing is the process of bringing earthwork to the desired shape and elevation. It involves smoothing slopes, shaping ditches, and bringing the earthwork to the elevation required by the specification. Motor grader is widely used for grading and finishing.

Rock Excavation. The process of rock moving may be considered in four phases: loosening, loading, hauling, and compacting. The traditional method for excavating rock involves drilling blast holes in the rock, loading the holes with explosives, detonating the explosives, loading the fractured rock into haul units with power shovels, and hauling the rock away in trucks or wagons.

Problem 18.1

In construction contract -

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A. consulting engineer has obligation to the owner and the general contractor only
B. consulting engineer has obligation to the owner, general contractor, and subcontractors.
C. consulting engineer has obligation to the general contractor only
D. consulting engineer has obligation to the owner only

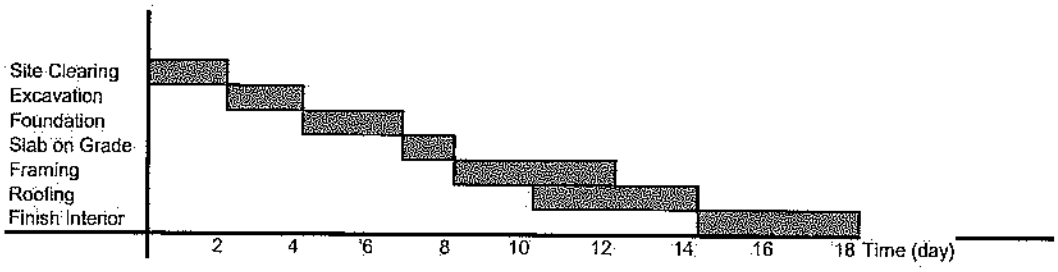
Solution: D

In customary practice, the owner hires engineer for the design and contract documents. The engineer works closely with general contractor and subcontractors but the engineer is only liable to the owner only.

Project Scheduling

Project scheduling is the determination of the timing and sequence of operations in the project and their assembly to give the overall completion time. There are different methods of project scheduling such as bar chart, activity-on-arrow, activity-on-node, precedence network, etc.

Bar Chart. Defined as a graphic representation of project activities shown in a time-scaled bar lines with no links between activities. It is also known as Gantt charts. An example of bar chart is shown below:

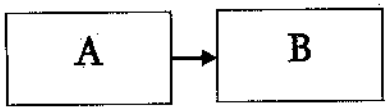


Activity-on-Arrow (AOA). In AOA network, the activity is represented by arrow between nodes; the nodes represent the events (start and finish) of the activity. Dummy activities are required in AOA networks for logic or identity. When one activity finishes, another activity starts, i.e. finish-start relationship. Due to limitations in AOA such as dummy activities, difficulty in lag adoption, etc. this AOA becomes obsolete.



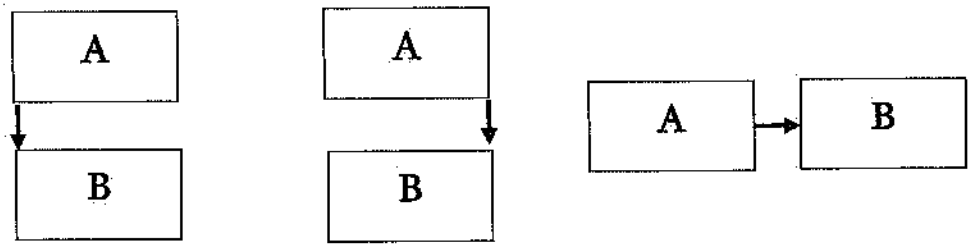
Activity-on-Node (AON). A node network is a network where nodes represent activities and arrows represent logic relationships (dependencies). When one activity finishes, another activity starts, i.e. finish-start relationship. It is better than AOA network but cannot accommodate start to start, finish to finish relationship, etc.

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Precedence Network. Precedence network uses node diagrams with four types of logical relationships:

1. Start-to-Start, S-S,
2. Finish-to-Finish, F-F,
3. Finish-to-Start, F-S,
4. Start-to-Finish, S-F. (not used in real life)



1. Start-to-Start: start of B depends on the start of A
2. Finish-to-Finish: finish of B depends on the finish of A
3. Finish-to-Start: start of B depends on the finish of A

Some common terms used in the scheduling are defined below:

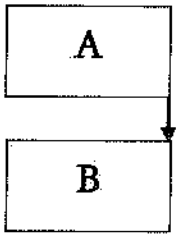
- Activity: A basic unit of work as part of the total project that is easily measured and controlled. It is also called Task. It is time and resource consuming.
- Event: a point that is usually the start or finish of a certain activity(s) with zero duration.
- Critical Path: The longest continuous path in a network from start to finish. It represents the summation of the durations of activities and lags along that path, taking in consideration calendars, constraints, resources, and other impacting factors.
- Predecessor Activity: An activity that must finish (or start) in order for the succeeding activity(s) to start (or finish). It has been also defined as an activity that has some measurable portion of its duration logically restraining a subsequent activity or activities.
- Successor Activity: An activity that cannot start (or finish) until another activity (predecessor) has finished (or started).
- Early Start, ES: The earliest date the activity can start within project constraints.
- Early Finish, EF: The earliest date the activity can finish within project constraints.
- Late Start, LS: The latest date of start without delaying the completion of the project.
- Late Finish, LF: The latest date of finish without delaying the completion of the project.

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- Total Float, TF: The maximum amount of time an activity can be delayed from its early start (ES) without delaying the entire project. TF = LS – ES or TF = LF – EF.
- Lag: A lag is defined as a minimum waiting period between the finish (or start) of an activity and the start (or finish) of its successor.

Problem 18.2

Which of the following statements is true for the following activity-on-node?



A. Start-to-Start: start of B depends on the start of A
B. Finish-to-Start: start of B depends on the finish of A
C. Finish-to-Finish: finish of B depends on the finish of A
D. None of the above

Solution: C

From the FE Ref. Handbook, Finish-to-Finish: finish of B depends on the finish of A.

Problem 18.3

Dummy activities in arrow diagrams are:

A. real but unimportant activities
B. real activities with no budget assigned
C. fictitious activities used for accurate drawing of the network
D. fictitious activities used to extend the duration of real activities

Solution: C

Dummy activities in arrow diagrams are fictitious activities used for accurate drawing of the network. It has no duration and does not consume any resource. Sometimes, it cannot be avoided in AOA network. In AON and Precedence Network, we do not need any dummy activity; hence became more popular than AOA network.

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Problem 18.4

A lag is waiting periods between two activities. They:

A. can be incorporated in arrow networks but not in node networks
B. can be incorporated in node networks but not in arrow networks
C. can be incorporated in both arrow and node networks
D. cannot be incorporated in arrow and node networks

Solution: B

A lag is waiting periods between two activities. They can be incorporated in node networks but not in arrow networks. Example of lag – few days of waiting after concrete pouring for curing operation.

Problem 18.5

A logical relationship shows:

A. activities that are to be performed by the same subcontractor or crew.
B. activities that make sense.
C. dependencies between (among) activities.
D. the duration of the activity

Solution: C

Problem 18.6

Soft logic means the:

A. logical relationship that is not so important
B. logical relationship in the software
C. resource constraint
D. logical relationship coming from external activity

Solution: B

Problem 18.7

The forward pass is a:

A. Subnet of the CPM network that includes all activities to be done first.

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B. Process of going on a CPM network from start to finish calculating early dates for each activity, and estimated completion date for the entire project.
C. Assigning logic relationships for succeeding activities.
D. Both (A) and (C)

Solution: B

Problem 18.8

After performing forward and backward passes, you found that for a certain activity- the Early Start date to be later than the Late Start date. Your explanation is most nearly:

A. This is impossible. You must have done a mathematical error.
B. This activity is not critical it has some leeway (float).
C. This activity was placed in the wrong position in the network.
D. You have used an imposed finish date for the project or the activity that is earlier than the calculated finish date.

Solution: D

Problem 18.9

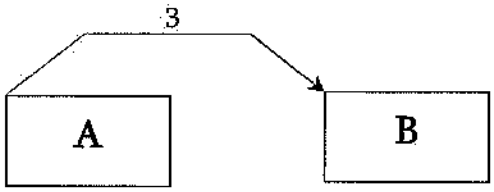
Free float represents:

A. The portion of total float where it is not influenced by preceding activities.
B. The portion of total float where it does not influence following activities.
C. The waiting period which does not affect the total duration of the project.
D. None of the above.

Solution: B

Problem 18.10

The following relationship means:



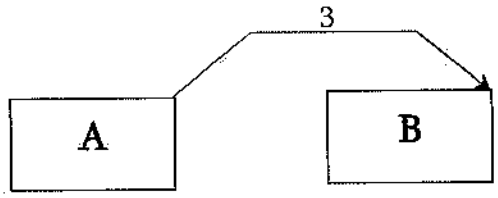
A. Activity B has to start exactly 3 days after the start of A.
B. Activity B can start at least 3 days after the start of A.
C. Activity B can start at most 3 days after the start of A.
D. None of the above.

Solution: B

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Problem 18.11

The following relationship means:



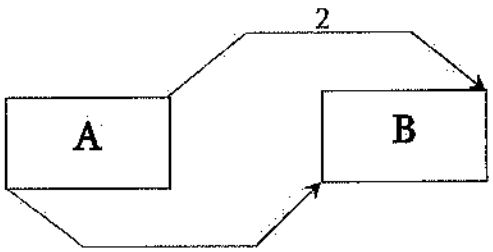
A. Activity B has to finish exactly 3 days after the finish of A.
B. Activity B can finish at least 3 days after the finish of A.
C. Activity B can finish at most 3 days after the finish of A.
D. None of the above.

Solution: B

The lag is 3 days, meaning the next action can start 3 days or later (simply at least 3 days).

Problem 18.12

The following relationship means:



Activities A and B must start simultaneously, but B has to finish 2 days after the finish of A.

A. Activity B can start after the start of A and but must finish at least 2 days before the finish of A.
B. Activity B can start before the start of A but must finish at most 3 days after the finish of A.
C. Activity B can start after the start of A and but must finish at least 2 days after the finish of A.

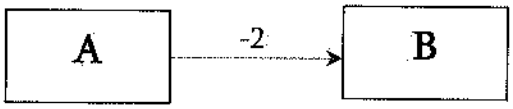
Solution: D

Both restrictions must be satisfied. Option D only does it.

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Problem 18.13

The following relationship means:



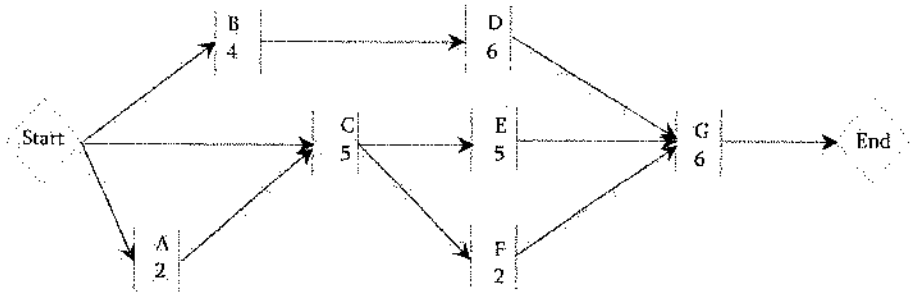
A. Activity A can start 2 days before the start of activity B.
B. Activity B can start 2 days after the completion of activity A.
C. Activity B can start 2 days before the completion of activity A.
D. Activity B has to start at least 2 days before the completion of activity A.

Solution: C

The lag is -2 days, meaning the next action can start -2 days or later; option D is not the answer.

Problem 18.14

An activity-on-node diagram for a construction project is given below. The minimum number of days to finish the project is most nearly:



A. 12 B. 15 C. 17 D. 13

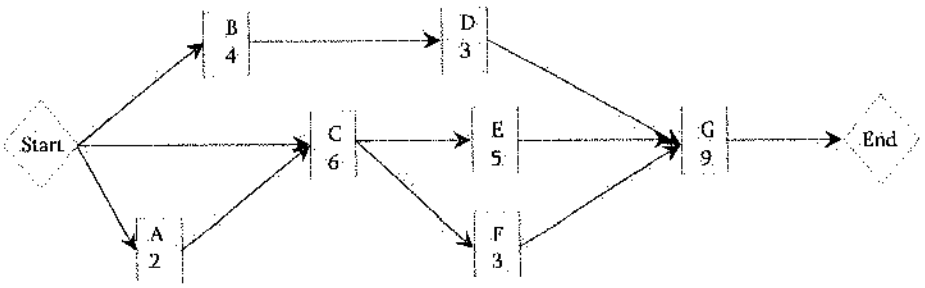
Solution: D

B-D-G path: 4 + 6 + 6 = 16 days
C-E-G path: 5 + 5 + 6 = 16 days
A-C-F-G path: 2 + 5 + 2 + 6 = 15 days
C-F-G path: 5 + 2 + 6 = 13 days **(Ans.)**

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Problem 18.15

An activity-on-node diagram for a construction project is given below. The minimum number of days to finish the project is most nearly:



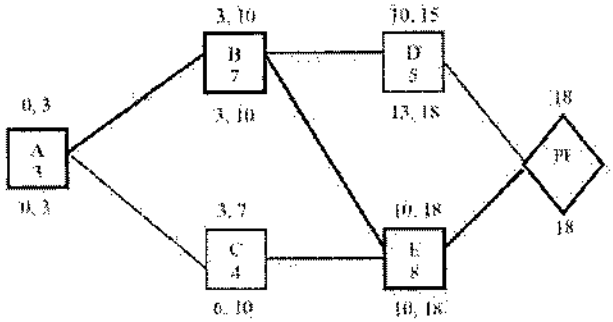
A. 19 B. 21 C. 18 D. 16

Solution: D

B-D-G path: 4 + 3 + 9 = 16 days
C-E-G path: 6 + 5 + 9 = 20 days
A-C-F-G path: 2 + 6 + 3 + 9 = 20 days
C-F-G path: 6 + 3 + 9 = 18 days

Problem 18.16

A node diagram is shown below with forward pass and backward pass calculations. Which of the following statements is true for this diagram?

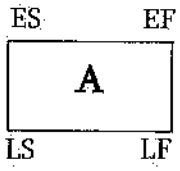


A. Total Float of C is 3 and Free Float of D is 0
B. Total Float of D is 3 and Free Float of C is 0
C. Total Float of C is 3 and Free Float of D is 3
D. Total Float of D is 0 and Free Float of C is 3

Solution: C

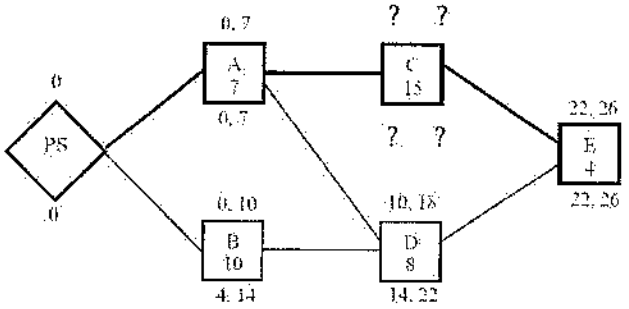
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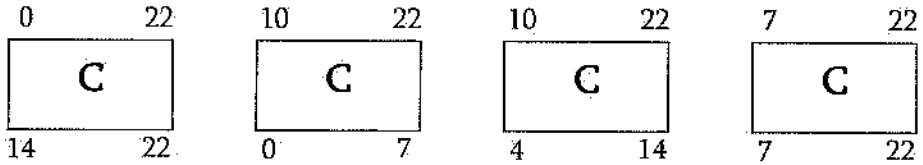
Total Float of C = LS – ES = 6 – 3 = 3
Total Float of D = LS – ES = 13 – 10 = 3
Free Float of C = ES of next activity – EF of current activity = 10 – 7= 3
Free Float of D = ES of next activity – EF of current activity = 18 – 15= 3



Problem 18.17

A node diagram is shown below with forward pass and backward pass calculations. Which of the following statements is true for the Activity C?

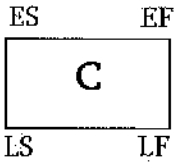




A. B. C. D.

Solution: D

Activity C is following Activity-A and Activity-E is following Activity-C.
Forward Pass:
ES of Activity C = 7 as it starts after Activity-A ends.
EF of Activity C = 7+15 = 22 as it has duration of 15 days.
Backward Pass:
LF of Activity C = 22 as E starts on 22
LS of Activity C = 22 – 15 = 7 (duration of 15 days)



Determining the Project Size Modifier

To determine the project size and initial cost estimating, the NCEES FE Ref. Handbook, Construction Section, listed different types of building, its median cost, and typical sizes. Review the Table and practice the following problem.

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Problem 18.18

The median cost per square foot of construction of Jails is most nearly:

A. $ 165.0 B. $ 81.60 C. $115.90 D. Unknown; depends on location

Solution: A

From the FE Ref. Handbook, Construction Section, $ 165.

Problem 18.19

The typical size of post office is most nearly:

A. 12,400 sft B. 7,500 sft C. 9,000 sft D. Unknown; depends on location

Solution: A

From the FE Ref. Handbook, Construction Section, 12,400 sft.

Earned-Value Analysis

Earned Value analysis is a standard way to measure a project's progress, forecast its completion date and final cost, and provide schedule and budget variances along the way. It compares the planned amount of work with what has actually been completed, to determine if cost, schedule, and work accomplished are progressing as planned. The following short forms are used in earned value analysis:

BCWS = Budgeted cost of work scheduled (Planned)
ACWP = Actual cost of work performed (Actual)
BCWP = Budgeted cost of work performed (Earned)
BAC = Budget at completion

Cost Variances is defined as:

CV = BCWP - ACWP (Cost variance = Earned - Actual)

It is a comparison of the budgeted cost of work performed with actual cost. A negative variance means the project is over budget.

Schedule variance (SV) is defined as –

SV = BCWP - BCWS (Schedule variance = Earned - Planned)

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It is a comparison of amount of work performed during a given period of time to what was scheduled to be performed. A negative variance means the project is behind schedule

Cost Performance Index (CPI) and Schedule Performance Index are defined as:

$$CPI = \frac{BCWP}{ACWP}, \text{ CPI} < 1 \text{ means project is over budget}$$

$$SPI = \frac{BCWP}{BCWS}, \text{ SPI} < 1 \text{ means project is behind schedule}$$

There are tools available that can be used to predict the Estimate at Completion (EAC) based upon the past history of contractor performance on the contract (e.g., cost and schedule efficiencies). The earned value EAC formula is based on the simple concept that the estimate at completion is equal to the amount of money already spent on the contract plus the amount of money it will take to complete the contract. Translating this into the language of earned value; money already spent is ACWP, and money need to complete the contract is based on budget cost of work remaining, or BCWR, divided by a performance factor.

The Estimate at Completion (EAC) is determined as, EAC = ACWP + ETC

The phrase budget at completion (often referred to as BAC) refers to the sum of all budget values that have been previously established for the work to be performed on a project, or on components within a project such as a schedule activity or work breakdown structure component. The budget at completion also can be referred to as the total planned value of the project.

Estimate to complete (ETC) is a forecast of how much more money will need to be spent to complete the project.

The Estimate to Complete (ETC) is determined as, $ETC = \frac{BAC - BCWP}{CPI}$

Problem 18.20

Suppose you had a budgeted cost of work performed (BCWP) of a project is $540,000. After construction it was found that Actual cost of work performed (ACWP) is 590,000. The cost performance index (CPI) is most nearly:

A. 1.125
B. 0.92
C. $100,000
D. None of the above

Solution: B

$$CPI = \frac{BCWP}{ACWP} = \frac{540,000}{590,000} = 0.92 \text{ (CPI less than 1.0 means over budget)}$$

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Problem 18.21

The budgeted cost of work performed (BCWP) of a part of a project was $100,000. The actual cost of work performed (ACWP) was $95,000. The Cost variance (CV) of this part of the project is most nearly:

A. 1.1 B. $5,000 C. $10,000 D. -$10,000

Solution: B

$$CV = BCWP - ACWP = 100{,}000 - 95{,}000 = \$5{,}000 \text{ (CV positive means under budget)}$$

Problem 18.22

Suppose you had a budgeted cost of work performed (BCWP) cost of a project at $900,000. Before the construction, the budgeted cost of work scheduled (BCWS) 750,000. The schedule performance index (SPI) is most nearly:

A. 1.125 B. 0.89 C. $100,000 D. 1.2

Solution: D

$$SPI = \frac{BCWP}{BCWS} = \frac{900{,}000}{750{,}000} = 1.2 \text{ (SPI more than 1.0 means ahead of schedule)}$$

Problem 18.23

_____not typically part of construction.

A. Engineers
B. Contractors
C. Architects
D. City officials

Solution: D

Construction Safety

The NCEES held FE Civil Exam does not exactly specify the topics to be covered in construction safety. However, the author recommends students to be aware of Occupational Safety and Health Administration (OSHA) health regulations for construction by online reading. Some important points are discussed in this section. Construction work is obviously a hazardous land-based job working with height, excavation, noise, dust, power tools, nails, falling body, and equipment. The

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most common fatalities are caused falls, being struck by an object, electrocutions, and being caught in between two objects. OSHA sets and enforces standards concerning workplace safety and health. By law, employers must provide their workers with a workplace that does not have serious hazards and must follow all OSHA safety and health standards. OSHA further requires that employers must first try to eliminate or reduce hazards by making feasible changes in working conditions rather than relying on personal protective equipment such as masks, gloves, or earplugs. Switching to safer chemicals, enclosing processes to trap harmful fumes, or using ventilation systems to clean the air are examples of effective ways to eliminate or reduce risks.

Employers must also:

- Inform workers about chemical hazards through training, labels, alarms, color-coded systems, chemical information sheets and other methods.
- Provide safety training to workers in a language and vocabulary they can understand
- Keep accurate records of work-related injuries and illnesses.
- Perform tests in the workplace, such as air sampling, required by some OSHA standards.
- Provide required personal protective equipment at no cost to workers.
- Provide hearing exams or other medical tests when required by OSHA standards.
- Post OSHA citations and injury and illness summary data where workers can see them
- Notify OSHA within eight hours of a workplace fatality.
- Not retaliate or discriminate against workers for using their rights under the law, including their right to report a work-related injury or illness.

Workers have the right to:

- Working conditions that do not pose a risk of serious harm.
- File a confidential complaint with OSHA to have their workplace inspected.
- Receive information and training about hazards, methods to prevent harm, and the OSHA standards that apply to their workplace. The training must be done in a language and vocabulary workers can understand.
- Receive copies of records of work-related injuries and illnesses occurred in the workplace.
- Receive copies of the results from tests done to find hazards in their workplace.
- Receive copies of their workplace medical records.
- Participate in an OSHA inspection and speak in private with the inspector.
- File a complaint with OSHA if they have been retaliated or discriminated against by their employer as the result of requesting an inspection or using any of their other rights.
- File a complaint if punished or retaliated against for acting as a whistleblower under the 21 additional federal laws for which OSHA has jurisdiction.

Problem 18.24

Whose responsibility is to provide a workplace that does not have serious hazards?

A. Engineers
B. City officials

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C. Employers
D. Employees

Solution: C

Employers must provide their workers with a workplace that does not have serious hazards and must follow all OSHA safety and health standards.

Problem 18.25

Which of the following is not the responsibility of employers?

A. Eliminate or reduce hazards for the working place
B. Keep accurate records of work-related injuries and illnesses
C. Notify OSHA of a workplace fatality
D. None of the above

Solution: D

Miscellaneous Problems

Problem 10.26

The budgeted labor amount for an excavation task is $6,000. The hourly labor cost is $50 per worker, and the workday is 8 hours. Two workers are assigned to excavate the material. The time (days) available for the workers to complete the task is most nearly:

A. 15 B. 8 C. 10 D. 6

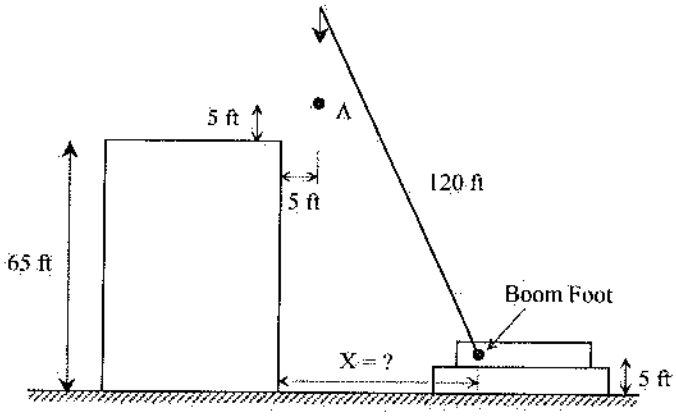
Solution: B

Total budgeted time = $6,000 / $50 = 120 hours = 120 / 8 = 15 days
There are two labors; therefore, it would take 15/2 = 7.5 days ≈ 8.0 days

Problem 10.27

A crane with a 120-ft boom is being used to install a small load on the roof of the 65-ft high building as shown below. The load is to be dropped at point, A, from where the crew on the top of building will take care. Point, A is 5-ft away and 5-ft upper than the top of building. What should be horizontal distance, X (ft) from the edge of the building to the boom foot? The boom foot is located 5-ft from the surface. The crane boom can make a minimum of 45° with the ground surface, and the load cannot be lifted up with cable.

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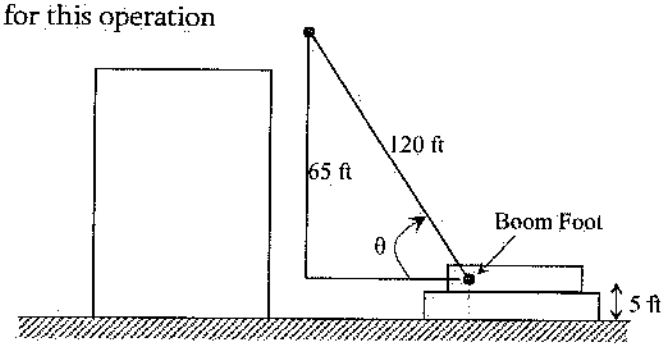
A. 45
B. 55
C. 35
D. The 120-ft crane is not suitable for this operation

Solution: D

Imagine a right-angle triangle
with corner points of A, and Boom foot.

$$\theta = \sin^{-1}\left(\frac{65}{120}\right) = 33^{o} \text{ less than } 45^{\circ}$$

The crane is unable to make this operation.



Problem 10.28

A bridge is to be jacked up to replace its bearings. The design requires a hydraulic ram with a minimum capacity of 2,000 kN. The hydraulic rams that are available are rated in tons. Note that 1 ton = 2,000 lb. The minimum size (tons) ram to use is most nearly:

A. 225 B. 450 C. 112.5 D. 450

Solution: A

2,000 kN = 2,000,000 N = 2,000,000 x 0.225 = 450,000 lb = 450,000/2,000 = 225 ton
[From the Unit Section of the NCEES Ref. Handbook, 1 N = 0.225 lb]

References:
Nunnaly, S.W. (2010). Construction Methods & Management, 8th Edition, Prentice Hall, Upper Saddle River, NJ.
Mubarak, S. (2015). Construction Project Scheduling and Control, 3rd Edition, Wiley.

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19 ENVIRONMENTAL ENGINEERING

Fundamentals of Engineering (FE) CIVIL Exam Specification mentions, 6-9 questions may appear in the exam based on Environmental Engineering as follows:

A. Water quality (ground and surface)
B. Basic tests (e.g., water, wastewater, air)
C. Environmental regulations
D. Water supply and treatment
E. Wastewater collection and treatment

NCEES FE Ref. Handbook, Environmental Engineering, listed the formulations related to Environmental Engineering. However, the listed topics are mainly intended for Environmental Engineering students, not purely for Civil Engineering students. Therefore, students are advised to study and practice this part as much as they studied in their degree program. In addition, do not try to master the whole topic. The author here discusses few topics which are more important for the Civil FE exam.

Selected Properties of Air

The amounts of major components of air, viscosity, and density of air are listed below:

Nitrogen (N_2) by volume	78.09%
Oxygen (O_2) by volume	20.94%
Argon (Ar) by volume	0.93%
Molecular weight of air	28.966 g/mol
Absolute viscosity, at 80 °F	0.045 lbm/(hr-ft)
Absolute viscosity, at 100 °F	0.047 lbm/(hr-ft)
Density, at 80 °F	0.0734 lbm/ft^3
Density, at 100 °F	0.0708 lbm/ft^3

Atmospheric Stability Under Various Conditions are listed in the NCEES FE Ref. Handbook, Environmental Engineering. Please have a glance at it.

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Problem 19.1

The density of air at 100 °F is most nearly:

A. 0.045 pcf B. 0.047 pcf C. 0.0708 pcf D. 0.0734 pcf

Solution: C

From the NCEES FE Ref. Handbook (Environmental Engineering), the density of air at 100 °F is 0.0708 pcf.

Problem 19.2

The amount of Oxygen (O_2) by the volume in air is most nearly:

A. 21 % B. 78 % C. 38 % D. 68 %

Solution: A

From the NCEES Ref. Handbook, the amount of Oxygen (O_2) by volume in air is 20.94%.

Problem 19.3

The atmospheric stability under surface wind speed of 5.5 m/s, and strong day solar insolation is most nearly:

A. Very Unstable
B. Neutral
C. Moderately Unstable
D. Slightly Unstable

Solution: D

From the NCEES FE Ref. Handbook (Environmental Engineering), the atmospheric stability under surface wind speed of 5.5 m/s, strong day solar insolation is Slightly Unstable.

Incineration Plant

Problem 19.4

In an Incineration plant, the principal organic hazardous constituent (POHC) input and output are 1,500 kg/h and 1,200 kg/h, respectively. The destruction and removal efficiency (DRE) of that plant is most nearly:

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A. 10% B. 20% C. 50% D. 80%

Solution: B

From the NCEES FE Ref. Handbook (Environmental Engineering),

$$DRE = \frac{W_{in} - W_{out}}{W_{in}}(100\%) = \frac{1,500 - 1,200}{1,500}(100\%) = 20\%$$

COD and BOD

COD is the amount of oxygen required to degrade the organic and inorganic matters; BOD is the amount of oxygen required to degrade the organic matter. Therefore, COD is higher than the BOD.

Problem 19.5

Which of the following statements is true regarding the Chemical Oxygen Demand (COD) and the Biological Oxygen Demand (BOD)?

A. COD is the amount of oxygen required to degrade the organic matter
B. BOD is the amount of oxygen required to degrade the inorganic matter
C. COD is the amount of oxygen required to degrade the organic and inorganic matters
D. BOD is higher than the COD

Solution: C

Fate and Transport

Mass Calculations

Mass balance: $\text{Mass}_{in} = \text{Mass}_{out}$
$M = CQ = CV$
Continuity equation = $Q = vA$

M = mass
C = concentration
Q = flow rate
V = volume
v = velocity
A = cross-sectional area of flow
M (lb/day) = C (mg/L) × Q (MGD) × 8.34 [lb-L/(mg-MG)]
MGD = million gallons per day
MG = million gallons

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Microbial Kinetics

BOD Exertion

$$y_t = L(1-10^{-kt})$$

k = BOD decay rate constant (base e, days^{-1})
L = ultimate BOD (mg/L)
t = time (days)
y_t = the amount of BOD exerted at time t (mg/L)
Kinetic Temperature Corrections, $k_T = k_{20}\theta^{T-20}$

Stream Modeling

$$D = \frac{k_1 L_o}{k_2 - k_1}\left[e^{-k_1 t} - e^{-k_2 t}\right] + D_o e^{-k_2 t}$$

$$t_c = \frac{1}{k_2 - k_1}\ln\left[\frac{k_2}{k_1}\left(1 - D_o\frac{(k_2 - k_1)}{k_1 L_o}\right)\right]$$

$$DO = DO_{sat} - D$$

D = dissolved oxygen deficit (mg/L)
DO = dissolved oxygen concentration (mg/L)
D_0 = initial dissolved oxygen deficit in mixing zone (mg/L)
DO_{sat} = saturated dissolved oxygen concentration (mg/L)
k_1 = deoxygenation rate constant, base e (days^{-1})
k_2 = reaeration rate constant, base e (days^{-1})
L_o = initial BOD ultimate in mixing zone (mg/L)
t = time (days)
t_c = time at which minimum dissolved oxygen occurs (days)

Problem 19.6

A wastewater treatment plant can treat 2 MGD of wastewater. At any time, the plant requires one unit to be taken out for maintenance. The wastewater has a total TSS of 300 mg/L. The maximum solids load on each clarifier is 600 lb-TSS/day. The number of clarifier units needed is most nearly:

A. 7 B. 9 C. 14 D. 18

Solution: B

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From the NCEES FE Ref. Handbook (Environmental Engineering),
M (lb/day) = C (mg/L)x Q (MGD)x 8.34 [lb-L/(mg-MG)] = 300 x 2 x 8.34 = 5,004 lb/day
Number of units required= 5,004/600 = 8.34 ≈ 9

Problem 19.7

The BOD of a waste sample incubated at 30 °C for 10 days was measured to be 120 mg/L. The deoxygenation rate constant (base-10, 20 °C) is 0.23 per day. The kinetic temperature correction factor is 1.047. The BOD of the sample after 15 days of incubation at 20 °C is most nearly:

A. 111.07 mg/L
B. 118.07 mg/L
C. 120.0 mg/L
D. 122.07 mg/L

Solution: C

From the FE Ref. Handbook, (Environmental Engineering),

$$k_{30^\circ C} = k_{20^\circ C}\,1.047^{30-20} = 0.23(1.047)^{10} = 0.36 \text{ per day}$$

$$y_t = L(1-10^{-kt})$$

$$120 = L(1-10^{-0.36(10)})$$

L = 120.03 mg/L

Therefore, $y_t = L(1-10^{-kt}) = 120.03(1-10^{-0.23(15)}) = 119.98$ mg/L

Problem 19.8

A dying factory discharges waste water into a river. The river-wastewater mixture has the following:

Temperature = 20 °C
Flow rate = 15 ft^3/sec
Average velocity = 5 fps
DO = 5 mg/L
Ultimate BOD = 20 mg/L
Saturation BOD at 20 °C = 12 mg/L
Deoxygenation rate constant = 0.3 per day
Reoxygenation rate constant = 0.4 per day

The DO of the stream (mg/L) at 10 miles downstream of the mixing point is most nearly:

A. 2.5 B. 3.5 C. 4.5 D. 7.36

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Solution: D

Initial oxygen deficit, $D_o = 12 - 5 = 7$ mg/L
10-mile travel time, $t = S/v = 10(5{,}280)$ ft / 5 fps = 10,560 s = 0.122 day
L_o = initial dissolved oxygen deficit in mixing zone = 20 mg/L
$k_1 = 0.3$; $k_2 = 0.4$

$$D = \frac{k_1 L_o}{k_2 - k_1}\left[e^{-k_1 t} - e^{-k_2 t}\right] + D_o e^{-k_2 t} = \frac{0.3(20)}{0.4 - 0.3}\left[e^{-0.3(0.122)} - e^{-0.4(0.122)}\right] + 7e^{-0.4(0.122)} = 7.36\frac{\text{mg}}{\text{L}}$$

Landfill

Break-Through Time for Leachate to Penetrate a Clay Liner, $t = \dfrac{d^2\eta}{K(d+h)}$

t = breakthrough time (yr)
d = thickness of clay liner (ft)
η = porosity
K = hydraulic conductivity (ft/yr)
h = hydraulic head (ft)
Typical porosity values for clays with a coefficient of permeability in the range of 10^{-6} to 10^{-8} cm/s vary from 0.1 to 0.3.

Soil Landfill Cover Water Balance

$$\Delta S_{LC} = P - R - \text{ET} - \text{PERsw}$$

ΔS_{LC} = change in water held in storage in a unit volume of landfill cover (in.)
P = amount of precipitation per unit area (in.)
R = amount of runoff per unit area (in.)
ET = amount of water lost through evapotranspiration per unit area (in.)
PERsw = amount of water percolating through the unit area of landfill cover into compacted solid waste (in.)

Problem 19.9

A 50 cm thick clay with permeability of 8×10^{-8} m/s is used as a liner material for a landfill project. The porosity of the clay is 0.4. if the hydraulic head is 3 m, the time in days it may take for the leachate material to penetrate the liner is most nearly:

A. 4.13 B. 10.1 C. 30.7 D. 60

Solution: A

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From the NCEES FE Ref. Handbook (Environmental Engineering),

$$\text{Time, } t = \frac{d^2\eta}{K(d+h)} = \frac{(0.5\text{ m})^2(0.4)}{8\text{x}10^{-8}\frac{\text{m}}{\text{s}}(0.5\text{ m} + 3\text{ m})} = 357{,}142\,s = 4.13\,\text{days}$$

Problem 19.10

In a 200 m^2 landfill, 80 inch of water was stored initially. In the meantime, there was a rainfall of 5.7 inch, and 12 inch of water was lost through evapotranspiration. If the percolated amount of water is 7 inch, the volume (m^3) of available water in the landfill is most nearly:

A. 60 B. 200 C. 240 D. 340

Solution: D

From the NCEES FE Ref. Handbook (Environmental Engineering),

$P = 5.7$ in ; $R = 0$ in ; ET = 12 in ;
$PERsw = 7$ in
$\Delta S_{LC} = P - R - ET - PERsw$
$\Delta S_{LC} = 5.7 - 0 - 12 - 7$
$\Delta S_{LC} = -13.3$ in

Final water height = 80 – 13.3 = 66.7 in

Final Volume = 200 m^2 (66.7 in)
= 200 m^2 (66.7/39.36 m)
= 338.9 $m^3 \approx 340$ m^3

Population Modeling

Linear Projection = Algebraic Projection: $P_t = P_0(1 + k\Delta t)$

P_t = population at time t
P_0 = population at time zero
K = growth rate
Δt = elapsed time in years relative to time zero

Log Growth = Exponential Growth = Geometric Growth: $P_t = P_0 e^{k\Delta t}$

$$\ln P_t = \ln P_0 + k\Delta t$$

P_t = population at time t
P_0 = population at time zero
k = growth rate
Δt = elapsed time in years relative to time zero

Compound Growth: $P_t = P_0(1 + k)^n$

n = number of periods

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Ratio and Correlation Growth: $\frac{P_2}{P_{2R}} = \frac{P_1}{P_{1R}} = k$

P_2 = projected population
P_{2R} = projected population of a larger region
P_1 = population at last census
P_{1R} = population of larger region at last census
k = growth ratio constant

Decreasing-Rate-of-Increase Growth: $P_t = P_o + (S - P_o)\left(1 - e^{-k(t-t_o)}\right)$

P_t = population at time t
P_0 = population at time zero
k = growth rate constant
S = saturation population
t, t_0 = future time, initial time

Problem 19.11

In 2015, the population in a city in Colorado was 80,000. The growth rate of that city is 3%. For this condition, the volume of population in 2040 is most nearly:

A. 140,000 using linear growth
B. 170,000 using log growth
C. 167,000 using compound growth
D. All of the above

Solution: D

2040 – 2015 = 25 years
Using Linear Growth, $P_t = P_0(1 + k\Delta t) = 80{,}000[1+0.03(25)] = 140{,}000$
Using log growth, $P_t = P_0 e^{k\Delta t} = 80{,}000\ e(0.03x25) = 169{,}360 \approx 170{,}000$
Compound growth, $P_t = P_0(1 + k)^n = 80{,}000(1 + 0.03)^{25} = 167{,}502 \approx 167{,}000$

Problem 19.12

In 2000, the counted population in a city in XY state was 80,000. The population in XY state in 2000 was 3 millions and the predicted population of XY state for 2020 is 3.2 millions. The projected population for the concern city in 2020 is most nearly:

A. 82,000 B. 83,000 C. 85,000 D. 88,000

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Solution: C

P_2 = projected population=?
P_{2R} = projected population of a larger region = 3.2 million
P_1 = population at last census = 80,000
P_{1R} = population of larger region at last census = 3 million

$$\frac{P_2}{P_{2R}} = \frac{P_1}{P_{1R}}$$

$$P_2 = \frac{80{,}000(3.2)}{3.0} = 85333$$

Radiation

Effective Half-Life. Effective half-life, τ_e is the combined radioactive and biological half-life.

$$\frac{1}{\tau_e} = \frac{1}{\tau_r} + \frac{1}{\tau_b}$$

where:

τ_r = radioactive half-life
τ_b = biological half-life

Half-Life. The half-life is calculated as, $N = N_o e^{\frac{-0.693t}{\tau}}$

N_0 = original number of atoms
N = final number of atoms
t = time
τ = half-life

Flux at distance 2 = (Flux at distance 1) $(r_1/r_2)^2$

where: r_1 and r_2 are distances from source

The half-life of a biologically degraded contaminant assuming a first-order rate constant is given by: $t_{1/2} = \frac{0.693}{k}$

where: k = rate constant (time^{-1}); $t_{1/2}$ = half-life (time)

Problem 19.13

The radioactive and the biological half-life of an element is 120 years and 170 years respectively. The effective half-life in years of that element is most nearly:

A. 145 B. 120 C. 156 D. 70

Solution: D

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$\tau_r = 120$ years
$\tau_b = 170$ years

$$\frac{1}{\tau_e} = \frac{1}{\tau_r} + \frac{1}{\tau_b} = \frac{1}{120} + \frac{1}{170} = 0.0142$$

$\Rightarrow \tau_e = 70$ years

Problem 19.14

The decay rate constant of an element is 0.0023 per year. The half-life of that element in years is most nearly:

A. 200 B. 300 C. 400 D. 500

Solution: B

$k = 0.0023$ per year
$t_{1/2} = ?$

$$t_{1/2} = \frac{0.693}{k} = \frac{0.693}{0.0023} = 301 \text{ years}$$

Problem 19.15

The decay rate constant of an element is 0.0023 per year. Most nearly at what time does its amount reduce by 80%?

A. 400 years B. 500 years C. 600 years D. 700 years

Solution: D

$k = 0.0023$ per year
$t = ?$

Half-life, $\tau = t_{1/2} = \frac{0.693}{k} = \frac{0.693}{0.0023} = 301$ years

Reduced by 80% means, remaining is 20% or $N/N_o = 0.2$

$$N = N_o e^{\frac{-0.693t}{\tau}} \Rightarrow \frac{N}{N_o} = e^{\frac{-0.693t}{\tau}} \Rightarrow 0.2 = e^{\frac{-0.693t}{301}} \Rightarrow \ln(0.2) = -\frac{0.693t}{301}$$

$\Rightarrow$ t = 699 years ≈ 700 years

Problem 19.16

An element has a half-life of approximately 30 years. If its initial mass is 300 mg, most nearly, how much it will be left after four years?

A. 121 mg B. 274 mg C. 294 mg D. 221 mg

Solution: B

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$$N = N_o e^{\frac{-0.693t}{t_c}} = 300 \text{ mg } e^{\frac{-0.693(4\text{years})}{30\text{ years}}} = 274 \text{ mg}$$

Water Quality

Hardness is an important water parameter. The procedure of determining hardness of water is explained here with an example. Note that it is not excluded in the most recent NCEES FE Ref. Handbook. However, you may expect few questions outside of the Handbook.

Wastewater Treatment and Technologies

Different wastewater treatment technologies are available such as activated sludge, facultative pond, biotower, etc. The author would like to advise students to review this topic if they studied this topic in their degree program. Otherwise, it may not be worthy to learn this topic only for the FE exam.

Facultative Pond

BOD Loading Total System ≤ 35 pounds BOD_5/(acre-day)
Minimum = 3 ponds
Depth = 3 - 8 ft
Minimum t = 90 - 120 days

Bio-tower

Fixed-Film Equation without Recycle, $\frac{S_e}{S_o} = e^{\frac{-kD}{q^n}}$

Fixed-Film Equation with Recycle, $\frac{S_e}{S_a} = \frac{e^{\frac{-kD}{q^n}}}{(1+R) - R\left(e^{\frac{-kD}{q^n}}\right)}$

S_e = effluent BOD_5 (mg/L)
S_o = influent BOD_5 (mg/L)
R = recycle ratio = Q_R/Q_o
Q_R = recycle flow rate

$$S_a = \frac{S_o + RS_e}{1+R}$$

D = depth of bio-tower media (m)
q = hydraulic loading [$m^3/(m^2 \cdot min)$]
k = treatability constant; functions of wastewater and medium (min^{-1}); range 0.01 - 0.1; for municipal wastewater and modular plastic media 0.06 min^{-1} @ 20°C
n = coefficient relating to media characteristics; modular plastic, n = 0.5

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Problem 19.17

In a hypothetical bio-tower with fixed film without recycle option, the effluent BOD_5 (mg/L) and the influent BOD_5 (mg/L) are 0.2 and 0.3 respectively. The treatability constant of the bio-tower is 0.06 min^{-1}. If the bio-tower height is 10 m, what is most nearly the hydraulic loading? Assume, $n = 1.0$.

A. 1 $m^3/(m^2.min)$
B. 1.5 $m^3/(m^2.min)$
C. 2.0 $m^3/(m^2.min)$
D. 2.5 $m^3/(m^2.min)$

Solution: B

Fixed-Film Equation without Recycle, $\frac{S_e}{S_o} = e^{-\frac{kD}{q^n}}$

$$\Rightarrow \frac{0.2}{0.3} = e^{-\frac{0.06(10)}{q^1}}$$

$$\Rightarrow \ln(0.67) = -\frac{0.6}{q}$$

$$\Rightarrow q = 1.48 \approx 1.50$$

Problem 19.18

When Nitrogen Dioxide (NO_2) is dissolved in water (H_2O), nitric acid and nitrous acid are formed as follows:

$$NO_2 + H_2O \longrightarrow HNO_3 + HNO_2$$

The atomic weights of H, O and N are 1, 16, and 14 respectively. The amount of water (g) required to completely degrade 200 g of Nitrogen Dioxide (NO_2) is most nearly:

A. 78 B. 39 C. 19.6 D. 9.8

Solution: B

$NO_2 + H_2O \rightarrow HNO_3 + HNO_2$
$2NO_2 + H_2O = HNO_3 + HNO_2$

$2NO_2 = 2(14 + 32) = 92$
$H_2O = 2x1 + 16 = 18$
Amount of water required $= \frac{18}{92}(200) = 39$ g

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Problem 19.19

For the following reaction, how much lead nitrate ($Pb(NO_3)_2$) is required to produce 500 g of Nitrogen Dioxide (NO_2)? [Pb = 207; N = 14; O = 16]

$$Pb(NO_3)_2 \rightarrow NO_2 + PbO + O_2$$

A. 70 B. 140 C. 35 D. 210

Solution: A

$Pb(NO_3)_2 \rightarrow NO_2 + PbO + O_2$

$2\ Pb(NO_3)_2 = 4\ NO_2 + 2\ PbO + O_2$

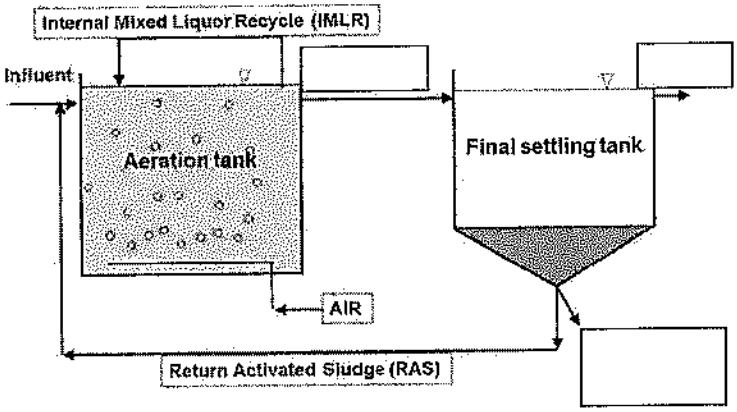
$2\ Pb(NO_3)_2 = 2 \times (207 + 2(14 + 48)) = 662$

$2NO_2 = 2(14 + 32) = 92$

$$\text{Amount of lead nitrate required} = \frac{92}{662}(500) = 69.5 \approx 70\,\text{g}$$

Problem 19.20

The activated sludge method for wastewater treatment process can be schematically shown below where three labels are missing. Fill in the boxes selecting proper words/phrases listed.

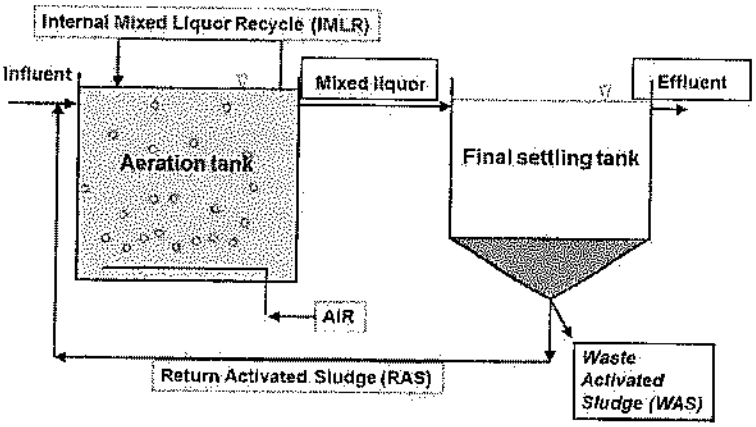


(Image: Courtesy of YSI Inc., Yellow Springs, Ohio. September 16, 2018)

1. Effluent
2. Waste Activated Sludge
3. Mixed Liquor
4. Settled Sludge
5. Leachate

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Solution:



Problem 19.21

Consider the reaction: 3A + 2B = C + 2D. The chemical equilibrium constant of the above reaction is most nearly:

A. $k = \dfrac{[C][D]}{[A][B]}$

B. $k = \dfrac{[A]^2[B]}{[C]^3[D]^2}$

C. $k = \dfrac{[C][D]^2}{[A]^3[B]^2}$

D. $k = \dfrac{[C]^3[D]^2}{[A]^2[B]}$

Solution: C

Problem 19.22

Match the numbers with the correct blanks in the equation to balance the reaction. Some numbers may be used more than once, some may not be used at all.

$$\text{HCL} + \_\_\_\_\text{Na} = \_\_\_\text{NaCL} + \_\_\_\_\text{H}_2$$

1 2 3 4

Solution:

$$2\,\text{HCl} + 2\,\text{Na} = 2\,\text{NaCl} + 1\,\text{H}_2$$

See "Chemical Reaction Equilibrium" in the Environmental Engineering Section.

The author is requesting the readers to report any concern about the book at books.mrislam@gmail.com. Any suggestion to improve this book, or any issue reported will be fixed in the next edition/printing with proper acknowledgement.

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Nonclassical Questions

Besides classical multiple-choice questions, sometimes, there will be options to select more than one option; drag and drop questions; point and click clickable area questions and fill in the blank questions. Some examples are given here.

Problem 1 [click clickable area question]

An economic analysis is required to compute an equivalent value today for an estimated constant monthly expense that is projected to occur each month over the next ten years. Mark the area of the single column of the factor table that contains the value that could be used for such an analysis.

Interest Rate Tables
Factor Table - $i = 0.50\%$

n	P/F	P/A	P/G	F/P	F/A	A/P	A/F	A/G
1	0.9950	0.9950	0.0000	1.0050	1.0000	1.0050	1.0000	0.0000
2	0.9901	1.9851	0.9901	1.0100	2.0050	0.5038	0.4988	0.4988
3	0.9851	2.9702	2.9604	1.0151	3.0150	0.3367	0.3317	0.9967
4	0.9802	3.9505	5.9011	1.0202	4.0301	0.2531	0.2481	1.4938
5	0.9754	4.9259	9.8026	1.0253	5.0503	0.2030	0.1980	1.9900
6	0.9705	5.8964	14.6552	1.0304	6.0755	0.1696	0.1646	2.4855
7	0.9657	6.8621	20.4493	1.0355	7.1059	0.1457	0.1407	2.9801
8	0.9609	7.8230	27.1755	1.0407	8.1414	0.1278	0.1228	3.4738
9	0.9561	8.7791	34.8244	1.0459	9.1821	0.1139	0.1089	3.9668
10	0.9513	9.7304	43.3865	1.0511	10.2280	0.1028	0.0978	4.4589
11	0.9466	10.6770	52.8526	1.0564	11.2792	0.0937	0.0887	4.9501
12	0.9419	11.6189	63.2136	1.0617	12.3356	0.0861	0.0811	5.4406

Solution:

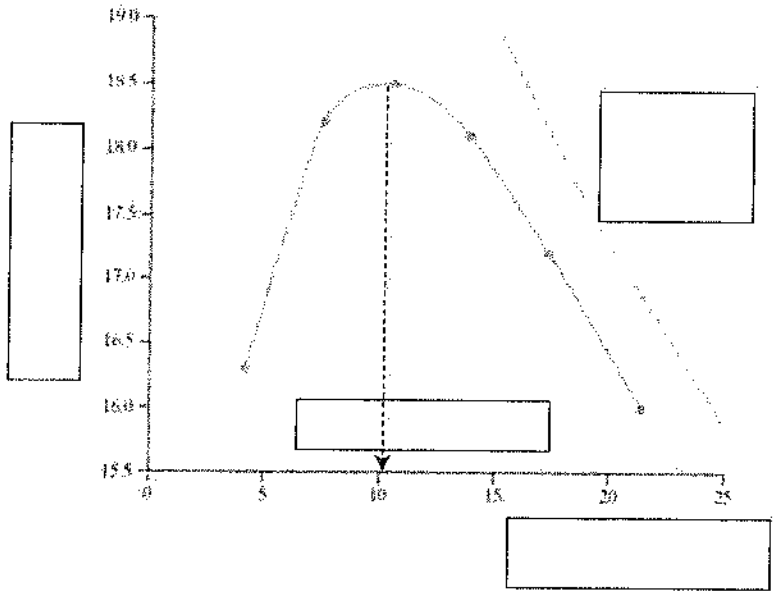
Interest Rate Tables
Factor Table - $i = 0.50\%$

n	P/F	P/A	P/G	F/P	F/A	A/P	A/F	A/G
1	0.9950	0.9950	0.0000	1.0050	1.0000	1.0050	1.0000	0.0000
2	0.9901	1.9851	0.9901	1.0100	2.0050	0.5038	0.4988	0.4988
3	0.9851	2.9702	2.9604	1.0151	3.0150	0.3367	0.3317	0.9967
4	0.9802	3.9505	5.9011	1.0202	4.0301	0.2531	0.2481	1.4938
5	0.9754	4.9259	9.8026	1.0253	5.0503	0.2030	0.1980	1.9900
6	0.9705	5.8964	14.6552	1.0304	6.0755	0.1696	0.1646	2.4855
7	0.9657	6.8621	20.4493	1.0355	7.1059	0.1457	0.1407	2.9801
8	0.9609	7.8230	27.1755	1.0407	8.1414	0.1278	0.1228	3.4738
9	0.9561	8.7791	34.8244	1.0459	9.1821	0.1139	0.1089	3.9668
10	0.9513	9.7304	43.3865	1.0511	10.2280	0.1028	0.0978	4.4589
11	0.9466	10.6770	52.8526	1.0564	11.2792	0.0937	0.0887	4.9501
12	0.9419	11.6189	63.2136	1.0617	12.3356	0.0861	0.0811	5.4406

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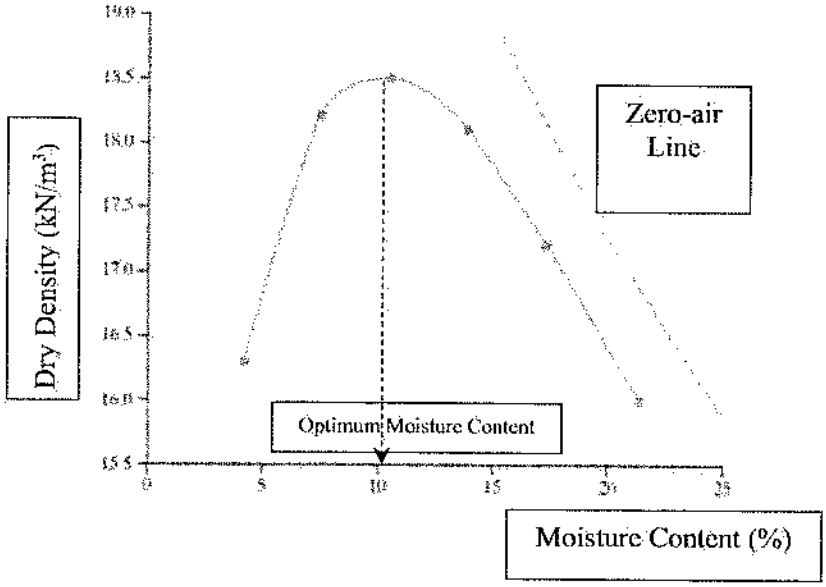
Problem 2 [drag and drop question]

The proctor test on a soil sample yields the following curve. Fill the four empty-boxes taking the most suitable phrases from the listed five phrases.



1. Dry Density (kN/m^3)
2. Dry Density (pcf)
3. Moisture Content (%)
4. Zero-air Line
5. Optimum Moisture Content

Solution:

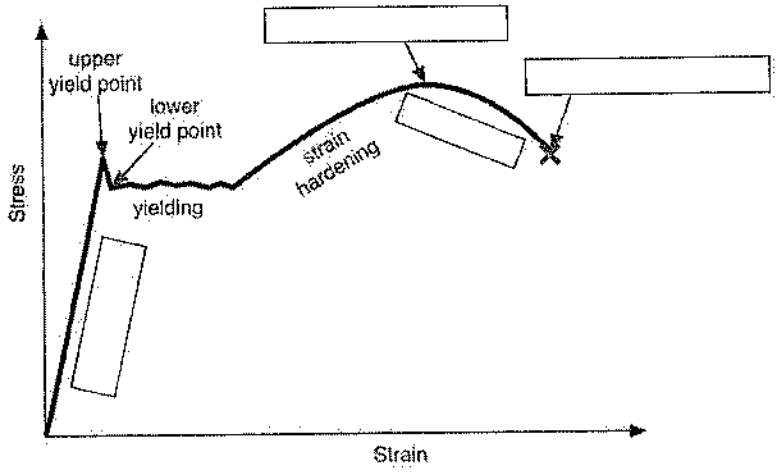


Dry density of soil commonly ranges 15 to 20 kN/m^3 or 90 to 130 pcf.

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Problem 3 [drag and drop question]

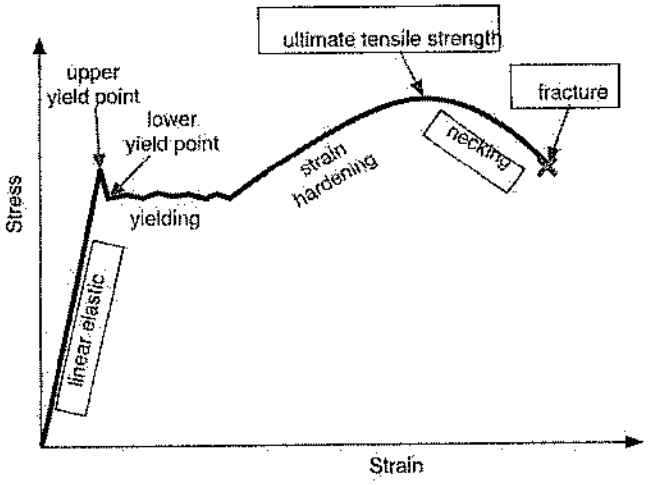
The stress-strain schematic curve of a tensile test on a steel specimen is shown below. Four parameters of the curve are missing and are shown by empty boxes. Fill the four empty-boxes taking the listed four properties.



(Image by Dr. Drang)

1. Ultimate tensile strength
2. Linear Elastic
3. Necking
4. Fracture

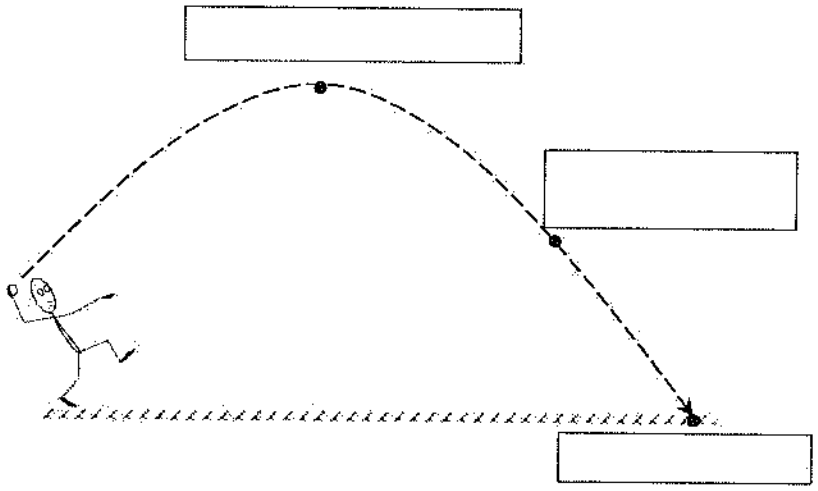
Solution:



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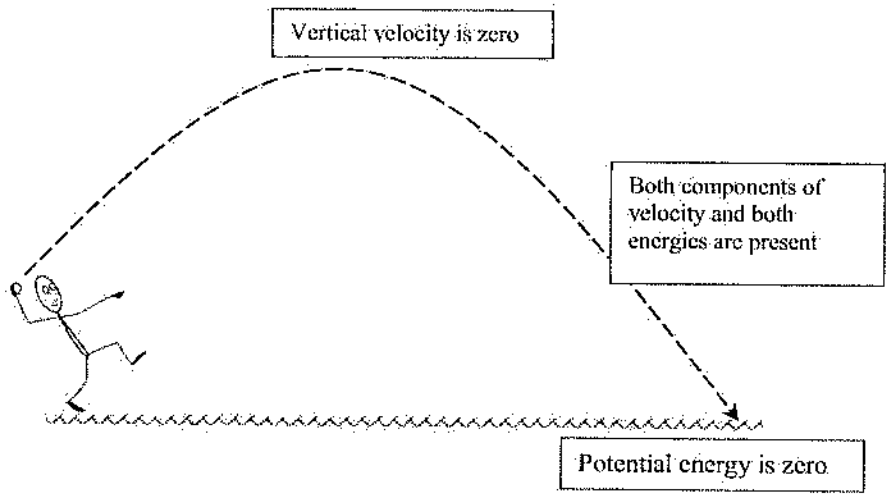
Problem 4 [drag and drop question]

A stone is thrown inclinedly with some angle with respect to the surface (less than 90°). Its travel path is shown below. Fill the three empty-boxes with appropriate statements from the listing below.



1. Vertical velocity is zero
2. Horizontal velocity is zero
3. Kinetic energy is zero
4. Potential energy is zero
5. Both components of velocity and both energies are present

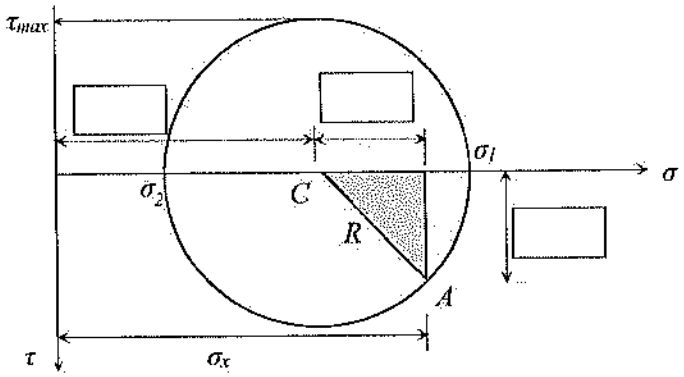
Solution:



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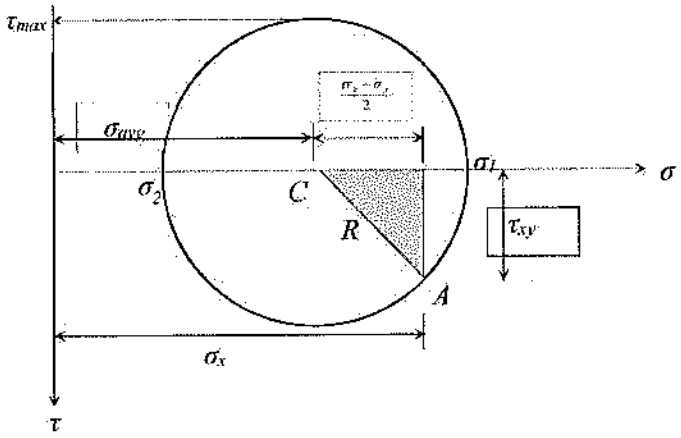
Problem 5 [drag and drop question]

The Mohr circle of an element of a structural member is shown below, where C is the center, R is the radius, σ is the axial stress, τ is the shear stress. Fill the three empty-boxes selecting appropriate items from the listing below.



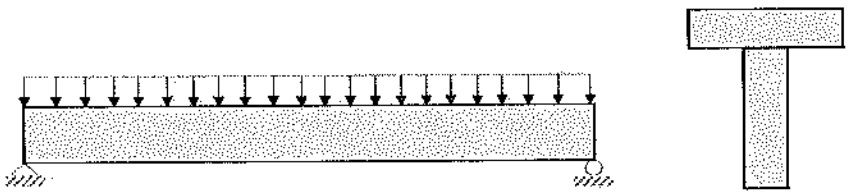
1) $\dfrac{\sigma_x - \sigma_y}{2}$
2) σ_{avg}
3) τ_{xy}

Solution:



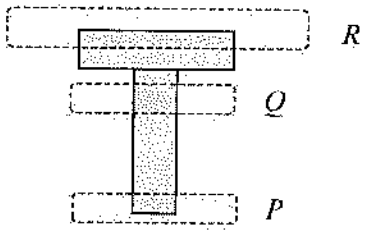
Problem 6 [click clickable area question]

A simply supported beam is having a uniformly distributed load as shown below. The cross-section of the beam is a T-section. The region having the most shear stress is most nearly:



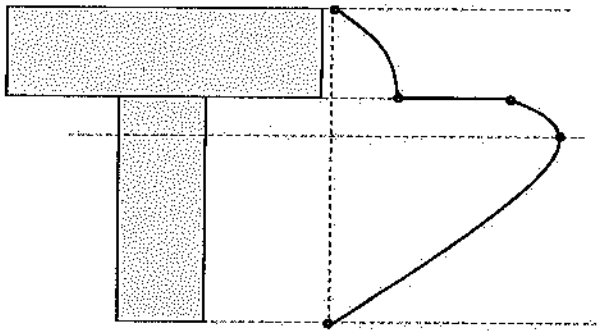
(the cursor will show region on the cross-section; select the most appropriate one)

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Solution:

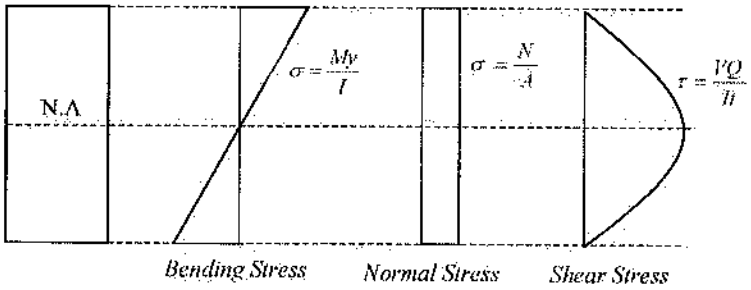
Q-Region (around the neutral axis w.r.t. the horizontal axis) is the region which gets the maximum shear stress. The schematic shear-stress diagram on T-section is shown below:



P-Region gets the maximum tensile bending stress; R-Region gets the maximum compressive bending stress.

Recall:

- For positive moment, the maximum tensile bending stress occurs at the bottom fiber, and the maximum compressive bending stress occurs at the top fiber regardless of shape (W, or T, or Rectangular). This is opposite is for negative moment.



- Shear stress is always the maximum at the neutral axis. It decreases away from the neutral axis. If sudden thickness changes (T, or W section), shear stress jumps inversely.
- If a torque is applied in a shaft, the maximum shear stress occurs at the outmost ring.

More resources are available at https://ncees.org/exams/exam-preparation-materials/

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CHAPTER

7

Fluid Mechanics

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Robert M. Scott

Introduction

Fluid mechanics is the study of the changes that occur in a fluid that is subjected to forces. There are two major divisions within the field of fluid mechanics: fluid statics and fluid dynamics. **Fluid statics** is the study of the state of a fluid subjected to balanced forces such that the fluid remains in an equilibrium condition. **Fluid dynamics** is the study of fluid in motion and the interactions of the fluid with the objects it may encounter.

7.1 Definitions
7.2 Fluid Properties
7.3 Fluid Statics
7.4 Fluid Dynamics
7.5 Internal Flow
7.6 External Flow
7.7 Devices
7.8 Fluid Measurements
7.9 Dimensional Analysis—Dimensional Homogeneity
7.10 Similitude

7.1 Definitions

As with any other field of study, fluid mechanics has a vocabulary of terms that must be understood before the basic principles can be introduced.

What Is a Fluid?

A **fluid** is a substance that will deform *continuously* when a shear stress is applied to it, no matter how small the shear stress is. A substance in a liquid or vapor phase is a fluid.

Control Volume

All of the principles of fluid mechanics are defined in relation to a control volume. The **control volume** is an arbitrary region in space defined by its boundaries, which may be stationary or moving. In general, material may cross a boundary of the control volume. Though the boundaries of a control volume are completely arbitrary, when possible they are chosen in a way that simplifies analysis.

Stress

The forces that act upon a fluid may be categorized as either body forces or surface forces. *Body forces* are forces that act on all fluid particles within the fluid and occur without physical contact. Examples include electromagnetic and gravitational forces. *Surface*

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8. A licensed professional engineer and land surveyor works 35 hours per week on a flex-time basis for a state governmental agency. In addition, he is associated with XYZ Engineering and Surveying as the PE in charge. He spends about 20 hours per week supervising engineering services at the firm plus an additional 12 hours of work on the weekends and is available for consultation 24 hours a day. XYZ has granted him 10% of the shares of the stock in the firm, and he receives 5% of the gross billings for his seal as extra compensation for taking charge of engineering. This agreement is contingent on the understanding that, if any of the three principals of XYZ becomes licensed as a PE, the agreement will become void, and the engineer will return the stock. Both organizations—the state governmental agency and XYZ—are aware that he is a dual employee and have no objections. If this arrangement comes to the attention of the Board of Ethical Review, the board will determine that
   (A) a flexible schedule is not sufficient grounds for dual employment
   (B) the engineer is stretching the role of responsible charge and cannot assume this role for two employers
   (C) it is acceptable and ethical to be involved with both the state governmental agency and XYZ Engineering and Surveying in the manner described
   (D) the engineer cannot work for a government agency and a private organization at the same time

Answer Key

1. D	5. A
2. A	6. C
3. B	7. A
4. B	8. C

Answers Explained

1. **D** The public good and the public welfare are practically synonymous; but since "public welfare is the term used in the Model Rules, it is the better choice.
2. **A** The code stresses that the order of responsibility is to society as a whole, the client or customer, and then to professional colleagues. The code makes no mention of self-interest. Obviously, then, that must come last.
3. **B** The code is very specific: "Registrants shall not reveal facts, data, or information obtained in a professional capacity. . . ." Even without a written nondisclosure agreement, the engineer is bound to maintain confidentiality.
4. **B** The code requires full disclosure and frankness with regard to possible future conflicts of interest. Current colleagues may be the second to know. There is no reason for the engineer to leave the job before the new business is in place so long as he does not use his current position as a springboard for lining up his own clients. Choice (D) represents a dishonest use of the employer's facilities and a form of unfair competition.
5. **A** Safety must come first. The engineer is required to report unsafe practices to the authorities. No company wants to expose itself to product liability lawsuits; but when an expensive recall might be involved, the company might well attempt to discount an alert engineer's fears. The engineer must first alert the company to the problem and cooperate in trying to find a solution consistent with maintaining the public welfare. However, if the company resists taking appropriate measures, the engineer must "blow the whistle."
6. **C** Choice (D) looks good, but (C) meets the confidentiality requirement. Both (A) and (B) additionally violate the requirement for fair dealing among contractors.
7. **A** Despite the mitigating language in some other choices, it should be obvious that (A) is the correct choice. The brochure constitutes false advertising at its worst.
8. **C** The problem does not say that XYZ Engineering and Surveying is doing any work for the state governmental agency, so there is no question of conflict of interest here. The engineer has two part-time jobs, which do not involve the same project, and both employers have agreed that this situation is acceptable.

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forces are forces that are applied directly to the surface of the fluid through physical contact.

The total force per unit area acting on the fluid at any point within its volume is an important parameter, and has been given the name stress. The **stress** at any point P is defined as follows:

$$\sigma(P) = \lim_{\delta A \to 0} \frac{\delta \mathbf{F}}{\delta \mathbf{A}} \qquad [7.1]$$

where $\sigma(P)$ = the vector stress at point P,
$\delta \mathbf{A}$ = an infinitesimal area at point P,
$\delta \mathbf{F}$ = the force acting on infinitesimal area $\delta \mathbf{A}$.

Both the area and the force in equation 7.1 are vector quantities; that is, they have magnitude and direction. These quantities are usually expressed by three components with respect to some coordinate system.

In general, to describe the stress of a fluid at any point with respect to a coordinate system, nine components are required. For convenience, stress is often expressed in terms of two vector components, in relation to the differential area ($\delta \mathbf{A}$). The component normal (or perpendicular) to $\delta \mathbf{A}$ is the normal stress (σ_n), and the component along $\delta \mathbf{A}$ is the shear stress (τ).

In a stationary fluid, the normal stress is the fluid pressure, whose magnitude is independent of the direction of the differential area used. The shear stress is related to fluid motion.

7.2 Fluid Properties

The **fluid properties** are the fluid characteristics whose values are used to predict the reaction of a fluid that is subjected to applied forces. The list that follows does not include all the possible fluid properties, but it defines the terms used throughout this chapter.

Velocity

Velocity is the time rate of change of the position of a fluid particle. The velocity of a fluid is a vector field quantity. For each point within the volume that makes up the fluid, a value can be assigned to the magnitude and direction of the fluid velocity. The velocity of a fluid will appear in three forms throughout this chapter. These forms are as follows:

- Velocity ($\mathbf{V}$) = the vector quantity of fluid velocity at a given point in the fluid. Both magnitude and direction are associated with this quantity.
- Speed (V) = the magnitude of the vector velocity at a given point in the fluid.
- Average speed ($\overline{V}$) = the average fluid speed through a surface of a control volume.

Mach Number

The **Mach number** (M_a) is a dimensionless parameter used to describe fluid speed. It is defined as the ratio of fluid speed to the speed of sound:

$$M_a = \frac{V}{c} \qquad [7.2]$$

where M_a = Mach number (dimensionless),
V = fluid speed (ft/sec or m/s),
c = speed of sound (ft/sec or m/s).

For a perfect gas, the speed of sound is defined as

$$c = \sqrt{kRT} \qquad [7.3]$$

where c = speed of sound (ft/sec or m/s),
k = specific heat ratio (c_p/c_v),
R = gas constant (ft-lbf/lb mole-°R or kJ/kg · K),
T = absolute temperature of the gas (K or °R).

In equation 7.3:

$$R = \frac{\overline{R}}{M} \qquad [7.4]$$

where $\overline{R}$ = universal gas constant:
= 1,545 ft-lbf/lb mole-°R,
= 8.314 kJ/(mol · K),
M = molecular weight of the gas.

Example 7.1

What is the Mach number of a jet of oxygen gas at standard conditions (1 atm and 25°C) with a speed of 450 m/s? (Assume that for oxygen k = 1.40 and R = 0.260 kJ/kg · K.)

Solution:

Oxygen at standard condition can be modeled as a perfect gas; therefore, from equation 7.3, the speed of sound is as follows:

$$c = \sqrt{kRT}$$

$$= \sqrt{(1.40)\left(0.260\frac{\text{kj}}{\text{kg} \cdot \text{K}}\right)[(25 + 273)\text{K}]\left(\frac{1{,}000\ \text{N} \cdot \text{m}}{\text{kJ}}\right)\left(\frac{\frac{\text{kg} \cdot \text{m}}{\text{s}^2}}{\text{N}}\right)}$$

$$= 329.35\ \text{m/s}$$

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Now determine the Mach number directly from equation 7.2:

$$M_a = \frac{V}{c}$$

$$= \frac{450 \text{ m/s}}{329.35 \text{ m/s}}$$

$$= 1.37$$

Acceleration

Acceleration is the time rate of change in the velocity of a fluid particle. Acceleration is also a vector field quantity, and can act to change either the magnitude or the direction of the velocity.

Pressure

There are two reference values for pressure to which all other values of pressure are related. **Gage pressure** is defined in reference to the ambient atmospheric pressure, and **absolute pressure** is defined in reference to true zero pressure. The two values for pressure are related by the equation:

$$p_a = p_g + p_{atm} \qquad [7.5]$$

where p_a = absolute pressure (lbf/in^2 = psia or N/m^2 = kPa),
p_g = gage pressure (lbf/in^2 = psig or N/m^2 = kPa),
p_{atm} = ambient atmospheric pressure:
= 14.696 psia or 101.3 kPa at standard sea-level conditions.

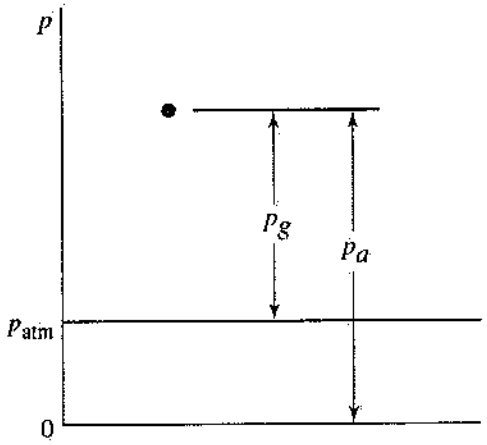


Figure 7.1 Definition of gage pressure.

Example 7.2

What will a pressure gauge read if the fluid in a vessel is at 200 psia? (Assume that the vessel is in an environment at standard sea-level pressure.)

Solution:

From equation 7.5:

$$p_g = p_a - p_{atm}$$

$$= 200 \text{ psi} - 14.7 \text{ psi}$$

$$= 185.3 \text{ psig}$$

Density

The **density** of a fluid (ρ) is the mass contained in one unit of volume. The density of a fluid at a given location is expressed as

$$\rho = \lim_{\Delta\mathcal{V} \to 0}\left(\frac{\Delta m}{\Delta\mathcal{V}}\right) \qquad [7.6]$$

where ρ = fluid density (lbm/ft^3 or kg/m^3),
Δm = mass contained in infinitesimal volume $\Delta\mathcal{V}$ (lbm or kg),
$\Delta\mathcal{V}$ = infinitesimal volume (ft^3 or m^3).

Analysis of fluid systems is often simplified by assuming that the density is the same for every point within the volume of the fluid, as in incompressible flow analysis.

Specific Volume

The **specific volume** of a fluid (v) is the volume of space occupied by a unit mass of the fluid. It is the reciprocal of the density and is expressed as:

$$v = \frac{1}{\rho} \qquad [7.7]$$

where v = specific volume (ft^3/lbm or m^3/kg).

Specific Weight

The **specific weight** (γ) of a fluid is defined as the weight of a unit volume of fluid. It is related to the fluid density as follows:

$$\gamma = \rho g \qquad [7.8]$$

where γ = specific weight (lbf/ft^3 or N/m^3),
g = acceleration due to gravity:
= 32.17 ft/sec^2 or 9.81 m/s^2 at sea level.

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Specific Gravity

Specific gravity is a dimensionless expression for the density of a fluid. Its value is the ratio of the fluid density to the density of liquid water at 4°C:

$$SG = \frac{\rho}{\rho_{water}} = \frac{\gamma}{\gamma_{water}} \quad [7.9]$$

where $\rho_{water} = 1{,}000\ kg/m^3$,
$= 62.4\ lbm/ft^3$,
$\gamma_{water} = 9{,}810\ N/m^3$,
$= 62.4\ lbf/ft^3$.

Example 7.3

What is the density of an oil if its specific gravity is 0.86?

Solution:

From equation 7.9:

$$\rho = (SG)(\rho_{water})$$
$$= (0.86)(62.4\ lbm/ft^3)$$
$$= 53.7\ lbm/ft^3$$

Viscosity

When a shear force is applied to a fluid, fluid motion will occur. The magnitude of the fluid velocity is related to the magnitude of the shear stress. For most common fluids the shear stress within the fluid is directly proportional to the velocity gradient. **Viscosity** (μ) is the name given to the proportionality constant:

$$\tau_n(P) = \mu \frac{\partial V}{\partial x_n} \quad [7.10]$$

where $\tau_n(P)$ = shear stress on a differential area normal to direction n,
μ = fluid viscosity (described below),
$\frac{\partial V}{\partial x_n}$ = velocity gradient in direction n.

Fluids of this type are called *Newtonian fluids*. It should be recognized from equation 7.10 that, for a given shear stress, as the viscosity of a fluid increases, the magnitude of the velocity gradient decreases.

There are many fluids, such as toothpaste, blood, and many paints, that cannot be classified as Newtonian fluids. For many non-Newtonian fluids the shear stress can be related to the velocity gradient of the fluid through a power law relationship. Two examples of such non-Newtonian fluids are *psuedoplastic* fluids, for which the viscosity decreases as the velocity gradient increases, and *dilitant* fluids, for which the viscosity increases as the velocity gradient increases.

For a power law (non-Newtonian) fluid:

$$\tau_i(P) = K\left(\frac{\partial V}{\partial X_i}\right)^n \quad [7.11]$$

where K = consistency index,
n = power law index: $n < 1$ for a psuedoplastic fluid, $n > 1$ for a dilatant fluid.

Surface Tension

At the surface of any fluid, there is a force that tends to hold the surface together. This force is called the **surface tension** of the fluid, which is defined as the tensile force between two points on the surface of the fluid, per unit length of distance between the two points:

$$\sigma = \frac{F}{L} \quad [7.12]$$

where σ = surface tension,
F = force along the fluid surface,
L = length of the fluid surface.

Capillarity

Capillary action or **capillarity** is observed as the change in shape of a fluid's free surface when in contact with a solid wall. It is a result of the interaction of the surface tension and the general cohesive forces between the molecules of the liquid.

The curved fluid surface that generally forms as a result of capillary action is called a *meniscus*, shown in Figure 7.2(a). If the area of a free surface is large, then, rather than forming a complete meniscus, the edges of the surface will be curved as shown in Figures 7.2(b) and (c). Whether or not the surface forms a complete meniscus, the slope of the curve at the wall of the tube is called the *angle of contact*, (β). The angle of contact is an indication of the relationship between the surface tension and the cohesive forces of the liquid. If the angle is less than 90°, as in Figure 7.2(b), the surface tension is stronger than the cohesive forces; if the angle is greater than 90°, as in Figure 7.2(c), the cohesive forces are stronger than the surface tension.

When a small-diameter tube is placed in a fluid for which the surface tension is stronger than the molecular cohesive forces, the fluid level inside the tube will rise a certain distance, as shown in Figure 7.2(a). The height of this column is given by these equations:

$$h = \frac{4\sigma \cos\beta}{\rho d g} \quad [7.13]$$

and

$$h = \frac{4\sigma \cos\beta}{\gamma d} \quad [7.14]$$

where h and d are as shown in Figure 7.2(a).

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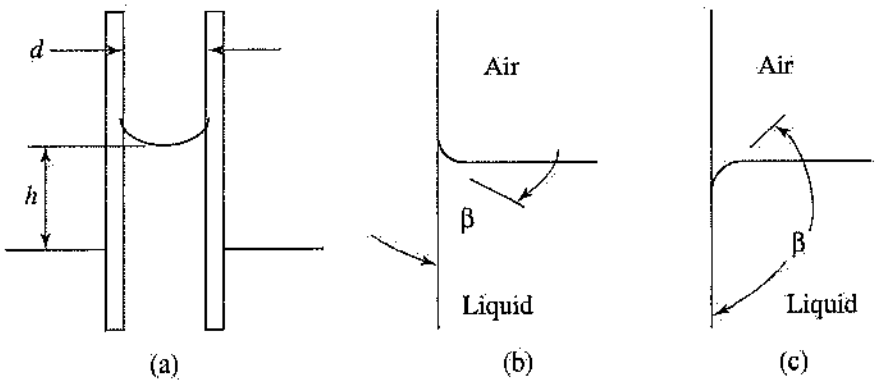


Figure 7.2 Illustration of a meniscus. (a) Meniscus in a small tube. (b) Surface tension dominates cohesive forces. (c) Cohesive forces dominate surface tension.

7.3 Fluid Statics

Fluid statics is the study of fluid systems with no fluid motion. The primary challenge of fluid statics is to determine the pressure distribution within a fluid.

Hydrostatic Pressure

When a fluid is not moving, the stress acting on each fluid particle becomes independent of the direction from which it is evaluated. In other words, for stress as defined in equation 7.1, the stress in a stationary fluid at some point has some value that is the same no matter in what direction the differential area (δA) is taken. This value is the **hydrostatic pressure** of the fluid at that point.

The hydrostatic pressure of a fluid is a function of height within the volume, regardless of the shape of the container. This is due to the fact that, at any point in a fluid, the fluid must support the weight of the fluid above it. For any two locations at the same height, the hydrostatic pressure will be the same. This is the principle behind the operation of two devices used to measure hydrostatic pressure, manometers and barometers.

For the control volume shown in Figure 7.3, the hydrostatic pressure at point B in the fluid is given by the equations:

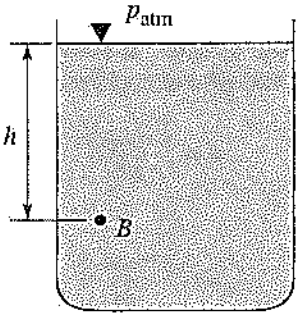


Figure 7.3 Hydrostatic pressure at point B.

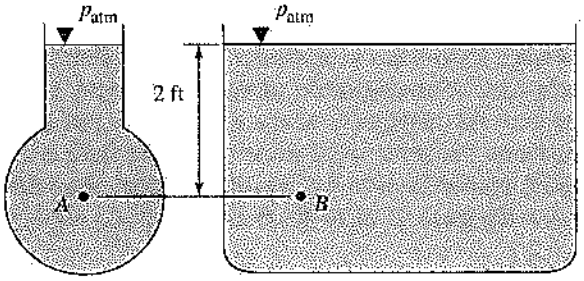
$$p_g = \rho g h \quad \text{or} \quad p_g = \gamma h \qquad [7.15]$$

and

$$p_a = p_{atm} + \rho g h \qquad [7.16]$$

Example 7.4

The two vessels shown in the accompanying figure are filled with water ($\rho = 62.4$ lbm/ft^3). What is the hydrostatic pressure in each container at points A and B 2 feet below the surface of the water?



Solution:

Since the hydrostatic pressure in a fluid is a function of its depth only and is independent of the shape of the container, the pressure at point A will equal the pressure at point B.

From equation 7.15:

$$p_g = \rho g h$$

$$= \left(62.4 \frac{\text{lbm}}{\text{ft}^3}\right)\left(32.17 \frac{\text{ft}}{\text{sec}^2}\right) \times (2\ \text{ft})\left(\frac{\text{ft}^2}{144\ \text{in}^2}\right)\left(\frac{\text{lbf}}{32.17 \frac{\text{lbm ft}}{\text{sec}^2}}\right)$$

$$= 0.866\ \text{lbf/in}^2 = 0.866\ \text{psig}$$

The term $\left(\frac{\text{lbf}}{32.17 \frac{\text{lbm ft}}{\text{sec}^2}}\right)$ in the above solution represents a unit conversion between pounds

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mass and pounds feet. This is the inverse of the quantity g_c.

Manometry

A **manometer**, illustrated in Figure 7.4, is a device used to measure a pressure difference. It consists of a U-shaped tube with each end open to one of the two environments whose pressure we wish to compare.

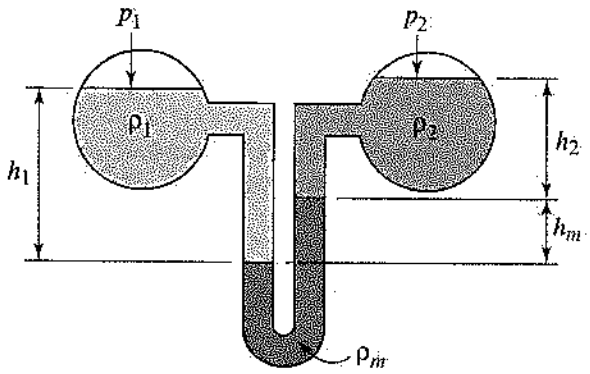


Figure 7.4 Diagram of a manometer, used to measure the difference in hydrostatic pressure between vessel 1 and vessel 2.

The difference in pressure between these two environments is related to the difference in height between the two legs of the manometer as follows:

$$p_1 - p_2 = \rho_m g h_m + (\rho_2 g h_2 - \rho_1 g h_1) \quad [7.17]$$

or

$$p_1 - p_2 = \gamma_m h_m + (\gamma_2 h_2 - \gamma_1 h_1) \quad [7.18]$$

If the density or specific weight of either fluid 1 or fluid 2 in Figure 7.4 is small compared to the density or specific weight of the manometer fluid, its contribution to equations 7.17 and 7.18 can be neglected. For example, if fluid 1 is air (density = 1.23 kg/m^3), fluid 2 is water (density = 1,000 kg/m^3), and the manometer fluid is oil (density = approx. 830 kg/m^3), then the effect of the air can be neglected, and equations 7.17 and 7.18 can be reduced to

$$p_1 - p_2 = \rho_m g h_m + \rho_2 g h_2 \quad [7.19]$$

and

$$p_1 - p_2 = \gamma_m h_m + \gamma_2 h_2 \quad [7.20]$$

If, however, the densities or specific weights of *both* fluids are small compared to the density or specific weight of the manometer fluid, equations 7.17 and 7.18 reduce to

$$p_1 - p_2 = \rho_m g h_m \quad [7.21]$$

and

$$p_1 - p_2 = \gamma_m h_m \quad [7.22]$$

When one end of the manometer is exposed directly to the atmosphere, the device is called an open manometer, illustrated in Figure 7.5. Equations 7.17 and 7.18 apply to an open manometer if p_2 is replaced with atmospheric pressure p_{atm}.

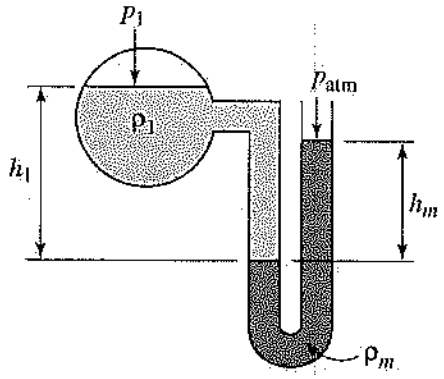


Figure 7.5 Diagram of an open manometer.

For the manometer shown in Figure 7.5, the pressure at the surface of fluid 1 (p_1) is given by

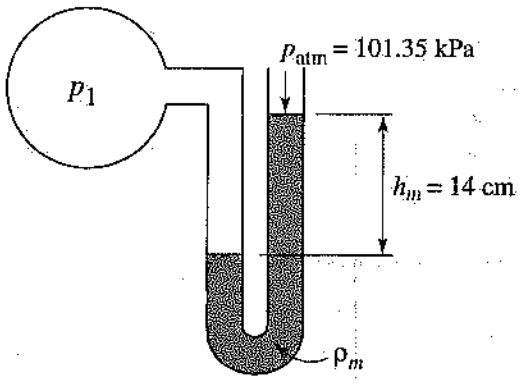
$$p_1 = p_{atm} + \rho_m g h_m - \rho_1 g h_1 \quad [7.23]$$

and

$$p_1 = p_{atm} + \gamma_m h_m - \gamma_1 h_1 \quad [7.24]$$

Example 7.5

The open manometer shown in the accompanying figure is used to measure the pressure (p_1) in an air tank. The manometer fluid is mercury, which has a density of 13,550 kg/m^3. What is the pressure in the vessel?



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Solution:

The density of air is much smaller than the density of mercury, so the contribution of air can be neglected. From equation 7.23:

$$p_1 = p_{atm} + \rho_m g h_m$$

$$= 101.35 \text{ kPa} + \left(13{,}550 \frac{\text{kg}}{\text{m}^3}\right)\left(9.81 \frac{\text{m}}{\text{s}^2}\right)$$

$$(14 \text{ cm})\left(\frac{\text{m}}{100 \text{ cm}}\right)\left(\frac{\text{N}}{\frac{\text{kg} \cdot \text{m}}{\text{s}^2}}\right)\left(\frac{\text{kPa}}{1{,}000 \frac{\text{N}}{\text{m}^2}}\right)$$

$$= 119.96 \text{ kPa}$$

Barometers

A **barometer,** illustrated in Figure 7.6, is a device used to measure atmospheric pressure. Atmospheric pressure acts on the exposed surface of the barometer fluid to maintain the height of a column of the fluid in a tube. The pressure in the empty section at the top of the tube is equal to the vapor pressure of the barometer fluid at the ambient temperature.

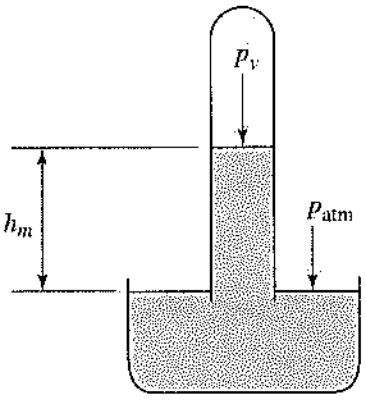


Figure 7.6 Diagram of a barometer.

The height of the fluid column is related to the atmospheric pressure as follows:

$$p_{atm} - p_v = \rho g h_m \qquad [7.25]$$

or

$$p_{atm} - p_v = \gamma h_m \qquad [7.26]$$

where p_v = vapor pressure of the manometer fluid at the ambient temperature.

The barometer fluid is usually chosen so that $p_v \approx 0$.

Forces on Submerged Surfaces

At any location on the surface of a submerged object, the pressure at that point is proportional to the point's distance below the surface. This pressure is determined as follows:

$$p = p_0 + \rho g z \qquad [7.27]$$

or

$$p = p_0 + \gamma z \qquad [7.28]$$

where p = pressure at the given location,
p_0 = pressure at the surface of the fluid,
z = vertical distance between the point and the surface of the fluid.

For an arbitrary region on the surface of a submerged object, each differential area, $d\mathbf{A}$, is acted upon by a force expressed as

$$d\mathbf{F} = p\, d\mathbf{A} \qquad [7.29]$$

where $d\mathbf{A}$ = differential area on the surface of a submerged object,
$d\mathbf{F}$ = force acting on $d\mathbf{A}$,
p = fluid pressure at the location of $d\mathbf{A}$.

The direction of the force is normal to and into the surface. If this force is integrated over a region, the total force acting on that region is obtained. If this region is a flat plate, as in the control volume shown in Figure 7.7, and the pressure at the surface of the fluid is zero, this force is expressed as follows:

$$\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y \qquad [7.30]$$

$$|\mathbf{F}_x| = \frac{p_1 + p_2}{2} A_v \qquad [7.31]$$

$$|\mathbf{F}_y| = \rho g \forall_f \qquad [7.32]$$

where $\mathbf{F}$ = total force exerted on the plate by the fluid, directed normal into the surface of the plate,
$|\mathbf{F}_x|$ = magnitude of the horizontal component of $\mathbf{F}$,
$|\mathbf{F}_y|$ = magnitude of the vertical component of $\mathbf{F}$,
p_1 = gage pressure at the top edge of the plate,
p_2 = gage pressure at the bottom edge of the plate,
A_v = vertical projection of the plate area,
$\forall_f$ = volume of fluid contained in the region bounded by the plate and the horizontal projection of the plate onto the free surface of the fluid.

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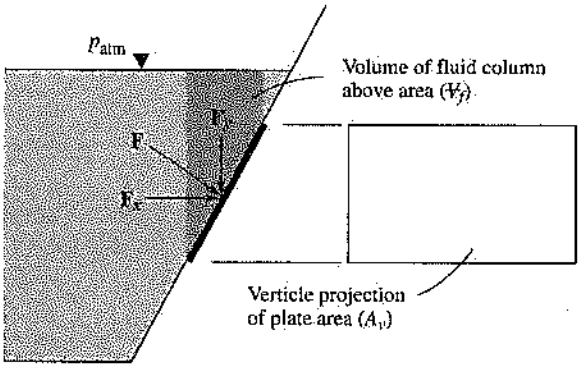
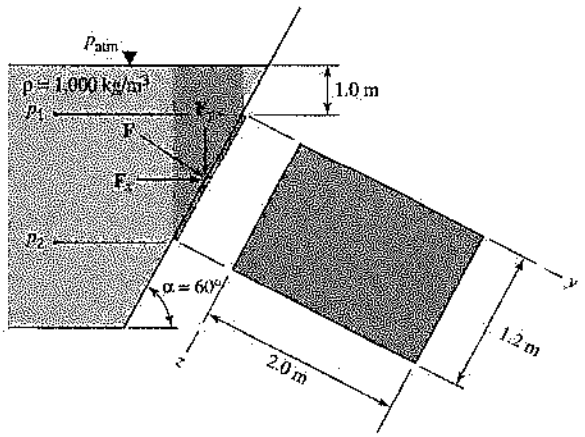


Figure 7.7 A flat-plate region of a submerged surface.

Equations 7.30 through 7.32 apply when the contribution of the atmospheric pressure to the force on a submerged surface is negligible. When this effect is not negligible, the gage pressures in equation 7.31 should be replaced with absolute pressures, and a surface-force term ($P_{atm}A_h$) should be added to equation 7.32. In this term, A_h is the horizontal projection of the submerged plate onto the free surface of the fluid.

Example 7.6

Find the total force acting on the submerged plate shown in the accompanying figure. Assume that the back side of the plate is also exposed to p_{atm}.



Solution:

The total force acting on the plate is directed normal to the plate, and into the plate, as shown in the figure. The magnitude of this force can be determined from equation 7.30:

$$\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y$$

From equation 7.31:

$$|\mathbf{F}_x| = \frac{p_1 + p_2}{2} A_v$$

$$p_1 = \rho g h_1$$

$$= \left(1{,}000 \frac{\text{kg}}{\text{m}^3}\right)\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(1.0 \text{ m}) \times \left(\frac{\text{N}}{\frac{\text{kb} \cdot \text{m}}{\text{s}^2}}\right)\left(\frac{\text{kPa}}{1{,}000 \frac{\text{N}}{\text{m}^2}}\right)$$

$$= 9.81 \text{ kPa}$$

$$p_2 = \rho g h_2$$

$$h_2 = 1.0 \text{ m} + (1.2 \text{ m}) \sin 60$$

$$= 2.04 \text{ m}$$

$$p_2 = \left(1{,}000 \frac{\text{kg}}{\text{m}^3}\right)\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(2.04 \text{ m}) \times \left(\frac{\text{N}}{\frac{\text{kg} \cdot \text{m}}{\text{s}^2}}\right)\left(\frac{\text{kPa}}{1{,}000 \frac{\text{N}}{\text{m}^2}}\right)$$

$$= 20.01 \text{ kPa}$$

$$A_v = (2.0 \text{ m})[(1.2 \text{ m}) \sin 60]$$

$$= 2.08 \text{ m}^2$$

$$|\mathbf{F}_x| = \frac{9.81 \text{ kPa} + 20.01 \text{ kPa}}{2} \times (2.08 \text{ m}^2)\left(\frac{1{,}000 \frac{\text{N}}{\text{m}^2}}{\text{kPa}}\right)$$

$$= 31{,}010 \text{ N}$$

From equation 7.33:

$$|\mathbf{F}_y| = \rho g V_f$$

$$V_f = \left(\underbrace{(1.2 \text{ m}) \cos 60}_{} \times \underbrace{1.0 \text{ m}}_{} + \frac{\overbrace{(1.2 \text{ m}) \cos 60}^{} \times \underbrace{(1.2 \text{ m}) \sin 60}_{}}{2} \right)(2.0 \text{ m})$$

$$V_f = \left\{[(1.0 \text{ m})(1.2 \text{ m}) \cos 60] + \frac{[(1.2 \text{ m}) \cos 60][(1.2 \text{ m}) \sin 60]}{2}\right\}(2.0 \text{ m})$$

$$= 1.82 \text{ m}^3$$

$$|\mathbf{F}_y| = \left(1{,}000 \frac{\text{kg}}{\text{m}^3}\right)\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(1.82 \text{ m}^3)\left(\frac{\text{N}}{\frac{\text{kg} \cdot \text{m}}{\text{s}^2}}\right)$$

$$= 17{,}850 \text{ N}$$

The total force can be expressed as

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$$|\mathbf{F}| = \sqrt{|\mathbf{F}_x|^2 + |\mathbf{F}_y|^2}$$
$$= \sqrt{(31{,}010\ \text{N})^2 + (17{,}850\ \text{N})^2}$$
$$= 35{,}780\ \text{N}$$

The Center of Pressure

For a region on the surface of a submerged rigid body, the **center of pressure** is the location in this region through which a concentrated point force (**F** from the preceding section) will have the same effect on the submerged body as the distributed pressure force over the entire region. This definition is illustrated in Figure 7.8.

The coordinates' center of pressure relative to the centroid of the submerged area (y^* and z^*) is given by the following equations:

$$y^* = \frac{\rho g I_{y_c z_c} \sin\alpha}{F}, \qquad z^* = \frac{\rho g I_{y_c} \sin\alpha}{F} \qquad [7.33]$$

or

$$y^* \frac{\gamma I_{y_c z_c} \sin\alpha}{F}, \qquad z^* = \frac{\gamma I_{y_c} \sin\alpha}{F} \qquad [7.34]$$

where I_{y_c} = area moment of inertia about an axis parallel to the y-axis and passing through the centroid of the area. The value of I_{y_c} is given by the equation

$$I_{y_c} = I_y - A y_c^2 \qquad [7.35]$$

from the parallel axis theorem, where

$$I_y = \int_A y^2\, dA \qquad [7.36]$$

In equations 7.33 and 7.34, $I_{y_c z_c}$ is the area product of inertia about an axis parallel to the y- and z-axes and passing through the centroid of the area. The value of $I_{y_c z_c}$ is given by the equation

$$I_{y_c z_c} = I_{yz} - A y_c z_c \qquad [7.37]$$

from the parallel axis theorem, where

$$I_{yz} = \int_A xy\, dA \qquad [7.38]$$

Values for the moment of inertia and product of inertia for several common shapes can be found in the dynamics chapter (Chapter 3) of this book.

Example 7.7

Determine the center of pressure for the submerged surface of Example 7.6, as described in the figure for that example.

Solution:

From equation 7.33:

$$y^* = \frac{\rho g I_{y_c z_c} \sin\alpha}{F}, \qquad z^* = \frac{\rho g I_{y_c} \sin\alpha}{F}$$

$$I_{y_c z_c} = I_{yz} - A y_c z_c$$
$$= \frac{b^2 h^2}{4} - (bh)\left(\frac{b}{2}\right)\left(\frac{h}{2}\right)$$
$$= \frac{b^2 h^2}{4} - \frac{b^2 h^2}{4}$$
$$= 0$$
$$y^* = 0\ \text{m}$$
$$I_{y_c} = I_y - A y_c^2$$
$$= \frac{b^3 h}{3} - (bh)\left(\frac{b}{2}\right)^2$$
$$= \frac{b^3 h}{3} - \frac{b^3 h}{4}$$
$$= \frac{b^3 h}{12}$$

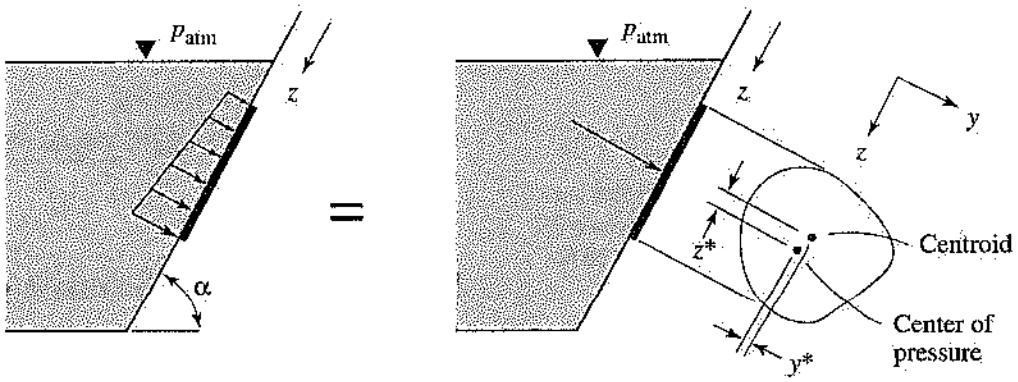


Figure 7.8 Distributed pressure force and equivalent point force on a submerged surface.

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$$= \frac{(0.20\ \text{m})^3(0.12\ \text{m})}{12}$$

$$= 8 \times 10^{-5}\ \text{m}^4$$

$$z^* = \frac{\left(1{,}000\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(8 \times 10^{-5}\ \text{m}^4)\sin 60}{36.06\ \text{N}} \times \left(\frac{\text{N}}{\frac{\text{kg} \cdot \text{m}}{\text{s}^2}}\right)$$

$$= 0.0188\ \text{m}$$

$$= 1.88\ \text{cm}$$

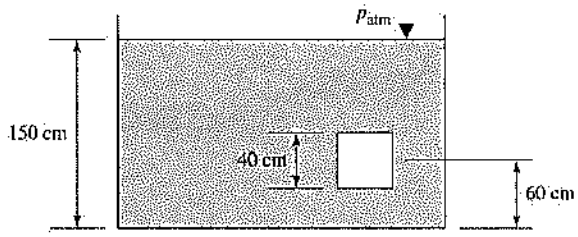
Archimedes' Principle and Buoyancy

Any object that is completely or partially submerged will experience a buoyant force. This is a force directed in a direction opposite to the weight of the object, which is the result of the total pressure force of the fluid acting on the submerged body.

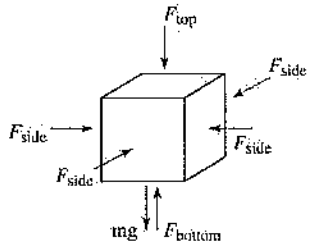
According to **Archimedes' principle,** the magnitude of the buoyant force is equal to the weight of the fluid displaced by the object. If the object is floating on the surface of a fluid, it will displace a volume of the fluid that has the same weight as the object.

Example 7.8

A 25-kg cube is submerged in water ($\rho = 1{,}000\ \text{kg/m}^3$), as shown in part (a) of the accompanying figure. What is the magnitude of the buoyant force acting on the cube?



(a)



(b)

Solution:

From the free-body diagram in part (b) of the figure, it can be seen that the force acting on each side of the cube is counteracted by the force on the opposite side; therefore the total force acting on the cube must be directed vertically. Take forces directed up as positive; then the total resultant force is given by the equation

$$F_{total} = F_{bottom} - F_{top} - mg$$

Calculate the force on each face of the cube by multiplying the hydrostatic pressure on that face by the area of the face:

$$F_{bottom} = A_{bottom}(\rho g h)_{water}$$

$$= (40\ \text{cm})^2\left(\frac{\text{m}}{100\ \text{cm}}\right)^2\left[\left(1{,}000\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right) \times (110\ \text{cm})\left(\frac{\text{m}}{100\ \text{cm}}\right)\left(\frac{\text{N}}{\frac{\text{kg} \cdot \text{m}}{\text{s}^2}}\right)\right]$$

$$= 1{,}727\ \text{N}$$

$$F_{top} = A_{top}(\rho g h)_{water}$$

$$= (40\ \text{cm})^2\left(\frac{\text{m}}{100\ \text{cm}}\right)^2\left[\left(1{,}000\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right) \times (70\ \text{cm})\left(\frac{\text{m}}{100\ \text{cm}}\right)\left(\frac{\text{N}}{\frac{\text{kg} \cdot \text{m}}{\text{s}^2}}\right)\right]$$

$$= 1{,}099\ \text{N}$$

The weight of the cube is

$$mg = (25\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\left(\frac{\text{N}}{\frac{\text{kg} \cdot \text{m}}{\text{s}^2}}\right)$$

$$= 245\ \text{N}$$

The total force is

$$F_{total} = F_{bottom} - F_{top} - mg$$

$$= 1{,}727\ \text{N} - 1{,}099\ \text{N} - 245\ \text{N}$$

$$= 383\ \text{N}$$

As an alternative to the approach shown above, Archimedes' principle may be used to solve the problem. The cube will experience a buoyant force equal to the weight of the water it displaces:

$$F_{buoyant} = \rho_{water} \forall_{cube} g$$

$$= \left(1{,}000\ \frac{\text{kg}}{\text{m}^3}\right)(0.40\ \text{m})^3\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\left(\frac{\text{N}}{\frac{\text{kg} \cdot \text{m}}{\text{s}^2}}\right)$$

$$= 628\ \text{N}$$

The weight of the cube will counteract the buoyant force, so the total resultant force acting on the cube is

$$F_{total} = F_{buoyant} - mg$$

$$= 628\ \text{N} - 245\ \text{N}$$

$$= 383\ \text{N}$$

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7.4 Fluid Dynamics

Fluid dynamics is the study of fluid in motion and the interactions of the fluid with the objects it may encounter. The basic principles of mass and energy conservation and Newton's second law of motion are applied to fluid systems to allow prediction of the flow characteristics.

Conservation of Mass (the Continuity Equation)

In every fluid system, mass is conserved. This conservation can be stated in terms of a mass balance equation:

$$\left(\frac{dm}{dt}\right)_{cv} = \dot{m}_{in} - \dot{m}_{out} \qquad [7.39]$$

A common class of fluid problem is the steady-state problem, in which the values of all of the fluid properties within the control volume do not change with time. For a steady-state control volume:

$$\left(\frac{dm}{dt}\right)_{cv} = 0 \qquad [7.40]$$

so that

$$\dot{m}_{in} = \dot{m}_{out} \qquad [7.41]$$

For the arbitrary control volume shown in Figure 7.9, conservation of mass is expressed as

$$\dot{m}_1 = \dot{m}_2 \qquad [7.42]$$

where $\dot{m}_1$ = mass flow rate through cross-sectional area of control volume at point 1,
$\dot{m}_2$ = mass flow rate through cross-sectional area of control volume at point 2.

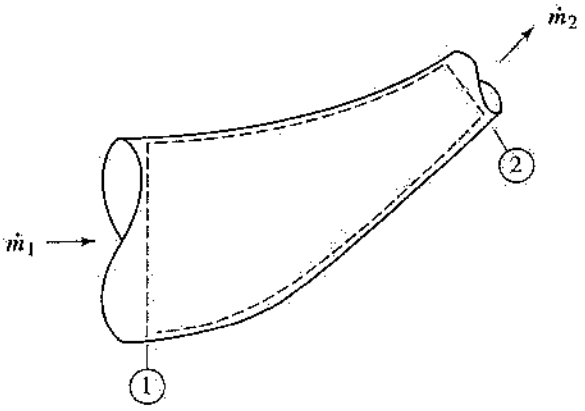


Figure 7.9 Conservation of mass in an arbitrary control volume.

Equivalent expressions of equation 7.42 are as follows:

$$\rho_1 A_1 \bar{V}_1 = \rho_2 A_2 \bar{V}_2 \qquad [7.43]$$

and

$$\rho_1 Q_1 = \rho_2 Q_2 \qquad [7.44]$$

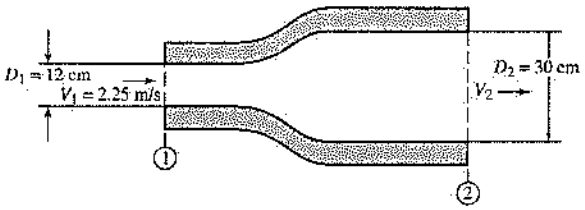
where A_1 = cross-sectional area of control volume at point 1,
A_2 = cross-sectional area of control volume at point 2,
$\bar{V}_1$ = average fluid speed at point 1,
$\bar{V}_2$ = average fluid speed at point 2,
Q_1 = volumetric flow rate of fluid entering control volume at point 1,
Q_2 = volumetric flow rate of fluid entering control volume at point 2.

If the flow is incompressible, the value of density is constant, and conservation of mass, or **continuity equation,** can be expressed as:

$$Q = A_1 \bar{V}_1 = A_2 \bar{V}_2 \qquad [7.45]$$

Example 7.9

What is the average speed at section 2 for the diffuser shown in the accompanying figure? (Assume that the flow is incompressible.)



Solution:

From equation 7.40:

$$A_1 \bar{V}_1 = A_2 \bar{V}_2$$

$$\bar{V}_2 = \frac{A_1}{A_2}\bar{V}_1 = \frac{D_1^2}{D_2^2}\bar{V}_1$$

$$= \frac{(0.12\ \text{m})^2}{(0.30\ \text{m})^2}\left(2.25\ \frac{\text{m}}{\text{s}}\right)$$

$$= 0.36\ \text{m/s}$$

Impulse-Momentum Principle

Newton's second law of motion ($\mathbf{F} = m\mathbf{a}$) can be applied to fluid systems through a principle known as

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the **impulse-momentum principle.** This principle is expressed mathematically for a control volume as follows:

$$\Sigma\left(\frac{d\mathbf{P}}{dt}\right)_{cv} = \Sigma\mathbf{F} \qquad [7.46]$$

where $\frac{d\mathbf{P}}{dt}$ = rate of momentum entering or leaving the control volume = $\dot{m}\mathbf{V}$,
$\Sigma\mathbf{F}$ = total external force acting on the control volume.

For a steady-state system, the change in momentum of a control volume can be expressed as

$$\left(\frac{d\mathbf{P}}{dt}\right)_{cv} = \left(\frac{d\mathbf{P}}{dt}\right)_{in} - \left(\frac{d\mathbf{P}}{dt}\right)_{out} = (\dot{m}\mathbf{V})_{in} - (\dot{m}\mathbf{V})_{out} \qquad [7.47]$$

so that

$$\Sigma\mathbf{F} = (\dot{m}\mathbf{V})_{in} - (\dot{m}\mathbf{V})_{out} \qquad [7.48]$$

The impulse-momentum principle can be applied to the case of a pipe bend, enlargement, or contraction to find the total force exerted on a fluid by such a device.

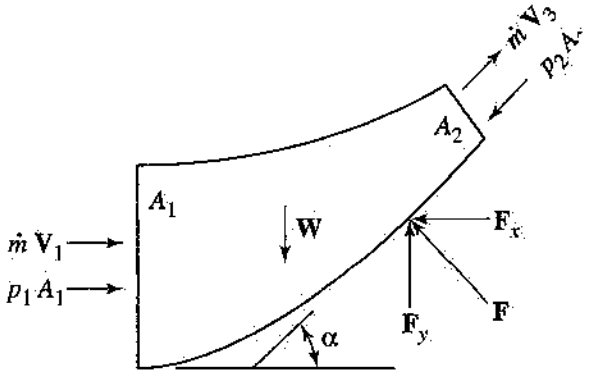


Figure 7.10 Free-body diagram of fluid in a general pipe bend, enlargement, or contraction.

Equation 7.46 can be applied to the control volume described in Figure 7.10 to produce these equations:

$$|\mathbf{F}_x| = (\dot{m}\bar{V}_1 + p_1A_1) + (\dot{m}\bar{V}_2 - p_2A_2)\cos\alpha \qquad [7.49]$$

$$|\mathbf{F}_y| = \mathbf{W} - (\dot{m}\bar{V}_2 - p_2A_2)\sin\alpha \qquad [7.50]$$

Example 7.10

For the control volume shown in Figure 7.10 with the parameters listed below, find the x- and y-components of the resultant force on the control volume.

$\rho = 1{,}000$ kg/m^3 $\quad \bar{V}_1 = 4$ m/s
$\forall_{cv} = 1.25$ m^3 $\quad p_1 = 12$ kPa $\quad p_2 = 10$ kPa
$\alpha = 40°$ $\quad A_1 = 0.50$ m^2 $\quad A_2 = 0.30$ m^2

Solution:

From equation 7.49:

$$|\mathbf{F}_x| = (\dot{m}\bar{V}_1 + p_1A_1) + (\dot{m}\bar{V}_2 - p_2A_2)\cos\alpha$$

$$\dot{m} = \rho\bar{V}_1A_1 = \left(1{,}000\ \frac{\text{kg}}{\text{m}^3}\right)\left(4\ \frac{\text{m}}{\text{s}}\right)(0.50\ \text{m}^2) = 2{,}000\ \text{kg/s}$$

$$\bar{V}_2 = \frac{\bar{V}_1A_1}{A_2} = \frac{\left(4\frac{\text{m}}{\text{s}}\right)(0.50\ \text{m}^2)}{0.30\ \text{m}^2} = 6.67\ \text{m/s}$$

$$|\mathbf{F}_x| = \left[\left(2{,}000\ \frac{\text{kg}}{\text{s}}\right)\left(4\ \frac{\text{m}}{\text{s}}\right)\left(\frac{\text{N}}{\frac{\text{kg}\cdot\text{m}}{\text{s}^2}}\right) + (12{,}000\ \text{Pa})(0.50\ \text{m}^2)\left(\frac{\text{N}}{\frac{\text{m}^2}{\text{Pa}}}\right)\right] + \left[\left(2{,}000\ \frac{\text{kg}}{\text{s}}\right)\left(6.67\ \frac{\text{m}}{\text{s}}\right)\left(\frac{\text{N}}{\frac{\text{kg}\cdot\text{m}}{\text{s}^2}}\right) - (10{,}000\ \text{Pa})(0.30\ \text{m}^2)\left(\frac{\text{N}}{\frac{\text{m}^2}{\text{Pa}}}\right)\right]\cos 40$$

$$= (8{,}000\ \text{N} + 6{,}000\ \text{N}) + (13{,}333\ \text{N} - 3{,}000\ \text{N})\cos 40 = 21{,}915\ \text{N}$$

From equation 7.50:

$$|\mathbf{F}_y| = |\mathbf{W}| - (\dot{m}\bar{V}_2 - p_2A_2)\sin\alpha$$

$$|\mathbf{W}| = \rho\forall_{cv}g = \left(1{,}000\ \frac{\text{kg}}{\text{m}^3}\right)(1.25\ \text{m}^3)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\left(\frac{\text{N}}{\frac{\text{kg}\cdot\text{m}}{\text{s}^2}}\right) = 12{,}263\ \text{N}$$

$$|\mathbf{F}_y| = 12{,}263\ \text{N} - (13{,}333\ \text{N} - 3{,}000\ \text{N})\sin 40 = 5{,}621\ \text{N}$$

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Conservation of Energy (the Bernoulli or Field Equation)

Where there is continuous flow, the idea of conservation of energy can be expressed using the Bernoulli equation. The **Bernoulli equation,** known also as the **field equation,** is an expression of the conservation of energy, neglecting frictional losses. It applies to changes in energy along the path of a fluid particle. The Bernoulli equation is expressed as:

$$\frac{p_1}{\gamma} + \frac{\bar{V}_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{\bar{V}_2^2}{2g} + z_2 \quad [7.51]$$

Any real flow will experience frictional losses, which are not accounted for in the Bernoulli equation. Equation 7.51 can be modified to include the effect of frictional losses as follows:

$$\frac{p_1}{\gamma} + \frac{\bar{V}_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{\bar{V}_2^2}{2g} + z_2 + h_f \quad [7.52]$$

Here, h_f is the major head loss, and its value is determined using empirical relationships. For flow in a horizontal pipe with constant cross-sectional area, equation 7.52 reduces to

$$h_f = \frac{p_1 - p_2}{\gamma} \quad [7.53]$$

Example 7.11

For the system described in Example 7.10, find the change in pressure between sections 1 and 2 using the Bernoulli equation. (Assume that frictional losses are negligible.)

Solution:

From equation 7.51:

$$\frac{p_1}{\rho} + \frac{\bar{V}_1^2}{2} = \frac{p_2}{\rho} + \frac{\bar{V}_2^2}{2}$$

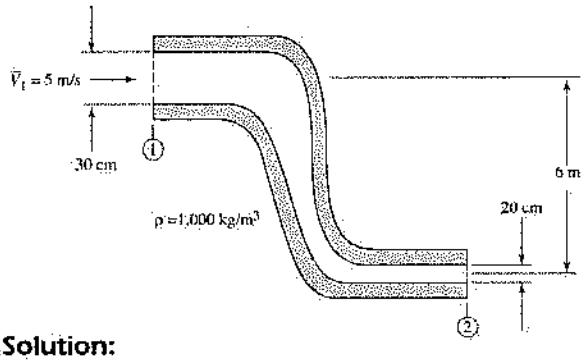
$$p_2 - p_1 = \frac{(\bar{V}_1^2 - \bar{V}_2^2)\rho}{2}$$

$$= \frac{\left[\left(2.25\frac{\text{m}}{\text{s}}\right)^2 - \left(0.36\frac{\text{m}}{\text{s}}\right)^2\right]\left(1{,}000\frac{\text{kg}}{\text{m}^3}\right)}{2} \times \left(\frac{\text{N}}{\frac{\text{kg}\cdot\text{m}}{\text{s}^2}}\right)\left(\frac{\text{kPa}}{1{,}000\frac{\text{N}}{\text{m}^2}}\right)$$

$$= 2.47 \text{ kPa}$$

Example 7.12

Water enters the control volume shown in the accompanying figure with a speed of 5 m/s. Assuming that the flow is incompressible and that friction is negligible, find the change in pressure between sections 1 and 2.



Solution:

From equation 7.51:

$$\frac{p_1}{\rho g} + \frac{\bar{V}_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{\bar{V}_2^2}{2g} + z_2$$

$$p_2 - p_1 = \rho g\left[\left(\frac{\bar{V}_1^2 - \bar{V}_2^2}{2g}\right) + (z_1 - z_2)\right]$$

$$= \frac{\rho(\bar{V}_1^2 - \bar{V}_2^2)}{2} + \rho g(z_1 - z_2)$$

Calculate $\bar{V}_2$ using the continuity equation (as expressed in equation 7.45):

$$A_1\bar{V}_1 = A_2\bar{V}_2$$

$$\bar{V}_2 = \frac{A_1}{A_2}\bar{V}_1 = \frac{D_1^2}{D_2^2}\bar{V}_1$$

$$= \frac{(0.30 \text{ m})^2}{(0.20 \text{ m})^2}\left(5\frac{\text{m}}{\text{s}}\right)$$

$$= 11.25 \text{ m/s}$$

The change in pressure is:

$$p_2 - p_1 = \frac{\rho(\bar{V}_1^2 - \bar{V}_2^2)}{2} + \rho g(z_1 - z_2)$$

$$= \frac{\left(1{,}000\frac{\text{kg}}{\text{m}^3}\right)\left[\left(5\frac{\text{m}}{\text{s}}\right)^2 - \left(11.25\frac{\text{m}}{\text{s}}\right)^2\right]}{2} + \left(1{,}000\frac{\text{kg}}{\text{m}^3}\right)\left(9.81\frac{\text{m}}{\text{s}^2}\right)(6 \text{ m})$$

$$= \left(-50{,}781\frac{\text{kg}}{\text{m}\cdot\text{s}^2} + 41{,}880\frac{\text{kg}}{\text{m}\cdot\text{s}^2}\right) \times \left(\frac{\text{N}}{\frac{\text{kg}\cdot\text{m}}{\text{s}^2}}\right)\left(\frac{\text{kPa}}{1{,}000\frac{\text{N}}{\text{m}^2}}\right)$$

$$= -8.90 \text{ kPa}$$

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Static Versus Stagnation Pressure

The pressure used in the Bernoulli equation is the **static pressure** of a fluid. It is the pressure that would be measured by a device that moves with the fluid. A stationary device that measures pressure in a moving fluid would read a different pressure, known as the **stagnation pressure.** A relationship between the static pressure and the stagnation pressure of a fluid can be derived from the Bernoulli equation:

$$\frac{p_0}{\rho} = \frac{p_s}{\rho} + \frac{V^2}{2} \qquad [7.54]$$

or

$$p_0 = p_s + \frac{1}{2}\rho V^2 \qquad [7.55]$$

where p_s = static pressure of the fluid,
p_0 = stagnation pressure of the fluid,
V = speed of the fluid at the point where the measurement is taken.

Grade Line (Hydraulic Gradient) and Energy Line

If a pipe is fitted with a series of open manometers along its length, the static fluid pressure at each location can be visualized. With flowing fluid, the pressure at each successive location will be lower, as shown in Figure 7.11. If an imaginary line is drawn through the meniscus of each manometer, that line is the **grade line** or **hydraulic gradient.**

Alternatively, the hydraulic gradient can be thought of as a plot of static pressure as a function of distance along the pipe. If the stagnation pressure rather than the static pressure of a fluid flowing through a pipe is plotted as a function of distance along the pipe, the line that results is the **energy line.**

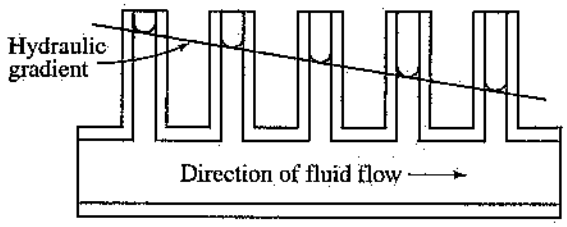


Figure 7.11 Description of the hydraulic gradient.

Reynolds Number

The **Reynolds number** (Re) is a nondimensional number used to characterize fluid flow. It is a ratio of the inertial forces to viscous forces acting on the fluid and is defined as follows:

$$\mathrm{Re} = \frac{\rho \bar{V} D}{\mu} \qquad [7.56]$$

or

$$\mathrm{Re} = \frac{\bar{V} D}{\nu} \qquad [7.57]$$

where ν = kinematic viscosity, defined as μ/ρ.

The parameter D used in equations 7.56 and 7.57 is a characteristic length. For flow in a tube the characteristic length is the pipe diameter. For flow over a flat plate, the characteristic length is the length of the plate in the direction of flow.

For a power-law non-Newtonian fluid, the Reynolds number is defined as

$$\mathrm{Re}' = \frac{\rho \bar{V}^{2-n} D^n}{K\left(\frac{3n+1}{4n}\right)^n 8^{n-1}} \qquad [7.58]$$

where K = consistency index,
n = power law index: $n < 1$ for a pseudoplastic fluid, $n > 1$ for a dilatant fluid.

Example 7.13

Find the Reynolds number for the water exiting the control volume of Example 7.12. Assume that the viscosity of water is 8×10^{-3} N · s/m^2.

Solution:

From equation 7.56:

$$\mathrm{Re} = \frac{\rho \bar{V} D}{\mu} = \frac{\left(1{,}000 \frac{\mathrm{kg}}{\mathrm{m}^3}\right)\left(11.25 \frac{\mathrm{m}}{\mathrm{s}}\right)(0.20\ \mathrm{m})}{8 \times 10^{-3} \frac{\mathrm{N \cdot s}}{\mathrm{m}^2}}$$

$$= 281{,}250$$

Laminar Flow

As fluid flow begins from rest, the flow is highly organized. All streamlines are smooth and continuous, and all fluid particles move in a path parallel to the mean bulk flow. This type of flow, called **laminar flow,** generally occurs in fluid flows with low Reynolds numbers, or in fluids with high viscosity and/or low density flowing at low speed. In laminar flow, the fluid flow field can be thought of as a collection of

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fluid layers. In each layer, all fluid particles are moving with the same velocity, and there is no macroscopic mixing of fluid layers.

Turbulent Flow

As the flow rate is increased, the general character of the flow begins to change significantly. Random, three-dimensional fluctuations in flow velocity will begin to appear, superimposed over the local mean flow velocity. This type of flow, called **turbulent flow,** results in significant fluid mixing.

The lowest Reynolds number for which turbulent flow can be expected is called the *critical Reynolds number* (Re_c). For flow in a pipe, the value of Re_c is approximately 2,300. For pipe flow at Reynolds numbers greater than approximately 4,000, the flow is considered fully turbulent. The range of Reynolds numbers between these limits is the transitional region, where it is difficult to predict which type of flow will occur.

7.5 Internal Flow

In this section we consider fluid flowing through a closed pipe or duct. Experimental observation has shown that fluid particles adjacent to the walls of the pipe have zero velocity. This is called the *no-slip condition* (no fluid is slipping against the wall of the pipe). The maximum fluid velocity occurs along the centerline of the pipe.

The variation in fluid velocity is caused by the viscous forces within the fluid, or the shear force between fluid particles. The shear forces perform work against the moving fluid, reducing the fluid energy as it travels through the pipe. This loss of energy, often called *head loss* (h_f), is observed as a drop in fluid pressure in the pipe.

As fluid enters or leaves a pipe or other internal passage, the velocity profile in the passage, is a function of the fluid's location along the length of the passage and the geometry of the opening. Far from the entrance and exit, however, the velocity profile becomes uniform for any position along the length of the pipe, and independent of the entrance and exit geometry. Flow in this region is fully developed.

For fully developed laminar flow, illustrated in Figure 7.12, the velocity of a fluid particle in a pipe is a function of the radial location of that particle in the pipe and of the pressure drop through the tube ($\Delta p/L$). Some relationships for flow in a pipe are as follows:

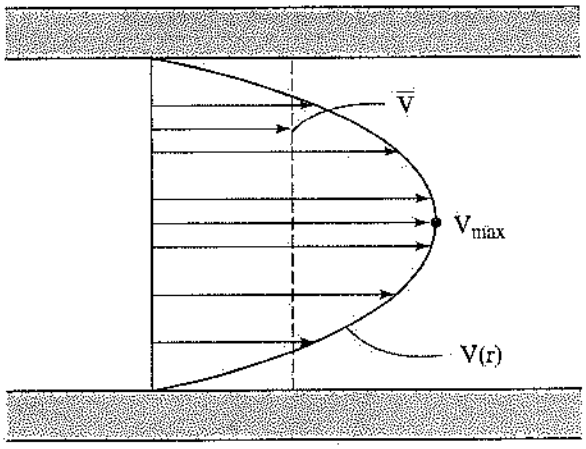


Figure 7.12 Fully developed laminar-flow velocity profile for flow in a pipe.

- Speed distribution:

$$V(r) = -\frac{R^2}{4\mu}\frac{\Delta p}{L}\left[1 - \left(\frac{r}{R}\right)^2\right] \qquad [7.59]$$

- Average speed:

$$\bar{V} = \frac{Q}{A} = -\frac{R^2}{8\mu}\frac{\Delta p}{L} \qquad [7.60]$$

- Maximum speed:

$$V_{max} = -\frac{R^2}{4\mu}\frac{\Delta p}{L} = 2\bar{V} \qquad [7.61]$$

- Volumetric flow rate:

$$Q = -\frac{\pi R^4}{8\mu}\frac{\Delta p}{L} \qquad [7.62]$$

- Shear stress distribution:

$$\tau(r) = \frac{r}{2}\frac{\Delta p}{L} \qquad [7.63]$$

and

$$\frac{\tau(r)}{\tau_w} = \frac{r}{R} \qquad [7.64]$$

where r = radial coordinate of a coordinate system with its origin on the centerline of the pipe,
R = inside radius of the pipe.

Darcy Equation

To estimate the frictional losses for flow in a horizontal pipe with a constant cross-sectional area, the Bernoulli equation (equation 7.51) reduces to

$$p_1 - p_2 = \rho g h_f = \gamma h_f \qquad [7.65]$$

The frictional head loss is estimated using the **Darcy equation:**

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$$h_f = f\frac{L}{D}\frac{\overline{V}^2}{2g} \qquad [7.66]$$

For laminar flow:

$$f = \frac{64}{\text{Re}}$$

The friction factor (f) for transitional or turbulent flow can be found in the Moody or Stanton diagram, a copy of which is included at the end of this chapter. Another parameter required for use of the Moody diagram is the relative roughness (e/D). The roughness (e) of a pipe is a function of the material and the process by which the pipe is manufactured. Some examples of roughness values are given in Table 7.1.

Table 7.1 Roughness Values for Selected Materials

Material	*e* (ft)	*e* (mm)
Riveted steel	0.003–0.03	0.9–9.0
Concrete	0.001–0.01	0.3–3.0
Cast iron	0.00085	0.25
Galvanized iron	0.0005	0.15
Galvanized steel or wrought iron	0.000015	0.046
Drawn tubing	0.000005	0.0015

Example 7.14

Using the Darcy equation, calculate the head loss in a horizontal pipe of constant cross-sectional area if the inside diameter of the pipe is 6 in and its length is 200 ft. Water with a density of 62.4 lbm/ft^3 and a viscosity of 1.67×10^{-5} lbf-sec/ft^2 flows into the pipe at a speed of 2 ft/sec. The roughness of the pipe is 0.01 in. (Assume that the flow is incompressible.)

Solution:

To use the Darcy equation, first calculate the Reynolds number and relative roughness.

$$\text{Re} = \frac{\rho \overline{V} D}{\mu}$$

$$= \frac{\left(62.4\ \frac{\text{lbm}}{\text{ft}^3}\right)\left(2\ \frac{\text{ft}}{\text{sec}}\right)(6\text{ in})\left(\frac{\text{ft}}{12\text{ in}}\right)}{1.67 \times 10^{-5}\ \frac{\text{lbf-sec}}{\text{ft}^2}} \times \left(\frac{\text{lbf}}{32.17\ \frac{\text{lbm-ft}}{\text{sec}^2}}\right)$$

$$= 9{,}670$$

$$\frac{e}{D} = \frac{0.01\text{ in}}{6\text{ in}} = 0.00167$$

Use the Moody diagram to determine that the friction factor is 0.024. Now use the Darcy equation to find the head loss.

$$h_f = f \cdot \frac{L}{D} \cdot \frac{\overline{V}^2}{2g}$$

$$= 0.024\left[\frac{200\text{ ft}}{6\text{ in}\left(\frac{\text{ft}}{12\text{ in}}\right)}\right]\left[\frac{\left(2\ \frac{\text{ft}}{\text{sec}}\right)^2}{2\left(32.17\ \frac{\text{ft}}{\text{sec}^2}\right)}\right]$$

$$= 0.596\text{ ft}$$

Hagen-Poiseuille Equation

Pressure loss for laminar flow in a circular pipe can be related to volumetric flow rate by using the **Hagen-Poiseuille equation:**

$$Q = \frac{\pi R^4 \Delta p_f}{8\mu L} \qquad [7.67]$$

or

$$Q = \frac{\pi D^4 \Delta p_f}{128\mu L} \qquad [7.68]$$

where Q = volumetric flow rate (ft^3/sec or m^3/s),
R = inside radius of the pipe,
D = inside diameter of the pipe,
Δp_f = pressure drop in the pipe due to frictional losses.

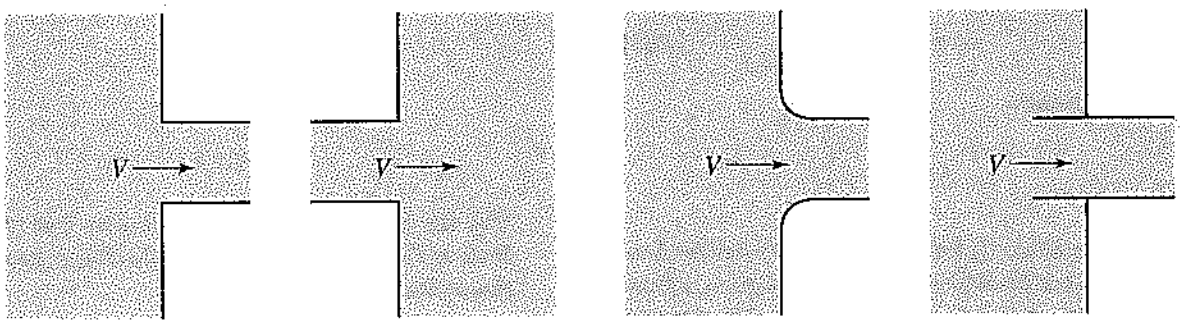
Head Losses in Pipe Bends, Enlargements, Contractions, Entrances, and Exits

Minor head losses (h_{fm}) occur any time that the size or direction of a pipe changes. The value of the minor head loss can be found by using the following equation:

$$h_{fm} = C\frac{\overline{V}^2}{2g} \qquad [7.69]$$

The loss coefficient (C) is determined by the geometry of the change and can vary greatly. A table is commonly used to determine the loss coefficient; Figure 7.13 gives the loss coefficients for some types of entrances and exits.

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(a) Sharp entrance: $C = 0.5$ (b) Sharp exit: $C = 1.0$ (c) Rounded entrance: $C = 0.1$ (d) Protruding-pipe entrance: $C = 0.8$

Figure 7.13 Loss coefficients for several types of pipe entrances and exits.

The Bernoulli equation is modified as follows to determine the change in energy of a fluid for flow through a pipe, including both major and minor head losses:

$$\frac{p_1}{\gamma} + \frac{\overline{V}_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{\overline{V}_2^2}{2g} + z_2 + h_f + h_{fm} \quad [7.70]$$

Example 7.15

If an elbow with loss coefficient $C = 0.75$ is placed at the end of the pipe of Example 7.14, what is the minor head loss through the elbow?

Solution:

From equation 7.69:

$$h_{fm} = C\frac{\overline{V}^2}{2g}$$

$$= 0.75\left[\frac{\left(2\frac{\text{ft}}{\text{sec}}\right)^2}{2\left(32.17\frac{\text{ft}}{\text{sec}^2}\right)}\right]$$

$$= 0.047 \text{ ft}$$

Multiple-Path Pipeline

A common technique used to increase the capacity of a pipe is to add another pipe in parallel.

It is possible to predict how much fluid will flow through each path. The fluid pressures before and after the branch must be equal. Therefore, the flow will divide itself in such a way that the head losses through each path will be equal.

For the multiple-path pipeline shown in Figure 7.14:

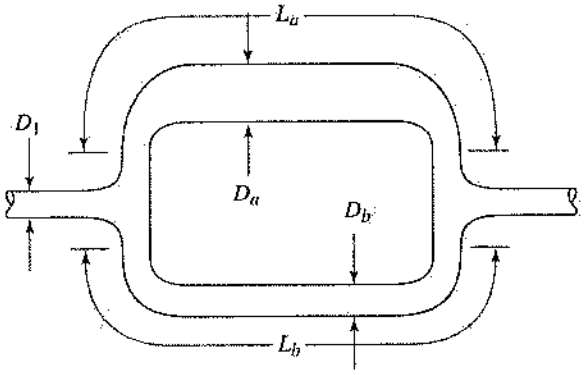


Figure 7.14 Diagram of a multiple-path pipeline.

$$h_{fa} = h_{fb} \quad [7.71]$$

$$f_a\frac{L_a}{D_a}\frac{\overline{V}_a^2}{2g} = f_b\frac{L_b}{D_b}\frac{\overline{V}_b^2}{2g} \quad [7.72]$$

From the continuity equation (equation 7.45):

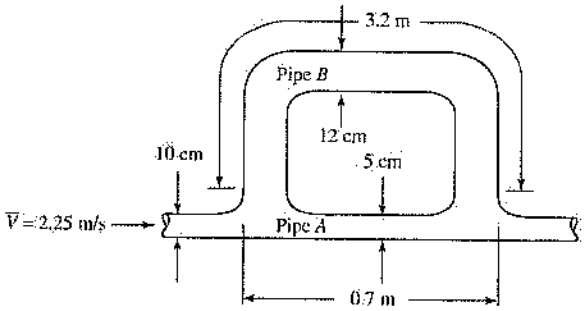
$$\frac{\pi}{4}D_1^2\overline{V}_1 = \frac{\pi}{4}D_a^2\overline{V}_a + \frac{\pi}{4}D_b^2\overline{V}_b \quad [7.73]$$

When the flow rate of the fluid entering the control volume and the diameters and lengths of the pipes are known, equations 7.72 and 7.73 can be solved simultaneously to determine $\overline{V}_a$ and $\overline{V}_b$.

Example 7.16

For the multiple-path pipeline shown in the accompanying figure, what is the average speed through pipe A ($\overline{V}_A$)? (Assume that the friction factor in pipe A is equal to the friction factor in pipe B.)

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Solution:

From equation 7.72:

$$f_A \frac{L_A}{D_A}\frac{\overline{V}_A^2}{2g} = f_B \frac{L_B}{D_B}\frac{\overline{V}_B^2}{2g}$$

$$\frac{\overline{V}_A}{\overline{V}_B} = \sqrt{\frac{D_A}{D_B}\frac{L_B}{L_A}}$$

$$= \sqrt{\frac{5\text{ cm}}{12\text{ cm}}\frac{3.2\text{ m}}{0.7\text{ m}}}$$

$$= 1.38$$

From equation 7.73:

$$\overline{V}A = \overline{V}_A A_A + \overline{V}_B A_B$$

$$= \overline{V}_A A_A + \frac{\overline{V}_A}{1.38}A_B$$

$$= \overline{V}_A\left(A_A + \frac{A_B}{1.38}\right)$$

$$\overline{V}_A = \overline{V}\left(\frac{\frac{\pi D^2}{4}}{\frac{\pi D_A^2}{4} + \frac{\pi D_B^2}{4(1.38)}}\right)$$

$$= \overline{V}\left(\frac{D^2}{D_A^2 + \frac{D_B^2}{1.38}}\right)$$

$$= 2.25\frac{\text{m}}{\text{s}}\left[\frac{(10\text{ cm})^2}{(5\text{ cm})^2 + \frac{(12\text{ cm})^2}{1.38}}\right]$$

$$= 1.74\text{ m/s}$$

Flow in Noncircular Ducts

Flow in noncircular ducts can be analyzed using the same techniques used for round ducts. All that is needed is to replace the tube diameter with a new parameter called the *hydraulic radius* (R_h) or *hydraulic diameter* (D_h). The equations for this parameter are as follows:

$$R_h = \frac{\text{cross-sectional area}}{\text{wetted perimeter}} \quad [7.74]$$

$$D_h = 4\left(\frac{\text{cross-sectional area}}{\text{wetted perimeter}}\right) = 4R_h \quad [7.75]$$

For instance, the Reynolds number for flow in a noncircular duct is expressed as:

$$\text{Re}_{D_h} = \frac{\rho \overline{V} D_h}{\mu} \quad [7.76]$$

Example 7.17

Find the hydraulic diameter of a rectangular duct 24 in wide and 15 in high.

Solution:

$$D_h = 4\left(\frac{\text{cross-sectional area}}{\text{wetted perimeter}}\right)$$

$$= 4\left[\frac{wh}{2(w+h)}\right]$$

$$= 4\left[\frac{(24\text{ in})(15\text{ in})}{2(24\text{ in} + 15\text{ in})}\right]$$

$$= 18.46\text{ in}$$

Open-Channel Flow

The relationships developed until now have been for flow inside a closed pipe. Most of these, such as the Darcy friction factor, have been determined through experimentation. As the basic geometry of the fluid system to be analyzed changes, we can expect to need new empirical relationships. Two such relationships for open-channel flow are Manning's equation and the Hazen-Williams equation.

Manning's equation:

$$\overline{V} = \frac{1}{n}R^{2/3}S^{1/2} \quad [7.77]$$

Hazen-Williams equation:

$$\overline{V} = 0.849CR^{0.63}S^{0.54} \quad [7.78]$$

where n = Manning roughness coefficient,
C = Hazen-Williams roughness coefficent,
R = hydraulic radius of the channel (ft or m),
S = slope of the energy grade line (ft/ft or m/m).

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7.6 External Flow

Often important in external flow analysis is the total drag force acting on the object of study. This drag is measured using a nondimensional coefficient of drag (C_D), which is defined as follows:

$$C_D = \frac{F_D}{\frac{1}{2}\rho V_\infty^2 A} \quad [7.79]$$

where C_D = nondimensional coefficient of drag,
F_D = total drag force exerted on the object by the fluid,
ρ = density of the fluid,
V_∞ = speed of the fluid far from the object,
A = area over which the drag force is acting.

If the coefficient of drag is given, the drag force can be calculated using this equation:

$$F_D = \frac{C_D \rho V_\infty^2 A}{2} \quad [7.80]$$

Uniform Flow over a Flat Plate

Flow over a flat plate demonstrates a concept important to the study of fluid dynamics, the boundary layer. As the fluid flow encounters the first edge of the plate, the vertical flow distribution is uniform, as shown in Figure 7.15.

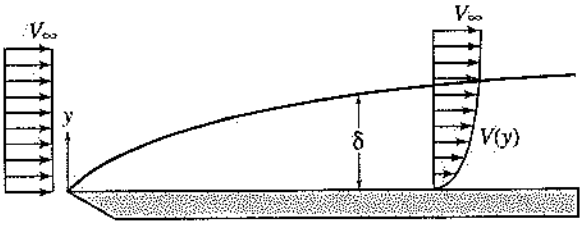


Figure 7.15 Flow over a flat plate.

As the flow progresses along the plate, the vertical distance above the plate at which fluid with a speed of V_∞ is first encountered increases. This distance is called the boundary layer thickness (δ). In the boundary layer, the fluid velocity and the shear stress in the fluid vary with distance above the surface of the plate.

The characteristic length used for uniform flow over a flat plate is the length of the plate. The Reynolds number for flow over a flat plate is expressed as follows:

$$\mathrm{Re}_L = \frac{\rho V_\infty L}{\mu} \quad [7.81]$$

The coefficient of drag for flow over a flat plate is given by these equations:

$$C_D = \frac{1.33}{\mathrm{Re}_L^{0.5}} \quad (10^4 < \mathrm{Re}_L < 5 \times 10^5)$$

and [7.82]

$$C_D = \frac{0.031}{\mathrm{Re}_L^{1/7}} \quad (10^6 < \mathrm{Re}_L < 10^9)$$

Example 7.18

A square plate with 50 cm on a side is placed in an airstream with a speed of 12 m/s.

A. What is the coefficient of drag for the plate?
B. What is the total drag force on the plate? (Assume that the density of air is 1.225 kg/m^3, and that the viscosity is 2×10^{-5} N · s/m^2.)

Solution:

A. From equation 7.81:

$$\mathrm{Re}_L = \frac{\rho V_\infty L}{\mu} = \frac{\left(1.225\frac{\mathrm{kg}}{\mathrm{m}^3}\right)\left(12\frac{\mathrm{m}}{\mathrm{s}}\right)(0.50\ \mathrm{m})}{2 \times 10^{-5}\frac{\mathrm{N \cdot s}}{\mathrm{m}^2}}\left(\frac{\mathrm{N}}{\frac{\mathrm{kg \cdot m}}{\mathrm{s}^2}}\right) = 3.67 \times 10^5$$

From equation 7.82:

$$C_D = \frac{1.33}{\mathrm{Re}_L^{0.5}} = \frac{1.33}{(3.67 \times 10^5)^{0.5}} = 2.19 \times 10^{-3}$$

B. The total force is given by equation 7.80:

$$F_D = \frac{C_D \rho V_\infty^2 A}{2} = \frac{(2.19 \times 10^{-3})\left(1.225\frac{\mathrm{kg}}{\mathrm{m}^3}\right)\left(12\frac{\mathrm{m}}{\mathrm{s}}\right)^2(0.50\ \mathrm{m})^2}{2} \times \left(\frac{\mathrm{N}}{\frac{\mathrm{kg \cdot m}}{\mathrm{s}^2}}\right) = 4.84 \times 10^{-2}\ \mathrm{N}$$

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7.7 Devices

The principles of fluid mechanics discussed so far can be used to describe the operation of many industrial devices. A few examples follow.

Deflector and Blades

Many devices utilize the kinetic energy of a high-velocity fluid to produce power, which is commonly transmitted to a series of curved surfaces or blades. The impulse-momentum principle is used to describe their operation. We will neglect the viscous drag of the fluid at the blade surface; therefore, the fluid leaving the trailing edge of the blade will have the same speed as the fluid approaching the leading edge of the blade.

Fixed Blade

Figure 7.16 shows a fluid jet acting on a stationary blade. Since the blade does not move, the direction of the fluid stream that leaves the blade is determined by the angle of the trailing edge of the blade. The blade's reaction force is related to the change in momentum of the fluid. It is assumed that the speed of the fluid stream leaving the blade is equal to the speed of the fluid stream entering the blade.

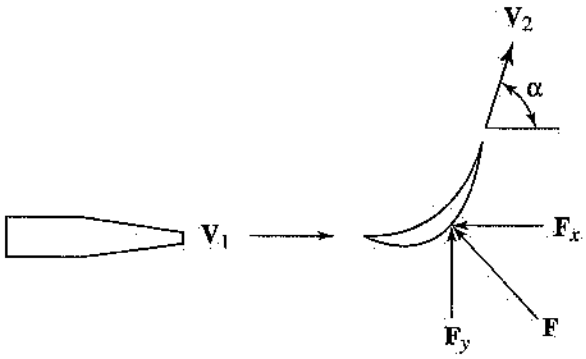


Figure 7.16 Fluid jet flowing over a stationary blade.

$$|\mathbf{V}_2| = |\mathbf{V}_1| \quad [7.83]$$

$$|\mathbf{F}_x| = \dot{m}(|\mathbf{V}_1| - |\mathbf{V}_2| \cdot \cos\alpha) \quad [7.84]$$

$$|\mathbf{F}_y| = \dot{m}|\mathbf{V}_2| \cdot \sin\alpha \quad [7.85]$$

where $|\mathbf{V}_1|$ = magnitude of the fluid velocity approaching the blade,
$|\mathbf{V}_2|$ = magnitude of the fluid velocity leaving the blade.

Example 7.19

Find the total reaction force on a fixed blade if 5 kg/s of water with a speed of 6 m/s enters the blade, and the blade angle α is 65°.

Solution:

From equation 7.84:

$$|\mathbf{F}_x| = \dot{m}(|\mathbf{V}_1| - |\mathbf{V}_2| \cdot \cos\alpha)$$

$$= \left(5\frac{\text{kg}}{\text{s}}\right)\left[6\frac{\text{m}}{\text{s}} - 6\frac{\text{m}}{\text{s}}(\cos 65)\right]\left(\frac{\text{N}}{\frac{\text{kg}\cdot\text{m}}{\text{s}^2}}\right)$$

$$= 17.32\ \text{N}$$

From equation 7.85:

$$|\mathbf{F}_y| = \dot{m}|\mathbf{V}_2| \cdot \sin\alpha$$

$$= \left(5\frac{\text{kg}}{\text{s}}\right)\left(6\frac{\text{m}}{\text{s}}\right)\sin 65\left(\frac{\text{N}}{\frac{\text{kg}\cdot\text{m}}{\text{s}^2}}\right)$$

$$= 27.19\ \text{N}$$

The magnitude of the reaction force is

$$|\mathbf{F}| = \sqrt{|\mathbf{F}_x|^2 + |\mathbf{F}_y|^2}$$

$$= \sqrt{(17.32\ \text{N})^2 + (27.19\ \text{N})^2}$$

$$= 32.23\ \text{N}$$

Moving Blade

When the blade is moving in the same direction as the fluid jet, as is the case in the rotating portions of turbomachinery, the velocity of the blade affects the change in fluid momentum that occurs. This situation is illustrated in Figure 7.17.

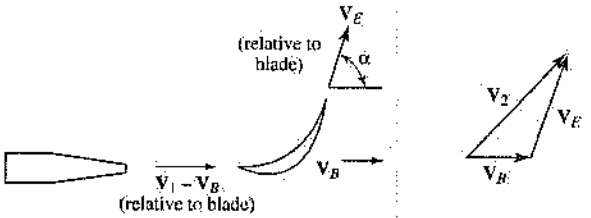


Figure 7.17 Fluid jet flowing over a moving blade.

The relationships between the velocities and the forces on the blade are given by the following equations:

$$\mathbf{V}_E = \mathbf{V}_1 - \mathbf{V}_B \quad [7.86]$$

$$\mathbf{V}_2 = \mathbf{V}_E + \mathbf{V}_B \quad [7.87]$$

$$|\mathbf{F}_x| = -\dot{m}(|\mathbf{V}_1| - |\mathbf{V}_B|)(1 - \cos\alpha) \quad [7.88]$$

$$|\mathbf{F}_y| = \dot{m}(|\mathbf{V}_1| - |\mathbf{V}_B|) \cdot \sin\alpha \quad [7.89]$$

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where $\mathbf{V}_1$ = velocity of the fluid jet approaching the blade,
$\mathbf{V}_B$ = velocity of blade,
$\mathbf{V}_E$ = velocity of fluid leaving the blade, relative to the blade,
$\mathbf{V}_2$ = true velocity of fluid leaving the blade.

Example 7.20

Find the total reaction force on a moving blade if 5 kg/s of water with a speed of 6 m/s enters the blade, the blade is moving at a speed of 1.4 m/s, and the blade angle α is 65°.

Solution:

From equation 7.88:

$$|\mathbf{F}_x| = \dot{m}(|\mathbf{V}_1| - |\mathbf{V}_B|)(1 - \cos\alpha)$$

$$= \left(5\frac{\text{kg}}{\text{s}}\right)\left(6\frac{\text{m}}{\text{s}} - 1.4\frac{\text{m}}{\text{s}}\right)(1 - \cos 65)\left(\frac{\text{N}}{\frac{\text{kg}\cdot\text{m}}{\text{s}^2}}\right)$$

$$= 13.28\ \text{N}$$

From equation 7.89:

$$|\mathbf{F}_y| = \dot{m}(|\mathbf{V}_1| - |\mathbf{V}_B|)(\sin\alpha)$$

$$= \left(5\frac{\text{kg}}{\text{s}}\right)\left(6\frac{\text{m}}{\text{s}} - 1.4\frac{\text{m}}{\text{s}}\right)(\sin 65)\left(\frac{\text{N}}{\frac{\text{kg}\cdot\text{m}}{\text{s}^2}}\right)$$

$$= 20.85\ \text{N}$$

The magnitude of the reaction force is:

$$|\mathbf{F}| = \sqrt{|\mathbf{F}_x|^2 + |\mathbf{F}_y|^2}$$

$$= \sqrt{(13.28\ \text{N})^2 + (20.85\ \text{N})^2}$$

$$= 24.72\ \text{N}$$

Impulse Turbine

An **impulse turbine,** shown schematically in Figure 7.18, is a disk with blades located radially around its edge. Fluid strikes the blades in a stream tangent to the turbine. The reaction force of the fluid against the blades begins to turn the turbine around its axis, and this motion can be used to generate power.

The amount of power generated is related to the angle of blade deflection (α). Maximum power is generated at $\alpha = 180°$. The turbine power is related to the mass flow rate of the fluid and to its velocity by the equation

$$\dot{W} = Q\rho(|\mathbf{V}_1 - \mathbf{V}_B|)(1 - \cos\alpha)|\mathbf{V}_B| \qquad [7.90]$$

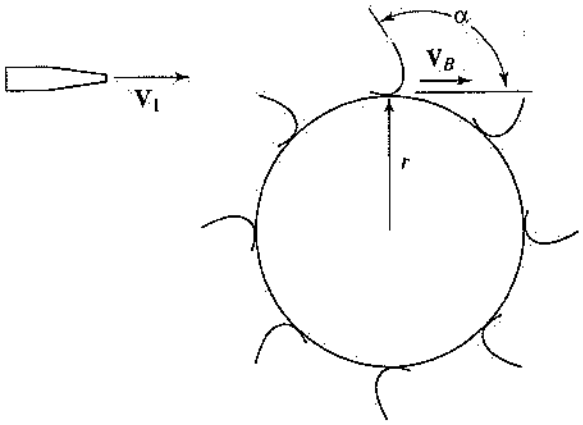


Figure 7.18 An impulse turbine.

Maximum power is expressed as

$$\dot{W}_{max} = Q\rho\left(\frac{|\mathbf{V}_1{}^2|}{4}\right)(1 - \cos\alpha) \qquad [7.91]$$

Or, when $\alpha = 180°$:

$$\dot{W}_{max} = \frac{Q\rho|\mathbf{V}_1|^2}{2} \qquad [7.92]$$

The speed of the blade is related to the rotational speed of the turbine by the equation

$$|\mathbf{V}_B| = r\omega \qquad [7.93]$$

where r = radius of the turbine (ft or m),
ω = rotational speed of the turbine (rad/s).

Example 7.21

Find the power of an impulse turbine if 25 kg/s of water leaves the nozzle with a speed of 65 m/s, and the turbine is spinning at 500 rpm. The radius of the turbine is 50 cm, and the blade angle α is 120°.

Solution:

The rotational speed of the turbine is

$$\omega = \left(500\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\ \text{rad}}{\text{rev}}\right)\left(\frac{\text{min}}{60\ \text{s}}\right)$$

$$= 52.36\ \text{rad/s}$$

From equation 7.93, the blade velocity is

$$|\mathbf{V}_B| = r\omega$$

$$= (0.50\ \text{m})\left(52.36\frac{\text{rad}}{\text{s}}\right)$$

$$= 26.18\ \text{m/s}$$

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From equation 7.90:

$$\dot{W} = \dot{m}(|\mathbf{V}_1 - \mathbf{V}_B|)(1 - \cos\alpha)|\mathbf{V}_B|$$

$$= \left(25\,\frac{\text{kg}}{\text{s}}\right)\left(65\,\frac{\text{m}}{\text{s}} - 26.18\,\frac{\text{m}}{\text{s}}\right)(1 - \cos 120)$$

$$\times \left(26.18\,\frac{\text{m}}{\text{s}}\right)\left(\frac{\text{N}}{\frac{\text{kg}\cdot\text{m}}{\text{s}^2}}\right)\left(\frac{\text{kW}}{1{,}000\,\frac{\text{N}\cdot\text{m}}{\text{s}}}\right)$$

$$= 38.11\ \text{kW}$$

Jet Propulsion

A fluid jet used for propulsion is an example of the practical use of the impulse-momentum principle. The momentum of the fluid jet leaving the control volume results in a force that acts on the control volume in a direction opposite to that of the fluid jet. In the system described in Figure 7.19, the potential energy of the fluid is converted to kinetic energy as the fluid flows out of the orifice. As the fluid is expelled, it leaves the system with a certain amount of momentum. As a result, a force is exerted on the control volume in the opposite direction to the flow out of the orifice. This force is given by the equation

$$F = Q\rho(\overline{V} - 0) \qquad [7.94]$$

or

$$F = \rho\overline{V}^2 A \qquad [7.95]$$

where F = force on the control volume,
$\overline{V}$ = average speed of the fluid through the nozzle,
A = cross-sectional area of the nozzle exit.

By applying the Bernoulli equation between the free surface and the outlet of this system, we calculate the force as follows:

$$\frac{p_1}{\gamma} + (z_2 + h) + \frac{0^2}{2g} = \frac{p_1}{\gamma} + z_2 + \frac{\overline{V}^2}{2g} \qquad [7.96]$$

$$\overline{V}^2 = 2gh \qquad [7.97]$$

$$F = 2\rho g h A \qquad [7.98]$$

Pumps

Pumps utilize mechanical energy to increase the energy of a fluid. The amount of energy that is added to the fluid per unit time is expressed as

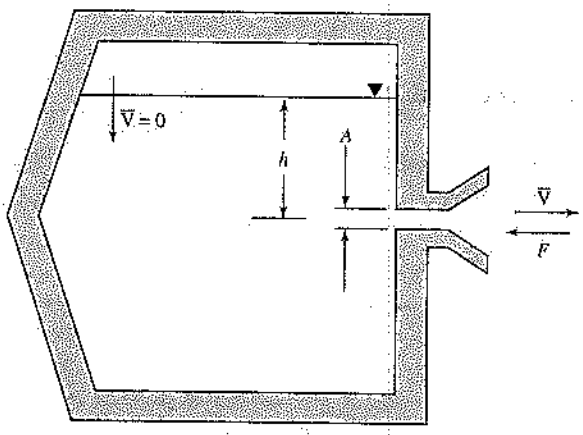


Figure 7.19 Jet of fluid exiting a container.

$$P = Q\gamma h \qquad [7.99]$$

where P = pump power (ft-lbf/sec or W),
Q = volumetric flow rate of fluid that is pumped (ft^3/sec or m^3/s),
h = change in fluid head (ft or m).

Because of inefficiencies in the pump, the amount of energy that must be supplied to the pump is somewhat greater than the amount of energy that is transferred to the fluid. The ratio of power transferred to the fluid to power supplied to the pump is the pump's efficiency (η). The amount of power that must be supplied to a pump can be found by using this equation:

$$P_{\text{req}} = \frac{Q\gamma h}{\eta} \qquad [7.100]$$

Example 7.22

A pump that is 63% efficient is required to lift 500 L/min of water from a stream into a tank that is 12 m above the surface of the stream. What is the minimum power input required for this pump?

Solution:

From equation 7.100:

$$P_{\text{req}} = \frac{Q\gamma h}{\eta}$$

$$= \frac{\left(500\,\frac{\text{L}}{\text{min}}\right)\left(9{,}810\,\frac{\text{N}}{\text{m}^3}\right)(12\ \text{m})}{0.63}$$

$$\times \left(\frac{\text{m}^3}{1{,}000\ \text{L}}\right)\left(\frac{\text{min}}{60\ \text{s}}\right)\left(\frac{\text{kW}}{1{,}000\,\frac{\text{N}\cdot\text{m}}{\text{s}}}\right)$$

$$= 1.56\ \text{kW}$$

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7.8 Fluid Measurements

A number of devices have been developed to allow measurement of fluid velocity and flow rate (Q). A few of the most common types are described below.

Pitot Tube

A **Pitot tube** is a device used to measure the stagnation pressure of a moving fluid. It is a tube with a small-diameter hole at the end that is pointed directly into a flowing fluid, as shown in Figure 7.20.

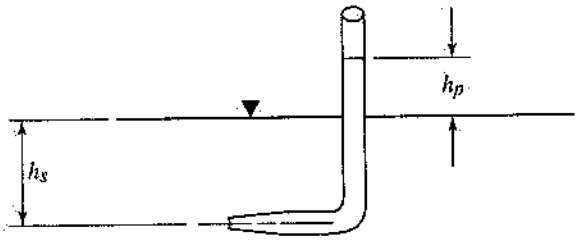


Figure 7.20 Illustration of a Pitot-tube flow-measurement device.

The speed of the fluid is computed from the stagnation pressure using one of these equations:

$$V = \sqrt{\frac{2(p_0 - p_s)}{\rho}} \qquad [7.101]$$

or

$$V = \sqrt{\frac{2g(p_0 - p_s)}{\gamma}} \qquad [7.102]$$

For the system shown in Figure 7.20, the static pressure and the stagnation pressure at the opening of the Pitot tube are given by the following relationships:

$$p_s = \rho g h_s + p_{atm} = \gamma h_s + p_{atm} \qquad [7.103]$$

and

$$p_0 = p_s + \rho g h_p \qquad [7.104]$$

By combining equations 7.102 and 7.104, the fluid speed can be related directly to h_p:

$$h_p = \frac{V^2}{2g} \qquad [7.105]$$

Although equation 7.101 was developed for an incompressible fluid, it may also be used with reasonable accuracy for a compressible fluid if the Mach number is less than approximately 0.3.

Example 7.23

A. For the Pitot tube shown in Figure 7.20, what is the stagnation pressure of the fluid if h_p = 13.6 cm and h_s = 8 cm? [Assume that the fluid is water (ρ = 1,000 kg/m^3), and that p_{atm} = 101.3 kPa.]

B. What is the fluid speed?

Solution:

A. From equation 7.103:

$$p_s = \rho g h_s + p_{atm}$$

$$= \left(1{,}000\ \frac{kg}{m^3}\right)\left(9.81\ \frac{m}{s^2}\right)(8\ cm)$$

$$\times \left(\frac{m}{100\ cm}\right)\left(\frac{N}{\frac{kg \cdot m}{s^2}}\right)\left(\frac{kPa}{1{,}000\ \frac{N}{m^2}}\right) + 101.3\ kPa$$

$$= 102.1\ Pa$$

From equation 7.104:

$$p_0 = p_s + \rho g h_p$$

$$= 102.1\ kPa + \left(1{,}000\ \frac{kg}{m^3}\right)\left(9.81\ \frac{m}{s^2}\right)(13.6\ cm)$$

$$\times \left(\frac{m}{100\ cm}\right)\left(\frac{N}{\frac{kg \cdot m}{s^2}}\right)\left(\frac{kPa}{1{,}000\ \frac{N}{m^2}}\right)$$

$$= 103.4\ kPa$$

B. From equation 7.101:

$$V = \sqrt{\frac{2(p_0 - p_s)}{\rho}}$$

$$= \sqrt{\frac{2(103.4\ kPa - 102.1\ kPa)}{1{,}000\ \frac{kg}{m^3}}\left(\frac{1{,}000\ \frac{N}{m^2}}{kPa}\right)\left(\frac{\frac{kg \cdot m}{s^2}}{N}\right)}$$

$$= 1.61\ m/s$$

Orifice Flow Meter

The **orifice flow meter**, illustrated in Figure 7.21, is a flat plate, with a small hole in the center, placed in a pipe. Pressure measurements are taken at fixed locations upstream and downstream of the orifice. The flow rate of the fluid is related to the difference in value between the upstream and downstream pressure measurements.

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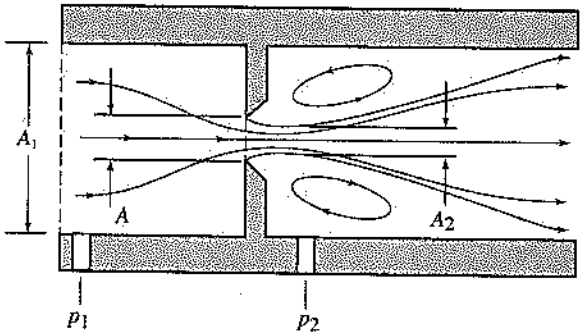


Figure 7.21 Diagram of an orifice flow meter.

The volumetric flow rate through an orifice flow meter is given by the following equation:

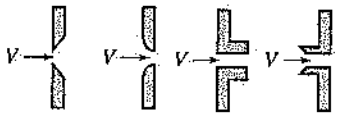
$$Q = CA\sqrt{2g\left[\left(\frac{p_1}{\gamma} + z_1\right) - \left(\frac{p_2}{\gamma} + z_2\right)\right]} \qquad [7.106]$$

The flow meter coefficient (C) is given by the equation

$$C = \frac{C_v C_c}{\sqrt{1 - C_c^2\left(\frac{A}{A_1}\right)^2}} \qquad [7.107]$$

The coefficients C_v and C_c are given in Table 7.2.

Table 7.2 Various Types of Orifices and Their Nominal Coefficient Values

	Sharp Edge	*Rounded*	*Short Tube*	*Borda*
				
Minimum value of C (meter coefficient)	0.61	0.98	0.80	0.51
C_c (coefficient of contraction)	0.62	1.00	1.00	0.52
C_v (coefficient of velocity)	0.98	0.98	0.80	0.98

An orifice flow meter provides a simple and efficient means to measure the flow rate. Unfortunately, however, this simplicity comes at a price; the sudden contraction of the flow produces large head losses.

Example 7.24

Water flows through a rounded orifice flow meter in a pipe having a 20-cm diameter. The orifice is in a horizontal section of the pipe and has an area of 0.03 m^2. The pressure just upstream of the orifice is 75 kPa, and the pressure just downstream is 15 kPa. What is the volumetric flow rate of the water?

Solution:

First determine the meter coefficient. From equation 7.107:

$$C = \frac{C_v C_c}{\sqrt{1 - C_c^2\left(\frac{A}{A_1}\right)^2}} = \frac{(0.98)(1.00)}{\sqrt{1 - (1.00)^2\left(\frac{0.03\ \text{m}^2}{\pi(0.25\ \text{m})^2}\right)}} = 1.065$$

Then use equation 7.106:

$$Q = CA\sqrt{2\left(\frac{p_1 - p_2}{\rho}\right)}$$

$$= (1.065)(0.03\ \text{m}^2) \times \sqrt{2\left[\left(\frac{75\ \text{kPa} - 15\ \text{kPa}}{1{,}000\ \frac{\text{kg}}{\text{m}^3}}\right)\right]\left(\frac{1{,}000\ \frac{\text{N}}{\text{m}^2}}{\text{kPa}}\right)\left(\frac{\frac{\text{kg}\cdot\text{m}}{\text{s}^2}}{\text{N}}\right)}$$

$$= 0.35\ \text{m}^3/\text{s}$$

Venturi Flow Meter

Like the orifice flow meter, the **venturi flow meter**, illustrated in Figure 7.22, uses a pressure drop across a flow restriction of known geometry to measure the flow rate.

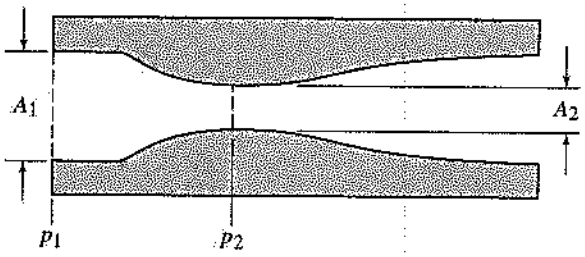


Figure 7.22 Diagram of a venturi flow meter.

The volumetric flow rate through a venturi flow meter is given by the following equation:

$$Q = \frac{C_v A_2}{\sqrt{1 - (A_2/A_1)^2}}\sqrt{2g\left[\left(\frac{p_1}{\gamma} + z_1\right) - \left(\frac{p_2}{\gamma} + z_2\right)\right]} \qquad [7.108]$$

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The coefficient of velocity (C_v) is obtained through experimentation.

The gradual contraction of the venturi flow meter largely reduces the head loss to the fluid stream. The shape of this device, however, is much more complicated to manufacture than is an orifice.

Submerged Orifice

The rate of flow through a submerged orifice, illustrated in Figure 7.23, is related to the hydrostatic pressure at each side of the orifice as shown in equation 7.106 or 7.107.

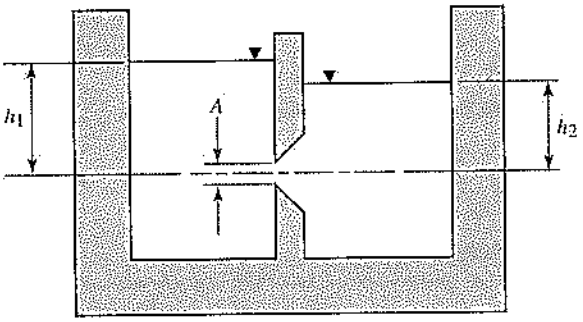


Figure 7.23 An orifice submerged below the surface of a fluid.

$$Q = C_c C_v A\sqrt{2g(h_1 - h_2)} \qquad [7.109]$$

or

$$Q = CA\sqrt{2g(h_1 - h_2)} \qquad [7.110]$$

Orifice Discharging into Atmosphere

If the fluid on the downstream side of an orifice discharges into the ambient atmosphere, as illustrated in Figure 7.24, the atmosphere's contribution to equations 7.108 and 7.109 can be neglected, and the flow rate can be expressed as follows:

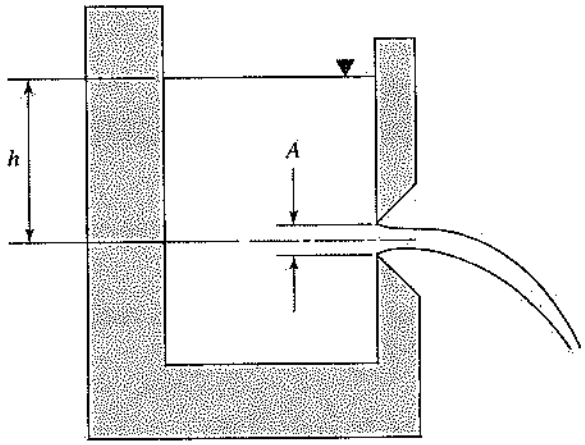


Figure 7.24 Diagram of an orifice discharging into the atmosphere.

$$Q = CA\sqrt{2gh} \qquad [7.111]$$

7.9 Dimensional Analysis—Dimensional Homogeneity

A dimensionally homogeneous relationship is one whose value is independent of the fundamental dimensions used (the units used to measure mass, length, or time). Dimensional homogeneity is assured for relationships that are placed in a dimensionless form. The *Buckingham PI* theorem tells how many dimensionless groups are required to describe a relationship. It states that, if the relationship has n independent variables, involving r different fundamental dimensions (mass, length, or time), then $n - r$ dimensionless groups are required.

7.10 Similitude

Model testing plays a large role in fluid dynamic analysis. The reason is that the equations of fluid flow are sufficiently complex that it is impracticable to solve them for all but the simplest flow conditions. Furthermore, testing of a full-size prototype is often prohibitively expensive.

We can be assured that the model we use for testing predicts accurately the fluid interaction of the full-scale prototype if we can prove that there is similarity between the two. Required are three kinds of similarity:

- *Geometric similarity:* The model must have the same shape as the full-scale prototype. Each of the geometric parameters that describe the shape and size of the model differs from the corresponding geometric parameter of the full-scale prototype by a constant scale factor.
- *Kinematic similarity:* The character of the fluid flow must be the same for the model and the full-scale prototype. In other words, the direction of flow of each fluid particle near the model must be the same as that of the fluid particle at the corresponding location near the full-scale prototype. The velocities of the two corresponding fluid particles must be related by a constant scale factor.

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- *Dynamic similarity:* The forces that interact with the model must be similar to the forces that act on the full-scale prototype. For each force that acts on the full-scale prototype, there must be a corresponding force acting on the model. Each corresponding force must act in the same direction, through the corresponding location, and with a magnitude that differs by a constant scale factor. The model and the prototype must have geometric and kinematic similarity if dynamic similarity is to be achieved.

  Dynamic similarity is achieved when the following conditions are met:

$$\left[\frac{F_I}{F_p}\right]_p = \left[\frac{F_I}{F_p}\right]_m \quad \text{or} \quad \left[\frac{\rho \bar{V}^2}{p}\right]_p = \left[\frac{\rho \bar{V}^2}{p}\right]_m \qquad [7.112]$$

$$\left[\frac{F_I}{F_V}\right]_p = \left[\frac{F_I}{F_V}\right]_m \quad \text{or} \quad \left[\frac{\rho \bar{V} l}{\mu}\right]_p = \left[\frac{\rho \bar{V} l}{\mu}\right]_m \quad \text{or} \quad [\mathrm{Re}]_p = [\mathrm{Re}]_m \qquad [7.113]$$

$$\left[\frac{F_I}{F_G}\right]_p = \left[\frac{F_I}{F_G}\right]_m \quad \text{or} \quad \left[\frac{\bar{V}^2}{lg}\right]_p = \left[\frac{\bar{V}^2}{lg}\right]_m \quad \text{or} \quad [\mathrm{Fr}]_p = [\mathrm{Fr}]_m \qquad [7.114]$$

$$\left[\frac{F_I}{F_E}\right]_p = \left[\frac{F_I}{F_E}\right]_m \quad \text{or} \quad \left[\frac{\rho \bar{V}^2}{E}\right]_p = \left[\frac{\rho \bar{V}^2}{E}\right]_m \quad \text{or} \quad [\mathrm{Ca}]_p = [\mathrm{Ca}]_m \qquad [7.115]$$

$$\left[\frac{F_I}{F_T}\right]_p = \left[\frac{F_I}{F_T}\right]_m \quad \text{or} \quad \left[\frac{\rho l \bar{V}^2}{\sigma}\right]_p = \left[\frac{\rho l \bar{V}^2}{\sigma}\right]_m \quad \text{or} \quad [\mathrm{We}]_p = [\mathrm{We}]_m \qquad [7.116]$$

where F_I = inertia force,
F_p = pressure force,
F_V = viscous force,
F_G = gravity force,
F_E = elastic force,
F_T = surface-tension force,
l = characteristic length,
Re = Reynolds number,
Fr = Froude number,
Ca = Cauchy number,
E = modulus of elasticity,
σ = surface tension,
We = Weber number.

The subscript p refers to the full-scale prototype, and the subscript m refers to the model.

Example 7.25

If a one-fifth-scale boat model is to be tested in a tow tank, at what speed should the model be towed if a speed of 15 knots (7.72 m/s) is to be simulated?

Solution:

From equation 7.113:

$$\left[\frac{\rho \bar{V} l}{\mu}\right]_p = \left[\frac{\rho \bar{V} l}{\mu}\right]_m$$

$$\bar{V}_m = \left(\frac{\rho_p}{\rho_m}\right)\left(\frac{\mu_m}{\mu_p}\right)\left(\frac{l_p}{l_m}\right)\bar{V}_p$$

$$= (1)(1)\left(\frac{1}{5}\right)\left(7.72\,\frac{\mathrm{m}}{\mathrm{s}}\right)$$

$$= 1.54 \text{ m/s}$$

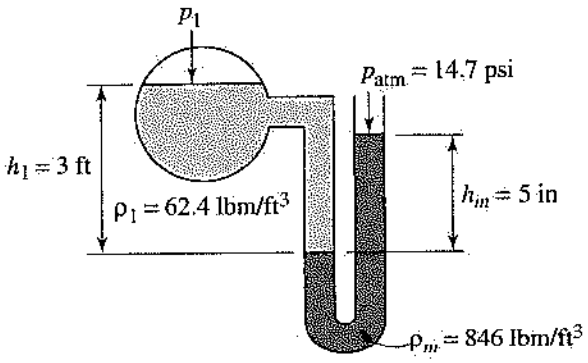
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PRACTICE PROBLEMS

1. What is the Mach number of a fluid stream flowing at a velocity of 370 ft/sec? (c = 1,080 ft/sec)
   (A) 0.66 **(B)** 2.92
   (C) 0.34 **(D)** 3.4
2. What is the gage pressure of air in a vessel if the absolute pressure is 340 kPa, and the ambient atmospheric pressure is 101 kPa?
   (A) 441 kPa **(B)** 105 kPa
   (C) 340 kPa **(D)** 239 kPa
3. The specific weight of a fluid is:
   (A) the ratio of the density of the fluid to the density of water
   (B) the mass of the fluid per unit volume
   (C) the weight of the fluid per unit volume
   (D) the ratio of the weight of the fluid to the weight of water
4. The viscosity of a fluid:
   (A) relates an applied normal stress to the velocity gradient in the fluid
   (B) relates an applied shear stress to the velocity gradient in the fluid
   (C) appears constant for all values of the velocity gradient in a dilatant fluid
   (D) appears constant for all values of the velocity gradient in a pseudoplastic fluid
5. What is the pressure at a point 18,000 ft below the surface of the ocean?

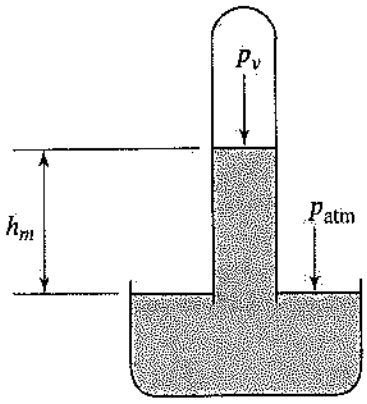
   ($\gamma = 64.0$ lb/ft^3)

   (A) 1,150,000 psig
   (B) 250 psig
   (C) 8,000 psig
   (D) 258,000 psig
6. In the figure, what is the absolute pressure (p_A) in vessel A?

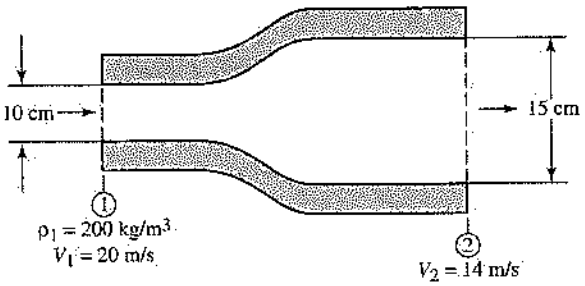


   (A) 15.8 psia **(B)** 18.4 psia
   (C) 13.6 psia **(D)** 11.0 psia

7. For the barometer shown in the figure, what is the atmospheric pressure if h_m = 20 cm, ρ_m = 13,550 kg/m^3, and p_v = 0 kPa?



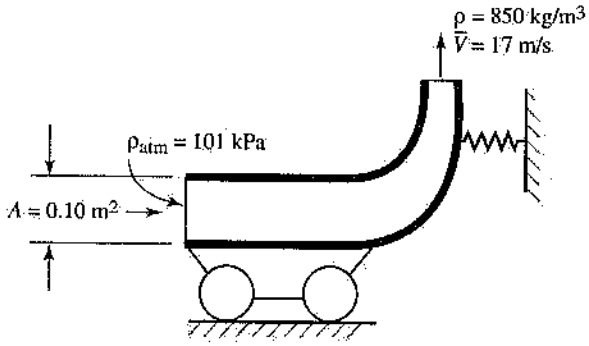
   (A) 128 kPa
   (B) 26.6 kPa
   (C) 4.36 kPa
   (D) 82.4 kPa
8. A cylindrical bar with a density of 41.6 lbm/ft^3 and a length of 3 ft floats in water (ρ = 62.4 lbm/ft^3) in such a way that the end surfaces of the bar are parallel to the surface of the water. How high will the top of the bar extend above the surface of the water?
   (A) 1 ft
   (B) 2 ft
   (C) 1.5 f
   (D) 0.5 ft
9. For steady flow of a compressible fluid in the expander shown in the figure, what is the density at the exit section (ρ_2)?



   (A) 190 kg/m^3
   (B) 62 kg/m^3
   (C) 642 kg/m^3
   (D) 127 kg/m^3

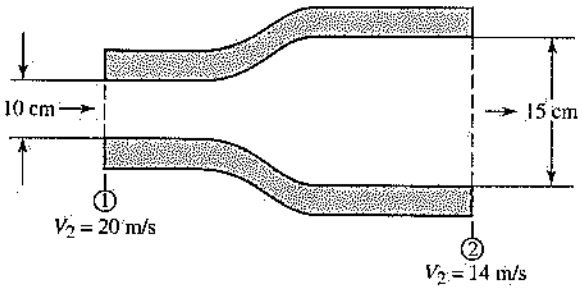
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10. For the steady-state system shown in the figure, what is the force in the spring?



(A) 43,495 N
(B) 24,575 N
(C) 16,895 N
(D) 24,565 N

11. For the control volume shown in the figure, what is the change in pressure ($p_1 - p_2$) if the specific weight of the fluid is 9,810 N/m^3? (Assume that frictional losses are negligible.)



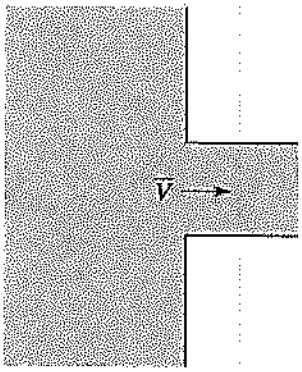
(A) −102 kPa
(B) −204 kPa
(C) 102 kPa
(D) 204 kPa

12. What is the stagnation pressure of water (ρ = 1,000 kg/m^3) flowing with a velocity of 28 m/s if the static pressure is 300 kPa?
(A) −92 kPa
(B) 692 kPa
(C) 92 kPa
(D) 1,084 kPa

13. Water flows through a 20-cm-diameter pipe. If the critical Reynolds number is 2,300, what is the maximum value of the average velocity for which laminar flow can be assumed?
(A) 0.047 m/s
(B) 1.2 m/s
(C) 0.092 m/s
(D) 0.75 m/s

14. Water flows through a 17-cm-diameter horizontal pipe (f = 0.017) at a rate of 0.34 m^3/s. What is the frictional head loss per meter of pipe length?
(A) 1.31 m
(B) 1.14 m
(C) 0.02 m
(D) 0.78 m

15. What is the minor head loss for the pipe entrance shown in the figure?



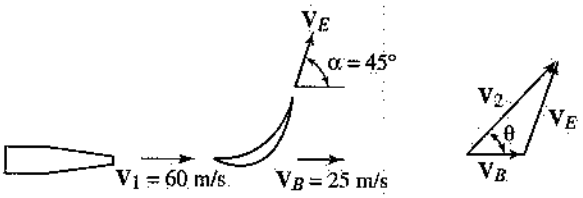
V = 165 m/s
C = 0.5

(A) 1.28 m
(B) 16.81 m
(C) 2,775 m
(D) 693.81 m

16. What is the hydraulic diameter of a square duct that measures 80 cm on a side?
(A) 2.4 m
(B) 1.2 m
(C) 0.80 m
(D) 0.40 m

17. What is the coefficient of drag for a flow of air (ρ = 1.225 kg/m^3, $\mu = 1.8 \times 10^{-5}$ N · s/m^2) over a flat plate that is 93 cm long, if V_∞ = 86 m/s?
(A) 3.38×10^{-3}
(B) 5.70×10^{-3}
(C) 2.19×10^{-3}
(D) 8.27×10^{-3}

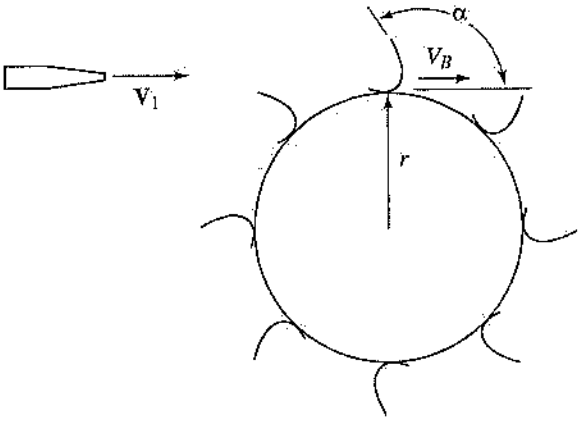
18. What is angle θ for the moving blade shown in the figure?



(A) 57.82°
(B) 63.55°
(C) 32.18°
(D) 26.45°

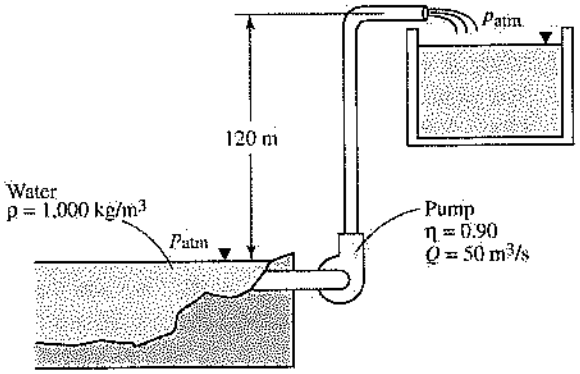
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19. If 16 kg/s of water, traveling at 67 m/s, enters the impulse turbine shown in the figure, which has a 26-cm diameter and a blade angle α of 120°, what is the maximum power that can be generated?



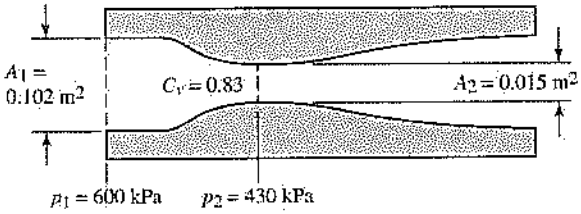
(A) 134 W
(B) 402 W
(C) 26.93 kW
(D) 8.98 kW

20. What is the pumping power required for the system shown in the figure? (Neglect friction in the pipes.)



(A) 58,900 kW (B) 65,400 kW
(C) 82,300 kW (D) 53,000 kW

21. What is the volumetric flow rate (Q) of water (γ = 9,810 N/m^3) for the venturi flow meter shown in the figure?



(A) 0.232 m^3/s (B) 0.248 m^3/s
(C) 0.733 m^3/s (D) 0.467 m^3/s

Answer Key

1. C	8. A	15. D
2. D	9. D	16. C
3. C	10. D	17. A
4. B	11. A	18. C
5. C	12. B	19. C
6. A	13. C	20. B
7. B	14. B	21. A

Answers Explained

1. **C** From equation 7.2:

$$M = \frac{\overline{V}}{c}$$
$$= \frac{370\ \frac{\text{ft}}{\text{sec}}}{1{,}080\ \frac{\text{ft}}{\text{sec}}}$$
$$= 0.343$$

2. **D** From equation 7.5:

$$p_a = p_g + p_{atm}$$

or

$$p_g = p_a - p_{atm}$$
$$= (340\ \text{kPa}) - (101\ \text{kPa})$$
$$= 239\ \text{kPa}$$

3. **C** The specific weight of a fluid is defined as the weight of a unit volume of fluid. It is related to the fluid density by the equation

$$\gamma = \rho g$$

where γ = specific weight (lbf/ft^3 or N/m^3),
g = acceleration due to gravity (32.17 ft/sec^2 or 9.81 m/s^2 at sea level),
ρ = density

4. **B** It can be shown that the shear force in most common fluids can be expressed using equation 7.10:

$$\tau_n(P) = \mu\frac{\partial V}{\partial x_n}$$

Therefore choice **B** is correct. In a dilatant fluid, the viscosity appears to increase as the velocity gradient increases. In a pseudoplastic fluid, the viscosity appears to decrease as the velocity gradient increases.

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5. C From equation 7.15: $p_g = \gamma h$

$$= \left(64.0 \frac{\text{lbf}}{\text{ft}^3}\right)(18{,}000 \text{ ft})\left(\frac{\text{ft}^2}{144 \text{ in}^2}\right)$$
$$= 8{,}000 \text{ lbf/in}^2 \text{ or psig}$$

6. A From equation 7.23:

$$p_1 = p_{\text{atm}} + \rho_m g h_m - \rho_1 g h_1$$

Then

$$\rho_m g h_m = \left(846 \frac{\text{lbm}}{\text{ft}^3}\right)\left(32.2 \frac{\text{ft}}{\text{sec}^2}\right)(5 \text{ in}) \times \left(\frac{\text{lbf}}{32.2 \frac{\text{lbm-ft}}{\text{sec}^2}}\right)\left(\frac{\text{ft}^3}{1{,}728 \text{ in}^3}\right)$$
$$= 2.45 \text{ lbf/in}^2$$

Also,

$$\rho_1 g h_1 = \left(62.4 \frac{\text{lbm}}{\text{ft}^3}\right)\left(32.2 \frac{\text{ft}}{\text{sec}^2}\right)(3 \text{ ft}) \times \left(\frac{\text{lbf}}{32.2 \frac{\text{lbm-ft}}{\text{sec}^2}}\right)\left(\frac{\text{ft}^2}{144 \text{ in}^2}\right)$$
$$= 1.30 \text{ lbf/in}^2$$

so

$$p_1 = 14.7 \frac{\text{lbf}}{\text{in}^2} + 2.45 \frac{\text{lbf}}{\text{in}^2} - 1.30 \frac{\text{lbf}}{\text{in}^2}$$
$$= 15.85 \text{ lbf/in}^2 \text{ or psia}$$

7. B From equation 7.25:

$$p_{\text{atm}} - p_v = \rho_m g h_m$$

Then

$$p_{\text{atm}} = \rho_m g h_m$$
$$= \left(13{,}550 \frac{\text{kg}}{\text{m}^3}\right)\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(0.20 \text{ m}) \times \left(\frac{\text{N}}{\frac{\text{kg·m}}{\text{s}^2}}\right)\left(\frac{\text{kPa}}{1{,}000 \frac{\text{N}}{\text{m}^2}}\right)$$
$$= 26.6 \text{ kPa}$$

8. A According to Archimedes' principle, an object floating in a fluid will displace an amount of fluid whose weight is equal to the weight of the object. The weight of the cylindrical bar is given by the equation

$$W_{\text{bar}} = m_{\text{bar}} g$$
$$= \rho_{\text{bar}} V_{\text{bar}} g$$
$$= \left(41.6 \frac{\text{lbm}}{\text{ft}^3}\right)[(\pi R^2)(3 \text{ ft})]\left(32.2 \frac{\text{ft}}{\text{sec}^2}\right) \times \left(\frac{\text{lbf}}{32.2 \frac{\text{lbm-ft}}{\text{sec}^2}}\right)$$
$$= 392 R^2 \text{ lbf/ft}^2$$

where R is the radius of the cylinder.

The weight of the fluid displaced is given by the equation

$$W_{\text{disp}} = m_{\text{disp}} g$$
$$= \rho_{\text{water}} V_{\text{disp}} g$$
$$= \left(62.4 \frac{\text{lbm}}{\text{ft}^3}\right)[(\pi R^2)(L)]\left(32.2 \frac{\text{ft}}{\text{sec}^2}\right)\left(\frac{\text{lbf}}{32.2 \frac{\text{lbm-ft}}{\text{sec}^2}}\right)$$
$$= 196 R^2 L \text{ lbf/ft}^3$$

where L is the length of the displaced cylinder.

Find the value of L by setting the weight of the bar and the weight of the fluid equal to each other, and solving for L:

$$W_{\text{bar}} = W_{\text{disp}}$$
$$392 R^2 \frac{\text{lbf}}{\text{ft}^2} = 196 R^2 L \frac{\text{lbf}}{\text{ft}^3}$$
$$L = \frac{392}{196} \frac{R^2}{R^2} \frac{\frac{\text{lbf}}{\text{ft}^2}}{\frac{\text{lbf}}{\text{ft}^3}}$$
$$= 2 \text{ ft}$$

This result is the amount of the bar that is submerged; therefore:

1 ft of bar will extend above the surface of the water.

9. D From the principle of mass conservation, equation 7.43 gives:

$$\rho_1 A_1 \bar{V}_1 = \rho_2 A_2 \bar{V}_2$$

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or

$$\rho_2 = \frac{\rho_1 A_1 \overline{V}_1}{A_2 \overline{V}_2}$$

$$= \left(200 \frac{\text{kg}}{\text{m}^3}\right)\left(\frac{\frac{\pi(0.10\ \text{m})^2}{4}}{\frac{\pi(0.15\ \text{m})^2}{4}}\right)\left(\frac{20 \frac{\text{m}}{\text{s}}}{14 \frac{\text{m}}{\text{s}}}\right)$$

$$= 127\ \text{kg/m}^3$$

10. D From application of the impulse-momentum principle and a summation of forces in the horizontal direction, the force in the spring is given by the equation

$$F_{\text{spring}} = \dot{m}\overline{V}$$

Also,

$$\dot{m} = \rho \overline{V} A$$

$$= \left(850 \frac{\text{kg}}{\text{m}^3}\right)\left(17 \frac{\text{m}}{\text{s}}\right)(0.10\ \text{m}^2)$$

$$= 1{,}445\ \text{kg/s}$$

so

$$F_{\text{spring}} = \left(1{,}445 \frac{\text{kg}}{\text{s}}\right)\left(17 \frac{\text{m}}{\text{s}}\right)\left(\frac{\text{N}}{\frac{\text{kg·m}}{\text{s}^2}}\right)$$

$$= 24{,}565\ \text{N}$$

11. A Use the Bernoulli equation (equation 7.51):

$$\frac{p_1}{\gamma} + \frac{\overline{V}_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{\overline{V}_2^2}{2g} + z_2$$

or

$$p_1 - p_2 = \gamma\left[\frac{(\overline{V}_2^2 - \overline{V}_1^2)}{2g} + (z_2 - z_1)\right]$$

$$= \left(9{,}810 \frac{\text{N}}{\text{m}^3}\right)\left[\frac{\left(14 \frac{\text{m}}{\text{s}}\right)^2 - \left(20 \frac{\text{m}}{\text{s}}\right)^2}{2\left(9.81 \frac{\text{m}}{\text{s}^2}\right)} + 0\ \text{m}\right]$$

$$\times\left(\frac{\text{kPa}}{1{,}000 \frac{\text{N}}{\text{m}^2}}\right)$$

$$= -102\ \text{kPa}$$

12. B From equation 7.55:

$$p_o = p_s + \frac{1}{2}\rho V^2$$

$$= (300\ \text{kPa}) + \frac{1}{2}\left(1{,}000 \frac{\text{kg}}{\text{m}^3}\right)\left(28 \frac{\text{m}}{\text{s}}\right)^2\left(\frac{\text{N}}{\frac{\text{kg·m}}{\text{s}^2}}\right)$$

$$\times\left(\frac{\text{kPa}}{1{,}000 \frac{\text{N}}{\text{m}^2}}\right)$$

$$= 692\ \text{kPa}$$

13. C From equation 7.56:

$$\text{Re} = \frac{\rho \overline{V} D}{\mu}$$

or

$$\overline{V} = \frac{\mu \text{Re}}{\rho D}$$

$$= \frac{\left(8 \times 10^{-3} \frac{\text{N·s}}{\text{m}^2}\right)(2{,}300)}{\left(1{,}000 \frac{\text{kg}}{\text{m}^3}\right)(0.20\ \text{m})}\left(\frac{\frac{\text{kg·m}}{\text{s}^2}}{\text{N}}\right)$$

$$= 0.092\ \text{m/s}$$

14. B From equation 7.66: $h_f = f\left(\frac{L}{D}\right)\left(\frac{\overline{V}^2}{2g}\right)$

Find the average velocity through the pipe as follows:

$$\overline{V} = \frac{Q}{A}$$

$$= \frac{0.34 \frac{\text{m}^3}{\text{s}}}{\frac{\pi(0.17\ \text{m})^2}{4}}$$

$$= 14.98\ \text{m/s}$$

Then

$$h_f = (0.017)\left[\frac{1\ \text{m}}{0.17\ \text{m}}\right]\left[\frac{\left(14.98 \frac{\text{m}}{\text{s}}\right)^2}{2\left(9.81 \frac{\text{m}}{\text{s}^2}\right)}\right]$$

$$= 1.14\ \text{m}$$

15. D From equation 7.69: $h_{fm} = C\frac{\overline{V}^2}{2g}$

$$= (0.5)\frac{\left(165 \frac{\text{m}}{\text{s}}\right)^2}{2\left(9.81 \frac{\text{m}}{\text{s}^2}\right)}$$

$$= 693.81\ \text{m}$$

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16. C From equation 7.75:

$$D_h = 4\left(\frac{\text{cross-sectional area}}{\text{wetted perimeter}}\right)$$
$$= 4\left(\frac{(0.80\text{ m})^2}{4(0.80\text{ m})}\right)$$
$$= 0.80\text{ m}$$

17. A From equation 7.82:

$$C_D = \frac{1.33}{\text{Re}_L^{0.5}} \quad (10^4 < \text{Re}_L < 5 \times 10^5)$$

or

$$C_D = \frac{0.031}{\text{Re}_L^{1/7}} \quad (10^6 < \text{Re}_L < 10^9)$$

The Reynolds number for this flow is given by equation 7.81:

$$\text{Re}_L = \frac{\rho V_\infty L}{\mu}$$
$$= \frac{\left(1.225\ \frac{\text{kg}}{\text{m}^3}\right)\left(86\ \frac{\text{m}}{\text{s}}\right)(0.93\text{ m})}{\left(1.8 \times 10^{-5}\ \frac{\text{N}\cdot\text{s}}{\text{m}^2}\right)}$$
$$= 5.443 \times 10^6$$

Therefore the second equation for the coefficient of drag is used:

$$C_D = \frac{0.031}{\text{Re}_L^{1/7}}$$
$$= \frac{0.031}{(5.443 \times 10^6)^{1/7}}$$
$$= 3.381 \times 10^{-3}$$

18. C If friction along the face of the blade is neglected, the velocity leaving the blade has the same magnitude as the velocity entering the blade, directed as shown in the diagram. Solve for angle θ as follows:

$$\mathbf{V}_{2x} = \mathbf{V}_B + \mathbf{V}_E \sin\alpha$$
$$= 25\frac{\text{m}}{\text{s}} + \left(60\ \frac{\text{m}}{\text{s}}\right)\sin(45°)$$
$$= 67.43\text{ m/s}$$

and

$$\mathbf{V}_{2y} = \mathbf{V}_E \cos\alpha$$
$$= \left(60\ \frac{\text{m}}{\text{s}}\right)\cos(45°)$$
$$= 42.43\text{ m/s}$$

Then:

$$\theta = \tan^{-1}\left(\frac{V_{2y}}{V_{2x}}\right)$$
$$= \tan^{-1}\left(\frac{42.43\ \frac{\text{m}}{\text{s}}}{67.43\ \frac{\text{m}}{\text{s}}}\right)$$
$$= 32.18°$$

19. C The maximum power that can be generated by the impulse turbine shown in the figure is given by equation 7.91:

$$\dot{W}_{\max} = Q\rho\left(\frac{|V_1^2|}{4}\right)(1 - \cos\alpha)$$
$$= \dot{m}\left(\frac{|V_1^2|}{4}\right)(1 - \cos\alpha)$$
$$= \left(16\ \frac{\text{kg}}{\text{s}}\right)\left[\frac{\left(67\ \frac{\text{m}}{\text{s}}\right)^2}{4}\right][1 - \cos(120°)]$$
$$\times\left(\frac{\text{N}}{\frac{\text{kg}\cdot\text{m}}{\text{s}^2}}\right)\left(\frac{\text{kW}}{1{,}000\ \frac{\text{N}\cdot\text{m}}{\text{s}}}\right)$$
$$= 26.93\text{ kW}$$

20. B Use the Bernoulli equation:

$$\frac{p_1}{\gamma} + \frac{\bar{V}_1^2}{2g} + z_1 + h_m = \frac{p_2}{\gamma} + \frac{\bar{V}_2^2}{2g} + z_2$$

In this case, $p_1 = p_2$ and $\bar{V}_1 = \bar{V}_2$, so

$$h_m = z_2 - z_1$$
$$= 120\text{ m}$$

The pumping power is then given by the equation

$$\dot{W} = \rho g h Q$$
$$= \left(1{,}000\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(120\text{ m})\left(50\ \frac{\text{m}^3}{\text{s}}\right)$$
$$\times\left(\frac{\text{N}}{\frac{\text{kg}\cdot\text{m}}{\text{s}^2}}\right)\left(\frac{\text{kW}}{1{,}000\ \frac{\text{N}\cdot\text{m}}{\text{s}}}\right)$$
$$= 58{,}860\text{ kW}$$

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But this is the power required to pump the water. The pump will need somewhat more power to account for its irreversibilities:

$$\eta = \frac{\dot{W}}{\dot{W}_{\text{supplied}}}$$

$$\dot{W}_{\text{supplied}} = \frac{\dot{W}}{\eta} = \frac{58{,}860 \text{ kW}}{0.90} = 65{,}400 \text{ kW}$$

21. A The volumetric flow rate through a venturi flow meter is given by equation 7.108:

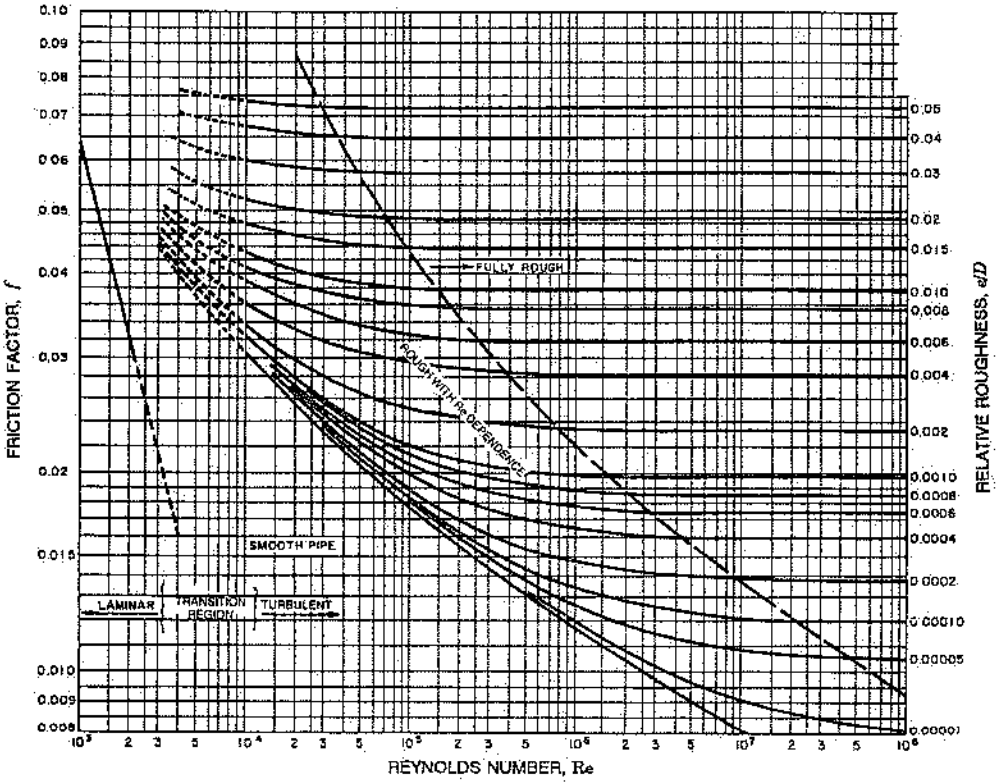
$$Q = \frac{C_v A_2}{\sqrt{1-(A_2/A_1)^2}}\sqrt{2g\left[\left(\frac{p_1}{\gamma}+z_1\right)-\left(\frac{p_2}{\gamma}+z_2\right)\right]}$$

$$= \frac{(0.83)(0.015 \text{ m}^2)}{\sqrt{1-\left(\frac{0.015 \text{ m}^2}{0.102 \text{ m}^2}\right)^2}} \times \sqrt{2\left(9.81 \frac{\text{m}}{\text{s}^2}\right)\left[\left(\frac{(600 \text{ kPa})}{9{,}810 \frac{\text{N}}{\text{m}^3}}\right)\left(\frac{1{,}000 \frac{\text{N}}{\text{m}^2}}{\text{kPa}}\right)-\left(\frac{(430 \text{ kPa})}{9{,}810 \frac{\text{N}}{\text{m}^3}}\right)\left(\frac{1{,}000 \frac{\text{N}}{\text{m}^2}}{\text{kPa}}\right)\right]}$$

$$= 0.232 \text{ m}^3/\text{s}$$

MOODY (STANTON) DIAGRAM

	ϵ, (ft)	ϵ, (mm)
Riveted steel	0.003-0.03	0.9-9.0
Concrete	0.001-0.01	0.3-3.0
Cast iron	0.00085	0.25
Galvanized iron	0.0005	0.15
Commercial steel or wrought iron	0.00015	0.046
Drawn tubing	0.000005	0.0015



Reproduced from "Friction Factors for Pipe Flow," *MOODY DIAGRAM*, in *Trans. ASME 66*, 1944. Used with permission of ASME.

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$$(x-1)^2+(y+3)^2+(z-2)^2=25$$

Its general-form equation is found by expanding and simplifying its standard-form equation:

$$x^2+y^2+z^2-2x+6y-4z-11=0$$

9.5 Logarithms

The **logarithmic function with base b** is defined by

$$y=\log_b x \quad \text{if and only if} \quad x=b^y \text{ for } x>0$$

A base-10 logarithm, written as $\log x$, is called a **common logarithm.** A base-e ($e \approx 2.718...$) logarithm, written as $\ln x$, is called a **natural logarithm.** To change from one logarithmic base to another, use the **change of base formula:**

$$\log_b x=\frac{\log_a x}{\log_a b},\ a>0,\ a\neq 1$$

Four **logarithmic identities** hold for all bases b, with $b>0$ and $b\neq 1$:

1. $\log_b 1=0$
2. $\log_b b=1$
3. $b^{\log_b u}=u,\ u>0$
4. $\log_b b^u=u$

If $b>0$, $b\neq 1$, then, for positive real numbers x and y, there are three **properties of logarithms:**

1. $\log_b xy=\log_b x+\log_b y$
2. $\log_b \frac{x}{y}=\log_b x-\log_b y$
3. $\log_b x^n=n\log_b x$

Example 9.9

Evaluate $\log_4 24$ using:

A. common logarithms **B.** natural logarithms

Solution:

A. Use the change of base formula with $a=10$, $b=4$, and the common logarithm key on a calculator:

$$\log_4 24=\frac{\log 24}{\log 4}\approx 2.29$$

B. Use the change of base formula with $a=e$, $b=4$, and the natural logarithm key on a calculator:

$$\log_4 24=\frac{\ln 24}{\ln 4}\approx 2.29$$

Example 9.10

Simplify each expression.

A. $e^{2\ln(x+1)}$ **B.** $\log_2 16^{x-2}$

Solution:

A. Apply logarithmic property 3, and use the fact that $b^{\log_b u}=u$, for $u>0$:

$$e^{2\ln(x+1)}=e^{\ln(x+1)^2}=(x+1)^2$$

provided that $x>-1$.

B. Begin by applying the properties of exponents and matching the base of the exponential expression to the base of the logarithm:

$$\log_2 16^{x-2}=\log_2 (2^4)^{x-2}=\log_2 2^{4x-8}$$

Now, since $\log_b b^u=u$ for any permissible base b and any real number u,

$$\log_2 16^{x-2}=\log_2 2^{4x-8}=4x-8$$

Example 9.11

Use the properties of logarithms to write each expression as a single logarithm.

A. $\log_6 50+2\log_6 4-3\log_6 2$

B. $\ln(x^2-1)-2\ln(x+1)$

Solution:

A. Begin with property 3 and then apply the other two logarithmic properties:

$$\begin{aligned}\log_6 50+2\log_6 4-3\log_6 2&=\log_6 50+\log_6 4^2-\log_6 2^3\\&=\log_6 50+\log_6 16-\log_6 8\\&=\log_6\frac{(50)(16)}{8}\\&=\log_6 100\end{aligned}$$

B. Assuming that $x>1$, begin with property 3 and then apply property 2, reducing the fraction to lowest terms:

$$\begin{aligned}\ln(x^2-1)-2\ln(x+1)&=\ln(x^2-1)-\ln(x+1)^2\\&=\ln\frac{x^2-1}{(x+1)^2}\\&=\ln\frac{x-1}{x+1}\end{aligned}$$

Example 9.12

Solve each equation for x.

A. $4e^{2x-1}=100$ **B.** $2-\log x=\log 3$

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Solution:

A. Begin by dividing both sides of the equation by 4 to isolate the exponential expression on one side:

$$e^{2x-1} = 25$$

Now, take the natural logarithm of each side, apply log property 3, and solve for x:

$$\ln e^{2x-1} = \ln 25$$
$$(2x-1)\ln e = \ln 25$$
$$x = \frac{1 + \ln 25}{2} \approx 2.109$$

B. Begin by grouping the logarithms on one side of the equation and then condense them into a single logarithm by applying the properties of logarithms:

$$2 - \log x = \log 3$$
$$\log 3 + \log x = 2$$
$$\log 3x = 2$$

Now, change to exponential form and solve for x:

$\log 3x = 2$ if and only if $10^2 = 3x$

implying that $x = \frac{100}{3}$.

9.6 Triangle Trigonometry

The interior angles of a polygon are usually measured in radians or degrees. The relationship between **radian measure** and **degree measure** is as follows:

$$\pi \text{ radians} = 180 \text{ degrees}$$

The **sum of the interior angles of an n-sided polygon** is

$$(n-2)180^\circ \quad \text{or} \quad (n-2)\pi$$

An angle whose measure is less than 90° ($\pi/2$ radians) is called an **acute angle,** and an angle whose measure is between 90° and 180° is an **obtuse angle.** If an angle measures 90°, it is called a **right angle.** The Pythagorean theorem states that in any right triangle

$$x^2 + y^2 = r^2$$

as shown in Figure 9.9.

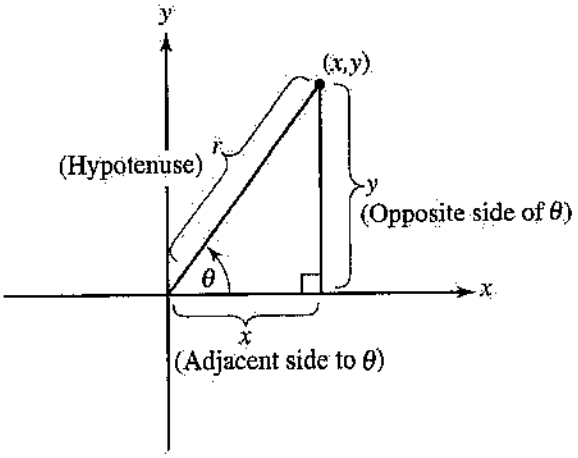


Figure 9.9

For Figure 9.9, there are six **trigonometric functions:**

1. $\sin\theta = \frac{y}{r} = \frac{\text{opp}}{\text{hyp}}$
2. $\cos\theta = \frac{x}{r} = \frac{\text{adj}}{\text{hyp}}$
3. $\tan\theta = \frac{y}{x} = \frac{\text{opp}}{\text{adj}}$
4. $\csc\theta = \frac{r}{y} = \frac{\text{hyp}}{\text{opp}}$
5. $\sec\theta = \frac{r}{x} = \frac{\text{hyp}}{\text{adj}}$
6. $\cot\theta = \frac{x}{y} = \frac{\text{adj}}{\text{opp}}$

Figure 9.10 shows in which quadrants the trigonometric functions are positive:

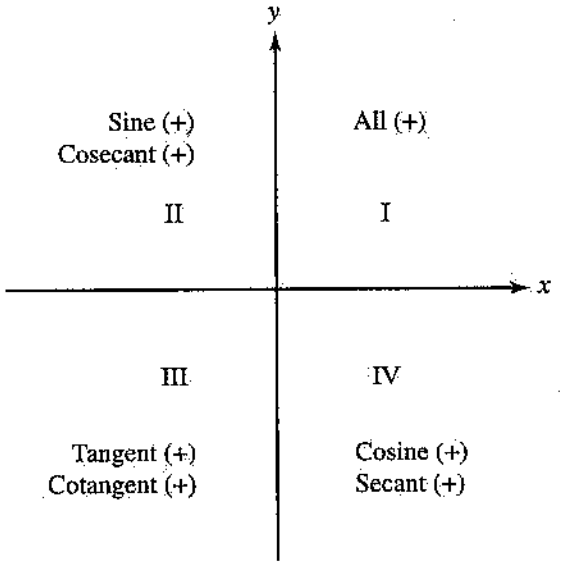


Figure 9.10

The **inverse trigonometric functions** are defined as follows:

1. $\sin^{-1} k$, with $-1 \le k \le 1$, denotes the angle between -90° and 90° whose sine is k.
2. $\cos^{-1} k$, with $-1 \le k \le 1$, denotes the angle between 0° and 180° whose cosine is k.

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3. $\tan^{-1} k$ denotes the angle between $-90°$ and $90°$ whose tangent is k.

The **law of sines** states that in a given triangle the ratio of the length of the side to the sine of the angle opposite that side has the same constant value:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The **law of cosines** states that the square of the length of any side of a triangle is the sum of the squares of the other two sides minus twice the product of those two sides and the cosine of the angle between them:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

If a and b are any two sides of a triangle and θ is the angle between those two sides, then the **area** A of the triangle is

$$A = \frac{ab \sin \theta}{2}$$

Example 9.13

Given that $\sin \theta = \frac{3}{5}$, find $\cos \theta$ if:

A. θ is acute. **B.** θ is obtuse.

Solution:

If $\sin \theta = \frac{3}{5}$, then, by the definition of sine, $y = 3$ and $r = 5$. Hence, by the Pythagorean theorem, $x^2 + 3^2 = 5^2$, implying that $x = \pm 4$.

A. If θ is acute, then θ is in quadrant I, where $\cos \theta > 0$. Hence:

$$\cos \theta = \frac{x}{r} = \frac{4}{5}$$

B. If θ is obtuse, then θ is in quadrant II, where $\cos \theta < 0$. Hence:

$$\cos \theta = \frac{x}{r} = \frac{-4}{5} = -\frac{4}{5}$$

Example 9.14

Given a right triangle with legs of lengths 5 meters and 12 meters, find:

A. the length of the hypotenuse
B. the angle (in radians) opposite the longer leg

Solution:

A. If z is the length of the hypotenuse, then, by the Pythagorean theorem,

$$z^2 = 5^2 + 12^2$$

implying that the length of the hypotenuse is $z = 13$ m.

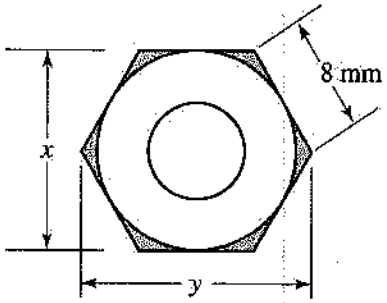


Figure 9.11

B. If θ is the angle opposite the longer leg, then, by the definition of tangent,

$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{12}{5}$$

To find θ, use the inverse tangent key on a calculator set in radian mode:

$$\theta = \tan^{-1} \frac{12}{5} \approx 1.176 \text{ rad}$$

Example 9.15

Each side of a hexagonal nut is 8 mm long, as shown in Figure 9.11.

A. Find distance x. **B.** Find distance y.

Solution:

A. Since a hexagonal nut has six sides, the sum of its interior angles is $(6 - 2)180° = 720°$. Therefore, each interior angle measures $720°/6 = 120°$, implying that triangle ABC is isosceles with $\theta = 60°$, as shown in Figure 9.12.

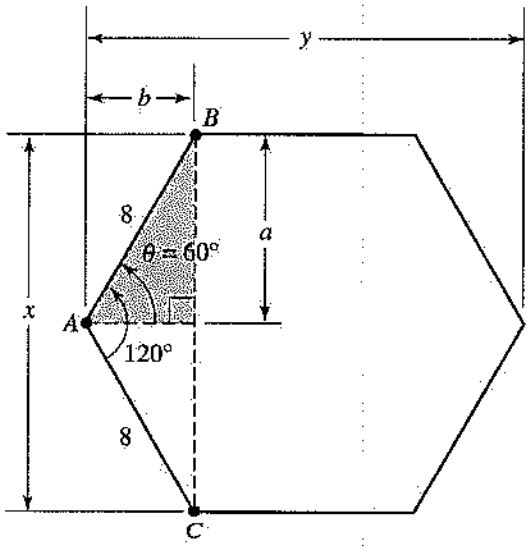


Figure 9.12

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In the right triangle in Figure 9.12,

$$\sin 60° = \frac{a}{8}$$

implying that $a = 8 \sin 60° \approx 6.93$ mm. Thus:

$$x = 2a \approx 13.9 \text{ mm}$$

B. Again, in the right triangle in Figure 9.12,

$$\cos 60° = \frac{b}{8}$$

implying that $b = 8 \cos 60° = 4$ mm. Therefore:

$$y = 2b + 8 = 16 \text{ mm}$$

Example 9.16

Two angles of a triangle measure 62° and 48°. The length of the side opposite the unknown angle is 52 cm.

A. Find the measure of the unknown angle.
B. Find the lengths of the unknown sides.

Solution:

A. The sum of the interior angles of a triangle is 180°. Hence, if θ is the unknown angle, then

$$\theta = 180° - (62° + 48°) = 70°$$

B. Let x and y be the sides opposite the angles that measure 48° and 62°, respectively. To find these unknown sides, apply the law of sines by matching each side with the sine of the angle opposite that side:

$$\frac{52}{\sin 70°} = \frac{x}{\sin 48°} = \frac{y}{\sin 62°}$$

Hence:

$$x = \frac{52 \sin 48°}{\sin 70°} \approx 41.1 \text{ cm}$$

and

$$y = \frac{52 \sin 62°}{\sin 70°} \approx 48.9 \text{ cm}$$

Example 9.17

The sides of a triangle have lengths 22.5 m, 16.2 m, and 14.9 m.

A. Determine the measure in degrees of the largest angle.
B. Find the area of the triangle.

Solution:

A. The largest angle in a triangle is always opposite its longest side. Hence, the largest angle is opposite the side of length 22.5 m. If θ is the largest angle, then, by the law of cosines,

$$22.5^2 = 16.2^2 + 14.9^2 - 2(16.2)(14.9) \cos \theta$$

Solving for $\cos \theta$ gives

$$\cos \theta = \frac{22.5^2 - 16.2^2 - 14.9^2}{-2(16.2)(14.9)}$$

The inverse cosine function in conjunction with a calculator set in degree mode shows that θ is obtuse:

$$\theta = \cos^{-1}\left[\frac{22.5^2 - 16.2^2 - 14.9^2}{-2(16.2)(14.9)}\right] \approx 92.6°$$

B. From part A, the angle between the sides of lengths 16.2 m and 14.9 m is 92.6°. Thus, the area A of this triangle is

$$A = \frac{ab \sin \theta}{2} = \frac{(16.2)(14.9)\sin 92.6°}{2} \approx 121 \text{ m}^2$$

9.7 Analytic Trigonometry

Below is a list of the most frequently used trigonometric identities. For additional trigonometric identities, refer to the *Fundamentals of Engineering Reference Handbook*.

1. $\sin \theta \csc \theta = 1$
2. $\cos \theta \sec \theta = 1$
3. $\tan \theta \cot \theta = 1$
4. $\tan \theta = \dfrac{\sin \theta}{\cos \theta}$
5. $\cot \theta = \dfrac{\cos \theta}{\sin \theta}$
6. $\sin^2 \theta + \cos^2 \theta = 1$
7. $1 + \tan^2 \theta = \sec^2 \theta$
8. $1 + \cot^2 \theta = \csc^2 \theta$
9. $\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$
10. $\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$
11. $\sin 2\theta = 2 \sin \theta \cos \theta$
12. $\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$

Example 9.18

Given that $\sin \theta = \frac{3}{5}$ and θ is acute, find:

A. $\sin 2\theta$ **B.** $\cos(\theta + \pi)$

Solution:

From Example 9.13, if $\sin \theta = \frac{3}{5}$ and θ is acute, then $\cos \theta = \frac{4}{5}$.

A. Use trigonometric identity 11; then

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$$\sin 2\theta = 2 \sin \theta \cos \theta = 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right) = \frac{24}{25}$$

B. Use trigonometric identity 9; then

$$\cos(\theta + \pi) = \cos \theta \cos \pi - \sin \theta \sin \pi$$
$$= \left(\frac{4}{5}\right)(-1) - \left(\frac{3}{5}\right)(0) = -\frac{4}{5}$$

Example 9.19

Simplify each expression to a single trigonometric function or a constant.

A. $\cos \theta \tan^2 \theta + \cos \theta$ **B.** $\dfrac{\cos^2 \theta}{1 - \sin \theta} - \sin \theta$

Solution:

A. Begin by factoring out $\cos \theta$ and then applying trigonometric identities 7 and 2:

$$\cos \theta \tan^2 \theta + \cos \theta = \cos \theta (\tan^2 \theta + 1)$$
$$= \cos \theta \sec^2 \theta$$
$$= (\cos \theta \sec \theta) \sec \theta$$
$$= (1) \sec \theta$$
$$= \sec \theta$$

B. Begin by subtracting fractions and then apply trigonometric identity 6:

$$\frac{\cos^2 \theta}{1 - \sin \theta} - \sin \theta = \frac{\cos^2 \theta - \sin \theta(1 - \sin \theta)}{1 - \sin \theta}$$
$$= \frac{\cos^2 \theta - \sin \theta + \sin^2 \theta}{1 - \sin \theta}$$
$$= \frac{1 - \sin \theta}{1 - \sin \theta}$$
$$= 1$$

provided that $1 - \sin \theta \neq 0$

Example 9.20

Write each expression as a first power of the cosine function.

A. $\cos^2 \theta$ **B.** $\sin^4 \theta$

Solution:

A. Use identity 12, $\cos 2\theta = 2 \cos^2 \theta - 1$, to solve for $\cos^2 \theta$:

$$\cos 2\theta = 2 \cos^2 \theta - 1$$
$$2 \cos^2 \theta = 1 + \cos 2\theta$$
$$\cos^2 \theta = \frac{1}{2}(1 + \cos 2\theta)$$

B. Use identity 12, $\cos 2\theta = 1 - 2\sin^2 \theta$, to solve for $\sin^2 \theta$:

$$\cos 2\theta = 1 - 2 \sin^2 \theta$$
$$2 \sin^2 \theta = 1 - \cos 2\theta$$
$$\sin^2 \theta = \frac{1}{2}(1 - \cos 2\theta)$$

Now, express $\sin^4 \theta$ in terms of the cosine function:

$$\sin^4 \theta = \sin^2 \theta \sin^2 \theta$$
$$= \frac{1}{2}(1 - \cos 2\theta)\frac{1}{2}(1 - \cos 2\theta)$$
$$= \frac{1}{4}(1 - 2 \cos 2\theta + \cos^2 2\theta)$$

Finally, using the results of part **A**, replace $\cos^2 2\theta$ with $\frac{1}{2}(1 + \cos 4\theta)$:

$$\sin^4 \theta = \frac{1}{4}\left[1 - 2 \cos 2\theta + \frac{1}{2}(1 + \cos 4\theta)\right]$$
$$= \frac{1}{4}\left(\frac{3}{2} - 2 \cos 2\theta + \frac{1}{2} \cos 4\theta\right)$$
$$= \frac{1}{8}(3 - 4 \cos 2\theta + \cos 4\theta)$$

9.8 Complex Numbers

The **imaginary unit** i is defined by

$$i = \sqrt{-1}$$

where $i^2 = -1$.

A complex number z in **rectangular form** is written as

$$z = ai + b$$

where a and b are real numbers and i is the imaginary unit, as shown in the complex plane in Figure 9.13.

A complex number z in **polar form** is given by

$$z = re^{i\theta} = r(\cos \theta + i \sin \theta)$$

where $a = r \cos \theta$, $b = r \sin \theta$, $r = \sqrt{a^2 + b^2}$, and $\tan \theta = \frac{b}{a}$, as shown in Figure 9.13.

The **complex conjugate** of $z = a + bi$ is $\bar{z} = a - bi$. The product $z\bar{z}$ is the nonnegative real number $a^2 + b^2$.

DeMoivre's theorem is used to raise a complex number z in polar form to an integer power:

$$z^k = r^k e^{ik\theta} = r^k(\cos k\theta + i \sin k\theta)$$

If $z = re^{i\theta} = r(\cos \theta + i \sin \theta)$, with $z \neq 0$ and n a positive integer, then z has exactly n distinct nth roots $w_0, w_1, w_2, \ldots, w_{n-1}$, which are given by the ***n*th root formula:**

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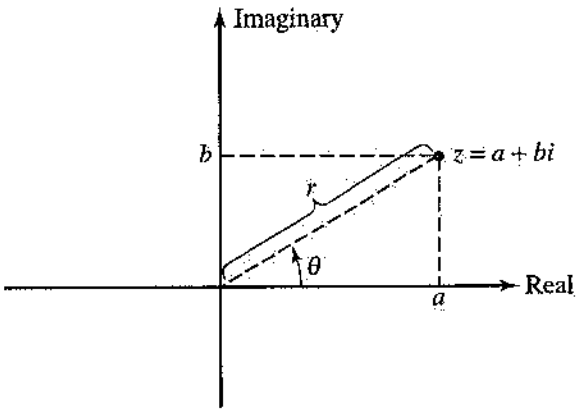


Figure 9.13

$$w_k = r^{1/n}e^{i(0+2\pi k)/n} = r^{1/n}\left[\cos\left(\frac{\theta+2\pi k}{n}\right) + i\sin\left(\frac{\theta+2\pi k}{n}\right)\right], \text{ for } k = 0, 1, 2, \ldots, n-1$$

Example 9.21

In each case, convert the given complex number to the indicated form.

A. $3 + 3i$ to polar form

B. $4.47e^{5.82i}$ to rectangular form

Solution:

A. For $3 + 3i$, $a = 3$ and $b = 3$. Thus:

$$r = \sqrt{a^2 + b^2} = \sqrt{3^2 + 3^2} = \sqrt{18} = 3\sqrt{2} \approx 4.24$$

Since $3 + 3i$ lies in quadrant I in the complex plane, θ must be a first-quadrant angle that satisfies the equation $\tan\theta = \frac{b}{a} = \frac{3}{3} = 1$. Hence,

$$\theta = \tan^{-1} 1 = \frac{\pi}{4} \approx 0.785$$

Thus, $3 + 3i = 3\sqrt{2}\, e^{(\pi/4)i} \approx 4.24e^{0.785i}$.

B. To express $4.47e^{5.82i}$ in rectangular form, use a calculator set in radian mode:

$$4.47e^{5.82i} = 4.47(\cos 5.82 + i \sin 5.82) \approx 4.00 - 2.00i$$

Example 9.22

In each case, given that $z_1 = 3 + 5i$ and $z_2 = 1 - 5i$, perform the indicated operation.

A. z_1z_2 **B.** $\frac{z_1}{z_2}$

Solution:

A. To find the product of two complex numbers in rectangular form, multiply each part of the first complex number by each part of the second complex number:

$$\begin{aligned} z_1z_2 &= (3 + 5i)(1 - 5i) \\ &= 3 - 15i + 5i - 25i^2 \\ &= 28 - 10i \end{aligned}$$

B. To find the quotient of two complex numbers in rectangular form, multiply by the complex conjugate of the denominator to obtain division by a nonnegative real number:

$$\begin{aligned} \frac{z_1}{z_2} &= \frac{3 + 5i}{1 - 5i} = \frac{(3 + 5i)(1 + 5i)}{(1 - 5i)(1 + 5i)} \\ &= \frac{-22 + 20i}{26} \\ &\approx -0.846 + 0.769i \end{aligned}$$

Example 9.23

In each case, given that $z_1 = 5e^{0.3i}$ and $z_2 = 3e^{2.5i}$, perform the indicated operation and write the answer in rectangular form:

A. z_1z_2 **B.** $z_1 + z_2$

Solution:

A. To multiply (or divide) complex numbers in polar form, add (or subtract) exponents:

$$z_1z_2 = (5e^{0.3i})(3e^{2.5i}) = 15e^{(0.3i+2.5i)} = 15e^{2.8i}$$

Now, convert to rectangular form:

$$15e^{2.8i} = 15(\cos 2.8 + i \sin 2.8) \approx -14.1 + 5.02i$$

B. To find the sum (or difference) of two complex numbers in polar form, convert the numbers to rectangular form and then add (or subtract) the real and imaginary parts of these numbers, separately. Hence:

$$\begin{aligned} z_1 + z_2 &= 5e^{0.3i} + 3e^{2.5i} \\ &= 5(\cos 0.3 + i \sin 0.3) + 3(\cos 2.5 + i \sin 2.5) \\ &= (5 \cos 0.3 + 3 \cos 2.5) + i\,(5 \sin 0.3 + 3 \sin 2.5) \\ &\approx 2.37 + 3.27i \end{aligned}$$

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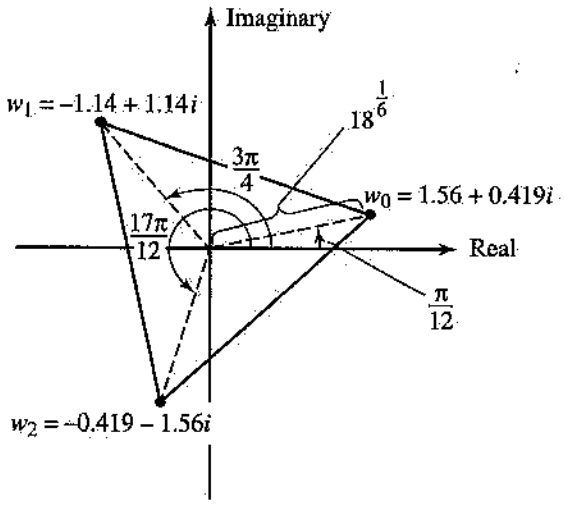


Figure 9.14

Example 9.24

In each case, given that $z = 3 + 3i$, find the indicated power or roots and write the answer in rectangular form.

A. z^4 **B.** the three cube roots of z

Solution:

A. From Example 9.21A, $z = 3 + 3i = 3\sqrt{2}e^{(\pi/4)i}$. Hence, by DeMoivre's theorem:

$$z^4 = (3\sqrt{2}e^{(\pi/4)i})^4 = (3\sqrt{2})^4 e^{4(\pi/4)i} = 324e^{\pi i}$$

Now, convert this answer to rectangular form:

$$324e^{\pi i} = 324(\cos\pi + i\sin\pi) = -324 + 0i = -324$$

B. Again, from Example 9.21A, $z = 3 + 3i = 3\sqrt{2}e^{(\pi/4)i}$. Hence, by the nth root formula, this complex number has three distinct cube roots given by

$$w_k = (3\sqrt{2})^{1/3} e^{i[(\pi/4)+2\pi k]/3}$$

for $k = 0$, 1, and 2. Hence, the three cube roots are

$$w_0 = (3\sqrt{2})^{1/3} e^{i(\pi/12)} = 18^{1/6}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$$
$$\approx 1.56 + 0.419i$$

$$w_1 = (3\sqrt{2})^{1/3} e^{i(3\pi/4)} = 18^{1/6}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$
$$\approx -1.14 + 1.14i$$

$$w_2 = (3\sqrt{2})^{1/3} e^{i(17\pi/12)} = 18^{1/6}\left(\cos\frac{17\pi}{12} + i\sin\frac{17\pi}{12}\right)$$
$$\approx -0.419 - 1.56i$$

Figure 9.14 shows the three cube roots of $3 + 3i$ in the complex plane.

Note that w_0, w_1, and w_2 form the vertices of an *equilateral triangle*. In general, the nth roots of a complex number form the vertices of a regular n-sided polygon in the complex plane.

9.9 Matrices

A **matrix** is a rectangular array of numbers enclosed by a pair of brackets. Each number in the matrix is called an *element*. Consider matrix A with m *rows* and n *columns*:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1j} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2j} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{ij} & \cdots & a_{in} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mj} & \cdots & a_{mn} \end{bmatrix}$$

This matrix is of *order* $m \times n$ and can be denoted more compactly by writing $A = [a_{ij}]$, where a_{ij} stands for the element in the ith row, jth column. Two matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ are equal if both are of order $m \times n$ and $a_{ij} = b_{ij}$ for all $i = 1, 2, 3, \ldots, m$ and $j = 1, 2, 3, \ldots, n$.

A **square matrix** is a matrix of order $n \times n$. The elements $a_{11}, a_{22}, \ldots, a_{nn}$ are called its *main diagonal elements*. A square matrix of order $n \times n$, with the digit 1 along its main diagonal and zeros elsewhere, is called an **identity matrix** of order $n \times n$. The **inverse** of a square matrix A of order $n \times n$ is denoted by A^{-1}. The product of a square matrix A and its inverse A^{-1} is the identity matrix I:

$$AA^{-1} = A^{-1}A = I$$

The **transpose** of matrix A, denoted by A^T, is formed by switching the rows of matrix A with its columns.

Three rules are used to add, subtract, and multiply matrices (matrix division is undefined):

1. **Matrix addition:** If $A = [a_{ij}]$ and $B = [b_{ij}]$ are matrices of order $m \times n$, then the sum $A + B$ is a matrix of order $m \times n$ defined by $A + B = [a_{ij} + b_{ij}]$.
2. **Scalar multiplication:** If $A = [a_{ij}]$ is a matrix of order $m \times n$ and k is a real number, then the scalar multiple kA is a matrix of order $m \times n$ defined by $kA = [ka_{ij}]$.

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3. **Matrix multiplication:** If $A = [a_{ij}]$ is a matrix of order $m \times n$ and $B = [b_{ij}]$ is a matrix of order $n \times p$, then the product AB is a matrix of order $m \times p$ defined by using a row times column procedure; that is, $AB = [c_{ij}]$, where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{in}b_{nj}$.

Example 9.25

In each case, given the matrices $A = \begin{bmatrix} 2 & 5 \\ -3 & 0 \\ 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 6 \\ -2 & -5 \\ 0 & 3 \end{bmatrix}$, perform the indicated operation.

A. $2A - B$ **B.** $A^T + B^T$

Solution:

A. Using the rules for scalar multiplication and matrix addition, proceed as follows:

$$2A - B = 2A + (-1)B = 2\begin{bmatrix} 2 & 5 \\ -3 & 0 \\ 1 & 4 \end{bmatrix} + (-1)\begin{bmatrix} 4 & 6 \\ -2 & -5 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 10 \\ -6 & 0 \\ 2 & 8 \end{bmatrix} + \begin{bmatrix} -4 & -6 \\ 2 & 5 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ -4 & 5 \\ 2 & 5 \end{bmatrix}$$

B. First, find the transpose matrices A^T and B^T by switching rows and columns:

$$A^T = \begin{bmatrix} 2 & -3 & 1 \\ 5 & 0 & 4 \end{bmatrix}, \text{ and } B^T = \begin{bmatrix} 4 & -2 & 0 \\ 6 & -5 & 3 \end{bmatrix}$$

Now, use the rule for matrix addition to obtain the sum of the transposes:

$$A^T + B^T = \begin{bmatrix} 2 & -3 & 1 \\ 5 & 0 & 4 \end{bmatrix} + \begin{bmatrix} 4 & -2 & 0 \\ 6 & -5 & 3 \end{bmatrix} = \begin{bmatrix} 6 & -5 & 1 \\ 11 & -5 & 7 \end{bmatrix}$$

Example 9.26

Given the matrices $A = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix}$, and $C = \begin{bmatrix} 2 & -1 \\ 3 & 4 \\ 0 & -2 \end{bmatrix}$, find each product.

A. AB **B.** CA

Solution:

A. Since matrices A and B are square matrices of order 2×2, the product AB is defined and is also of order 2×2. Using the rule for matrix multiplication, proceed as follows:

$$AB = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix} =$$

$$\begin{bmatrix} (3)(3) + (4)(-2) & (3)(-4) + (4)(3) \\ (2)(3) + (3)(-2) & (2)(-4) + (3)(3) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Since this product yields the identity matrix I of order 2×2, matrices A and B are inverses of each other.

B. Since matrix C has two columns and matrix A has two rows, the product CA is defined and is of order 3×2, since matrix C has three rows and matrix A has two columns. Using the rule for matrix multiplication, proceed as follows:

$$CA = \begin{bmatrix} 2 & -1 \\ 3 & 4 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix} =$$

$$\begin{bmatrix} (2)(3) + (-1)(2) & (2)(4) + (-1)(3) \\ (3)(3) + (4)(2) & (3)(4) + (4)(3) \\ (0)(3) + (-2)(2) & (0)(4) + (-2)(3) \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ 17 & 24 \\ -4 & -6 \end{bmatrix}$$

Note that the number of columns in matrix A is *not* the same as the number of rows in matrix C. Hence, the product AC is not defined. Unlike multiplication of real numbers, matrix multiplication, in general, is not commutative; that is, $AC \neq CA$.

9.10 Determinants and Inverses of Matrices

A **determinant** of order n contains n^2 elements arranged in n rows and n columns and enclosed by two vertical bars. The **minor** of an element a_{ij} in a determinant is denoted by M_{ij}, and represents the determinant that remains after deleting the row and column in which the element a_{ij} appears. The **cofactor** of an element a_{ij} in a determinant, denoted by C_{ij}, differs from M_{ij} at most in sign and is given by $C_{ij} = (-1)^{i+j}M_{ij}$.

The value of an nth-order determinant is a sum found by multiplying each element in any row or column by its corresponding cofactor, and then adding the products. This sum is called the **expansion of the determinant** by the specified row or column. The

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expansion of a second-order determinant by any row or column is

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

The inverse of an $n \times n$ matrix A can be found by using the **inverse formula:**

$$A^{-1} = \frac{\text{adj}(A)}{|A|}$$

where adj(A) is the *adjoint* of matrix A, obtained by replacing each element in the transpose matrix A^T with the value of its corresponding cofactor. If $|A| = 0$, then matrix A does not have an inverse.

A **system of n linear equations in n variables** $(x_1, x_2, \ldots, x_n)$, with each equation written in the form $a_1x_1 + a_2x_2 + \ldots + a_nx_n = k$, may be represented by the matrix equation

$$AX = K$$

where A is an $n \times n$ matrix containing the coefficients of the variables, X is an $n \times 1$ matrix containing the variables, and K is an $n \times 1$ matrix containing the constants on the right-hand sides of the equations. The **solution** of the $n \times n$ linear system of equations is given by $X = A^{-1}K$, provided that matrix A has an inverse.

Example 9.27

In each case, given the determinant $|A| = \begin{vmatrix} 1 & 3 & -2 \\ 3 & 0 & 1 \\ -1 & 2 & -4 \end{vmatrix}$, determine the value of (1) the minor and (2) the cofactor of the given element.

A. a_{31} **B.** a_{23}

Solution:

A. (1) To find M_{31}, the minor of element a_{31}, delete the third row and first column of $|A|$ and evaluate the remaining determinant:

$$M_{31} = \begin{vmatrix} 3 & -2 \\ 0 & 1 \end{vmatrix} = (3)(1) - (-2)(0) = 3$$

(2) The cofactor C_{31} of a_{31} is given by

$$C_{31} = (-1)^{3+1}M_{31} = M_{31} = 3$$

B. (1) To find M_{23}, the minor of element a_{23}, delete the second row and third column of $|A|$ and evaluate the remaining determinant:

$$M_{23} = \begin{vmatrix} 1 & 3 \\ -1 & 2 \end{vmatrix} = (1)(2) - (3)(-1) = 5$$

(2) The cofactor C_{23} of a_{23} is given by

$$C_{23} = (-1)^{2+3}M_{23} = -M_{23} = -5$$

Example 9.28

Evaluate each determinant.

A. $|A| = \begin{vmatrix} 1 & 3 & -2 \\ 3 & 0 & 1 \\ -1 & 2 & -4 \end{vmatrix}$ **B.** $|B| = \begin{vmatrix} 1 & -2 & 3 & -1 \\ 0 & 2 & 0 & 3 \\ 0 & 4 & 0 & -2 \\ 2 & 0 & 0 & 4 \end{vmatrix}$

Solution:

A. Since element $a_{22} = 0$, it is best to expand about the second row or second column. Choose the second row:

$$|A| = 3C_{21} + 0C_{22} + 1C_{23} = 3C_{21} + 1C_{23}$$

Now,

$$C_{21} = (-1)^{2+1}\begin{vmatrix} 3 & -2 \\ 2 & -4 \end{vmatrix} = 8$$

and

$$C_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 3 \\ -1 & 2 \end{vmatrix} = -5$$

Hence:

$$|A| = 3C_{21} + 1C_{23} = 3(8) + 1(-5) = 19$$

B. Since the third column of this determinant contains three zeros, it is best to expand about this column:

$$|B| = 3C_{13} + 0C_{23} + 0C_{33} + 0C_{43} = 3C_{13}$$

Now,

$$C_{13} = (-1)^{1+3}\begin{vmatrix} 0 & 2 & 3 \\ 0 & 4 & -2 \\ 2 & 0 & 4 \end{vmatrix} = \begin{vmatrix} 0 & 2 & 3 \\ 0 & 4 & -2 \\ 2 & 0 & 4 \end{vmatrix}$$

Since the first column of this third-order determinant contains two zeros, it is best to expand about this column:

$$C_{13} = 2(-1)^{3+1}\begin{vmatrix} 2 & 3 \\ 4 & -2 \end{vmatrix} = -32$$

Hence:

$$|B| = 3C_{13} = 3(-32) = -96$$

Example 9.29

Find the inverse of each matrix.

A. $A = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$ **B.** $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & -4 \\ -1 & 0 & 3 \end{bmatrix}$

Solution:

A. The inverse of matrix A is given by $A^{-1} = \frac{\text{adj}(A)}{|A|}$. Begin by finding the determinant:

$$|A| = \begin{vmatrix} 3 & 4 \\ 2 & 3 \end{vmatrix} = (3)(3) - (4)(2) = 1$$

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To find adj(A), form A^T and then replace each element in A^T with its corresponding cofactor:

$$A^T = \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix} \quad \text{implies} \quad \text{adj}(A) = \begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix}$$

Hence:

$$A^{-1} = \frac{\text{adj}(A)}{|A|} = \frac{\begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix}}{1} = \begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix}$$

To verify this inverse, show that $AA^{-1} = A^{-1}A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, the identity matrix of order 2×2.

B. The inverse of matrix B is defined by $B^{-1} = \frac{\text{adj}(B)}{|B|}$. For the determinant, expand about the third row:

$$|B| = -1(-1)^{3+1}\begin{vmatrix} 2 & -3 \\ 3 & -4 \end{vmatrix} + 3(-1)^{3+3}\begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = -4$$

To find adj(B), form B^T and then replace each element in B^T with its corresponding cofactor:

$$B^T = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 3 & 0 \\ -3 & -4 & 3 \end{bmatrix} \quad \text{implies} \quad \text{adj}(B) = \begin{bmatrix} 9 & -6 & 1 \\ -2 & 0 & -2 \\ 3 & -2 & -1 \end{bmatrix}$$

Hence:

$$B^{-1} = \frac{\text{adj}(B)}{|B|} = -\frac{1}{4}\begin{bmatrix} 9 & -6 & 1 \\ -2 & 0 & -2 \\ 3 & -2 & -1 \end{bmatrix} = \begin{bmatrix} -\frac{9}{4} & \frac{3}{2} & -\frac{1}{4} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ -\frac{3}{4} & \frac{1}{2} & \frac{1}{4} \end{bmatrix}$$

To check, show that $BB^{-1} = B^{-1}B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, the identity matrix of order 3×3.

Example 9.30

Solve each system of linear equations by using the inverse method.

A. $3x + 4y = 6$
$2x + 3y = 5$

B. $x + 2y - 3z = -1$
$2x + 3y - 4z = 2$
$-x + 3z = 5$

Solution:

A. Let

$$A = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad \text{and} \quad K = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

Then, by the rules for matrix multiplication and equality of matrices, the matrix equation $AX = K$ is equivalent to the given system of linear equations:

$$\begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

The solution of this matrix equation is $X = A^{-1}K$, provided that A^{-1} exists. Use the results of Example 9.29**A** to find X:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}^{-1}\begin{bmatrix} 6 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 6 \\ 5 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

Hence, the solution of this 2×2 system of linear equations is $x = -2$ and $y = 3$.

B. Let

$$B = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & -4 \\ -1 & 0 & 3 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad \text{and} \quad K = \begin{bmatrix} -1 \\ 2 \\ 5 \end{bmatrix}$$

Then, by the rules for matrix multiplication and equality of matrices, the matrix equation $BX = K$ is equivalent to the given system of linear equations:

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & -4 \\ -1 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 5 \end{bmatrix}$$

The solution of this matrix equation is $X = B^{-1}K$, provided that B^{-1} exists. Use the results of Example 9.29**B** to find X:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & -4 \\ -1 & 0 & 3 \end{bmatrix}^{-1}\begin{bmatrix} -1 \\ 2 \\ 5 \end{bmatrix} = \begin{bmatrix} -\frac{9}{4} & \frac{3}{2} & -\frac{1}{4} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ -\frac{3}{4} & \frac{1}{2} & \frac{1}{4} \end{bmatrix}\begin{bmatrix} -1 \\ 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 3 \end{bmatrix}$$

Hence, the solution of this 3×3 system of linear equation is $x = 4$, $y = 2$, and $z = 3$.

9.11 Vectors

A **vector** is a directed line segment in space that represents a force, a velocity, or a displacement. The *basic vectors*, denoted by the boldface letters **i**, **j**, and **k**, are directed line segments in the rectangular coordinate system from the origin (0,0,0) to points (1,0,0), (0,1,0), and (0,0,1), respectively. The *position vector* of a point in space, denoted by $\overrightarrow{OP} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, is a directed line segment from the origin

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CHAPTER

10

Mechanics of Materials

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Introduction

Mechanics of materials is the branch of mechanics that relates external loads applied to a body in equilibrium to internal forces and the subsequent deformation of the body. In this chapter the concepts of stress and strain and their relationship are introduced. The deformation of axially loaded members is also presented. The torsion and the bending of structural members are analyzed. Structures under combined loading, as well as the design of columns, are discussed.

10.1 Normal Stress
10.2 Shear Stress
10.3 Normal Strain
10.4 Shear Strain
10.5 The Relationship of Stress and Strain
10.6 Thermal Strain
10.7 Poisson's Ratio
10.8 Deformation in Axially Loaded Members
10.9 Torsion
10.10 Bending
10.11 Transverse Shear Stress
10.12 Combined Loading
10.13 Principal and Maximum Shear Stresses
10.14 Thin-Walled Pressure Vessels
10.15 Columns

10.1 Normal Stress

Consider a bar with constant cross-sectional area (prismatic) under the action of a constant force that is applied at the centroid of the bar. The stress is defined as the load, P, divided by the area (load per unit area), A, and is called normal stress, denoted by σ. If the bar is in tension, the stress is called tensile stress; if the bar is compressed, the stress is called compressive stress. Stress has units of pounds per square inch (lb/in^2 or psi) in the English system and newtons per square meter (N/m^2) or pascals (Pa) in the metric system.

$$\sigma = \frac{P}{A} \qquad [10.1]$$

σ_t = tensile stress (+)

σ_c = compressive stress (−)

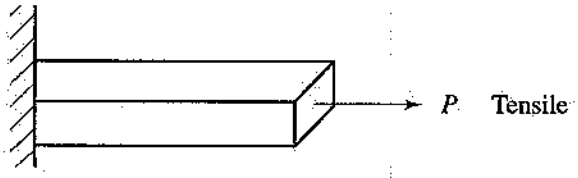


Figure 10.1

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Example 10.1

For the composite bar shown below, determine the normal stress in each section.

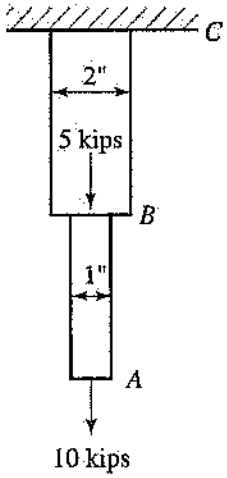


Figure 10.2

Solution:

$$\sigma_{AB} = \frac{10}{\frac{\pi}{4}(1)^2} = 12.73 \text{ ksi (tensile)}$$

$$\sigma_{BC} = \frac{15}{\frac{\pi}{4}(2)^2} = 4.77 \text{ ksi (tensile)}$$

10.2 Shear Stress

If two plates are bolted or glued to each other, the stress parallel to the area of contact, A_s, is called the shear stress and is denoted by τ.

$$\tau = \frac{V}{A_s} \qquad [10.2]$$

where V = shear load,

A_s = shear area.

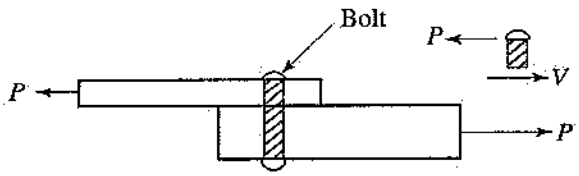


Figure 10.3

Example 10.2

If the allowable shearing stress in the bolt shown below is 350 MPa, determine the minimum diameter of the bolt.

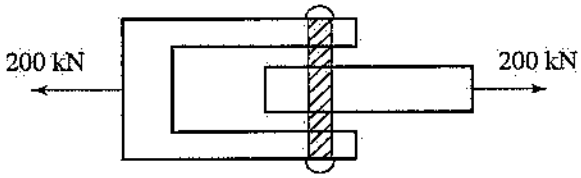


Figure 10.4

Solution:

This is a case of double shear.

$$V = \frac{P}{2} = \frac{200}{2} = 100 \text{ kN}$$

$$\tau = \frac{V}{A_s}, \quad A_s = \frac{V}{\tau_{all}} = \frac{100 \times 10^3}{350 \times 10^6} = 2.857 \times 10^{-4}$$

$$d = \sqrt{\frac{4A_s}{\pi}} = \sqrt{\frac{4(2.857 \times 10^{-4})}{\pi}} = 0.01907 \text{ m}$$

10.3 Normal Strain

Consider again a prismatic bar under the action of a tensile load. The initial length of the bar is l, and after the load is applied the bar elongates by δ. Normal strain is defined as the ratio of change in length to initial length and is denoted by ε. If the bar is in tension, the strain is called tensile strain. If the bar is in compression, the strain is called compressive strain. Note that strain is dimensionless.

$$\varepsilon = \frac{\delta}{\ell}$$

ε_t = tensile strain [10.3]

ε_c = compressive strain

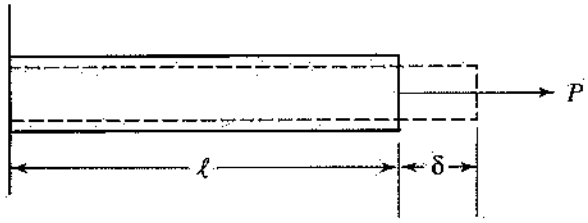


Figure 10.5

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10.4 Shear Strain

Consider the stress element under the action of the shear stresses shown in the drawing below. The shear stress on each side (face) must be the same in order for the stress element to remain in equilibrium. Shear strain, denoted by γ, is defined as the change in the angle between two sides of the stress element from the 90-degree initial angle. Shear strain is a measure of distortion of the element under the action of shear stresses. Shear strain is dimensionless, and its unit must be radians.

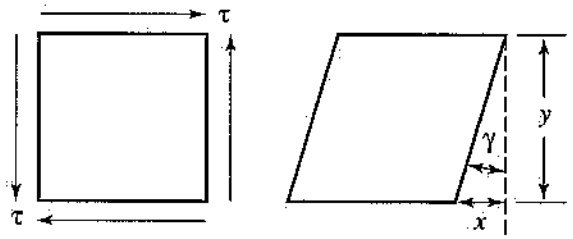


Figure 10.6

$$\tan\gamma \approx \gamma = \frac{x}{y} \qquad [10.5]$$

10.5 The Relationship of Stress and Strain

The relationship of elastic stress to elastic strain is based on a law known as Hooke's law:

$$\sigma = E\varepsilon \qquad [10.6]$$

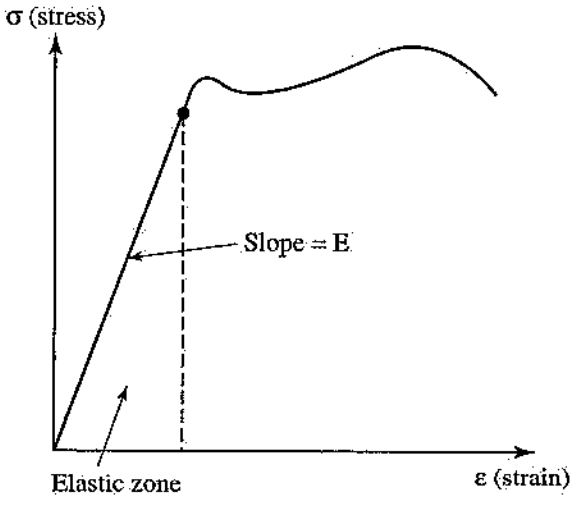


Figure 10.7

Here, E is a material property known as the modulus of elasticity or Young's modulus. For example, E for structural steel is about 30×10^6 psi or 200 GPa. If the relationship between stress and strain is represented graphically, then E is the slope of the straight line in the linear zone.

Hooke's law can be applied to a member in pure shear (i.e., a shaft being twisted) and relates shear stress, τ, to shear strain, γ:

$$\tau = G\gamma \qquad [10.7]$$

where G is a material property known as modulus of rigidity.

Example 10.3

For the rectangular block shown below, $G = 500$ MPa. If the lower plate is fixed and the upper plate moves through 0.7 mm as a result of applied force P, determine the value of force P.

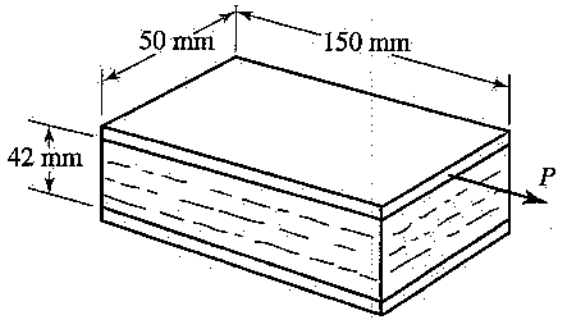


Figure 10.8

Solution:

$$\gamma = \frac{0.7 \text{ mm}}{42 \text{ mm}} = 0.0166 \text{ rad}$$

$$\tau = G\gamma = (500 \text{ MPa})(0.0166) = 8.3 \text{ MPa}$$

$$\tau = \frac{V}{A} = \frac{P}{A} \Rightarrow P = \tau A$$

$$P = (8.3 \times 10^6)(0.05 \times 0.15) = 62{,}250 \text{ N}$$

10.6 Thermal Strain

If a bar is constrained, as the temperature changes the bar will deform. The strain due to change in temperature is called thermal strain.

$$\varepsilon_T = \alpha\Delta T \qquad [10.8]$$

Here, α is the coefficient of thermal expansion, with units of 1/°F or 1/°C, and is defined as the strain

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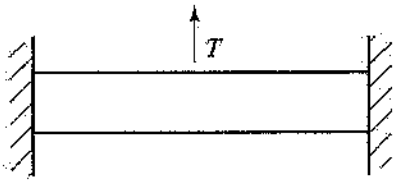


Figure 10.9

due to a 1-degree change in temperature. Note that α, like E and G, is a material property.

10.7 Poisson's Ratio

Consider a bar under the action of a load. The direction of the load is axial direction, and perpendicular to the axial direction is lateral direction. When the load is applied, the bar will elongate in the axial direction, but at the same time it will contract in the lateral direction. Poisson's ratio, denoted by ν, is defined as the negative of the ratio of the lateral strain to the axial strain.

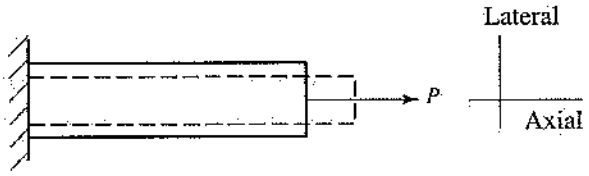


Figure 10.10

$$\nu = -\frac{\varepsilon_{lateral}}{\varepsilon_{axial}} \qquad [10.9]$$

The three properties E, G, and ν are related to each other as follows:

$$G = \frac{E}{2(1+\nu)} \qquad [10.10]$$

Example 10.4

The plastic rod shown below is 150 mm long and 12 mm in diameter. Determine the change in length and the change in diameter that occur when a 400-N load is applied if, for the plastic, E =2.5 GPa and $\nu = 0.4$.

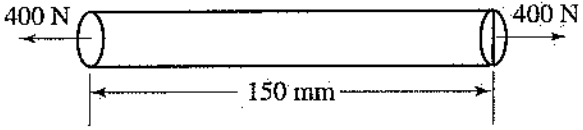


Figure 10.11

Solution:

$$\sigma = E\varepsilon_{axial}$$

$$\varepsilon_{axial} = \frac{\sigma}{E} = \frac{P}{EA} = \frac{400}{\frac{\pi}{4}(0.012)^2(2.5\times10^9)} = 0.0044$$

$$\delta = \ell\varepsilon_{axial} = 0.666\text{-mm increase in length}$$

$$\varepsilon_{lateral} = -\nu\varepsilon_{axial} = -0.4(0.0044) = -0.00177$$

$$\Delta d = d\varepsilon_{lateral} = (12\text{ mm})(-0.00177)$$

$$\Delta d = -0.0213\text{-mm decrease in diameter}$$

10.8 Deformation in Axially Loaded Members

For an axially loaded member (refer to Figure 10.10) the axial deformation can be determined by using the following equation:

$$\delta = \frac{P\ell}{EA} \qquad [10.11]$$

Example 10. 5

The circular steel rod ($E = 30\times10^6$ psi) shown in Figure 10.12, which is 2 inches in diameter, is loaded axially. Determine the load P that causes a deformation of 0.1 in in the rod.

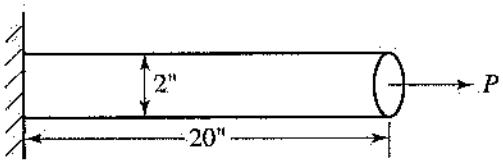


Figure 10.12

Solution:

$$\delta = \frac{P\ell}{EA} \Rightarrow P = \frac{\delta EA}{\ell}$$

$$P = \frac{(0.1)(30\times10^6)\left(\frac{\pi}{4}2^2\right)}{20} = 471.238\text{ kips}$$

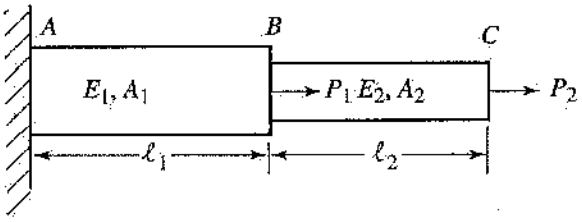


Figure 10.13

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For a composite structure (stepped shaft), the summation form of the equation can be used to determine the total axial deformation.

$$\delta = \Sigma \frac{P\ell}{EA} \qquad [10.12]$$

Example 10.6

Using Figure 10.13, determine the total elongation of the stepped shaft for the following values:

$$P_1 = 10 \text{ kips}, P_2 = 5 \text{ kips}$$
$$E_1 = E_2 = 30 \times 10^6 \text{ psi}$$
$$d_1 = 2 \text{ in}, d_2 = 1 \text{ in}$$
$$\ell_1 = 20 \text{ in}, \ell_2 = 15 \text{ in}$$

Solution:

$$\delta = \Sigma \frac{P\ell}{EA} = \left(\frac{P\ell}{EA}\right)_{AB} + \left(\frac{P\ell}{EA}\right)_{BC}$$

$$\delta = \left(\frac{(15 \times 10^3)(20)}{(30 \times 10^6)\left(\frac{\pi}{4}2^2\right)}\right) + \left(\frac{(5 \times 10^3)(15)}{(30 \times 10^6)\left(\frac{\pi}{4}1^2\right)}\right)$$

$$= 0.006366 \text{ in}$$

10.9 Torsion

If a straight bar with constant circular cross section (i.e., shaft) is fixed at one end and is twisted by a torque at the other end, such a member is said to be in pure torsion.

When the shaft is twisted, shear stresses are developed throughout the shaft as a result. The shear stress varies as a function of radial distance, r, and is maximum at the surface when r is maximum (i.e., $r = c$).

$$\tau = \frac{Tr}{J}, \qquad \tau_{max} = \frac{Tc}{J} \qquad [10.13]$$

Here, J is called the polar moment of inertia; for a circular cross section (shaft) $J = \pi d^4/32$ or $J = \pi c^4/2$ and has units of inch^4 or meter^4. (Refer to Table 2 in Chapter 11 for formulas to determine J.)

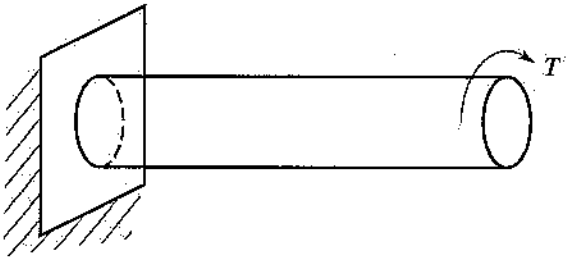


Figure 10.14

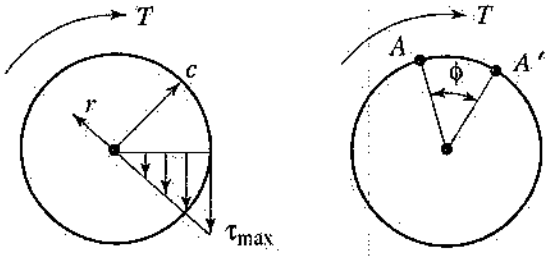


Figure 10.15

Twisting will also cause the shaft to deform (distortion). The angle of twist, (ϕ), for a shaft of length L, under the action of torque T, is given by the equation

$$\phi = \frac{TL}{GJ} \qquad [10.14]$$

Here G is the shear modulus (material property), and ϕ is dimensionless and should be expressed in radians.

Note that for a hollow shaft the polar moment of inertia, J, is given by

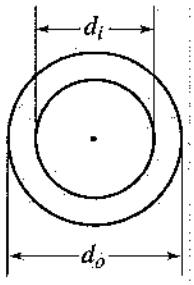


Figure 10.16

$$J = \frac{\pi(d_o^4 - d_i^4)}{32}, \quad J = \frac{\pi(c_o^4 - c_i^4)}{2} \qquad [10.15]$$

where d_o and c_o are the outside diameter and radius, respectively, and d_i and c_i are the inside diameter and radius, respectively.

Example 10.7

For the hollow cylinder shown in Figure 10.17, determine the torque, T, that causes a maximum shearing stress of 400 MPa.

Solution:

$$J = \frac{\pi(d_o^4 - d_i^4)}{32} = \frac{\pi(2^4 - 1.5^4)}{32}$$

$$J = 1.074 \text{ cm}^4 = 1.074 \times 10^{-8} \text{ m}^4$$

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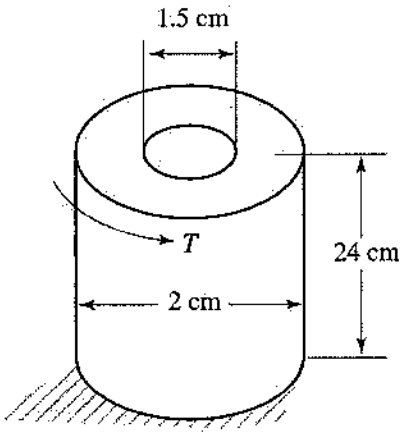


Figure 10.17

$$\tau_{max} = \frac{Tc_o}{J} \Rightarrow T = \frac{\tau_{max} J}{c_o}$$

$$T = \frac{(400 \times 10^6)(1.074 \times 10^{-8})}{0.01} = 429.6 \text{ N·m}$$

Example 10.8

Using Figure 10.17, determine the angle of twist for $T = 500$ N · m and $G = 100$ GPa.

Solution:

$$\phi = \frac{TL}{GJ} = \frac{(500)(0.24)}{(100 \times 10^9)(1.074 \times 10^{-8})} = 0.11 \text{ rad}$$

For hollow, thin-walled shafts, shear stress can be determined by using the equation

$$\tau = \frac{T}{2A_m t} \qquad [10.16]$$

where t = thickness of the wall,

A_m = total mean area enclosed by the shaft measured to the midpoint of the wall.

10.10 Bending

A beam is defined as a long, prismatical structure designed to support loads at various points along the member. In most cases, the loads are applied perpendicularly to the axis of the beam and cause only shear and bending in the beam.

Internal Forces

Consider the simply supported beam shown in Figure 10.18.

If the beam is cut at a certain section, it is observed that, for the left or right section to be in

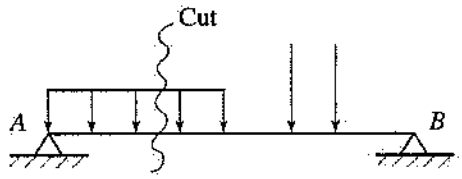


Figure 10.18

equilibrium, a shear load, V, and a bending moment, M, must be present at that section. The shear load and the bending moment are called internal forces. Note in Figure 10.19 that the directions of V and M for the left and right sections of the beam are indicated by the sign conventions used for positive shear and positive moment.

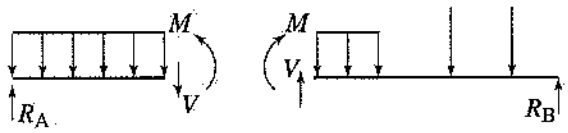


Figure 10.19

To determine the internal forces, do the following:

1. *Determine the reactions at the supports.*
2. *Cut the section where the internal forces are to be calculated, and draw the free-body diagram.*
3. *Use equilibrium equations to determine the internal forces.*

Note that the result is the same regardless of the section (right or left) used.

Example 10.9

For the beam shown below, determine the shear force and the bending moment at a point 2 m to the right of point A.

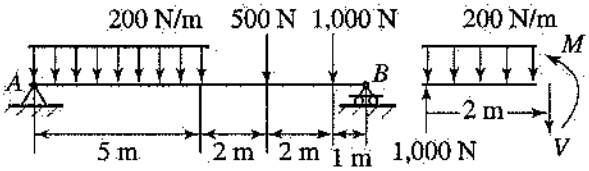


Figure 10.20

Solution:

First, calculate the reactions:

$$\Sigma M_A = 0$$

$$B(10) - (1000)(9) - (500)(7) - (1000)(2.5) = 0$$

$$B = 1{,}500 \text{ N}, \quad A = 1{,}000 \text{ N}$$

Now, determine the internal forces by cutting the beam at a point 2 m to the right of point A and applying the equilibrium equations:

$$V = 600 \text{ N}, \quad M = 1{,}600 \text{ N·m}$$

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Shear and Bending-Moment Diagrams

Since determination of maximum shear and bending moment is important in design, it is more convenient to plot shear and bending moment along the beam than to cut many sections to calculate these values. The graphs obtained are called the shear diagram and the bending-moment diagram, respectively.

Drawing Shear and Bending-Moment Diagrams

The following relations hold:

$$\frac{dV}{dx} = -w, \qquad \int_{V_1}^{V_2} dV = -\int_{x_1}^{x_2} w dx \qquad [10.17]$$

where w is the distributed load acting on the beam.
Net shear force = −(area under distributed load from x_1 to x_2)

$$\frac{dM}{dx} = V, \qquad \int_{M_1}^{M_2} dM = \int_{x_1}^{x_2} V dx \qquad [10.18]$$

Net bending moment = (area under shear load from x_1 to x_2)

Example 10.10

Draw the shear and bending-moment diagrams for the beam shown.

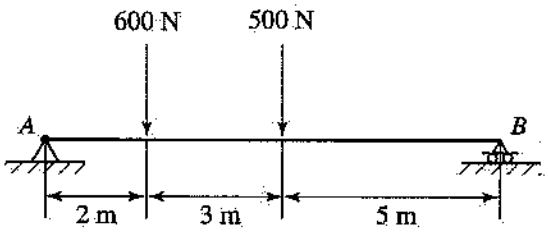


Figure 10.21

Solution:

First, determine the reactions.

$$\Sigma M_A = 0$$
$$B(10) - 500(5) - 600(2) = 370 \text{ N}.$$
$$\Sigma F_y = 0$$
$$A + B - 500 - 600 = 0$$
$$A = 730 \text{ N}$$

Then draw the diagrams.

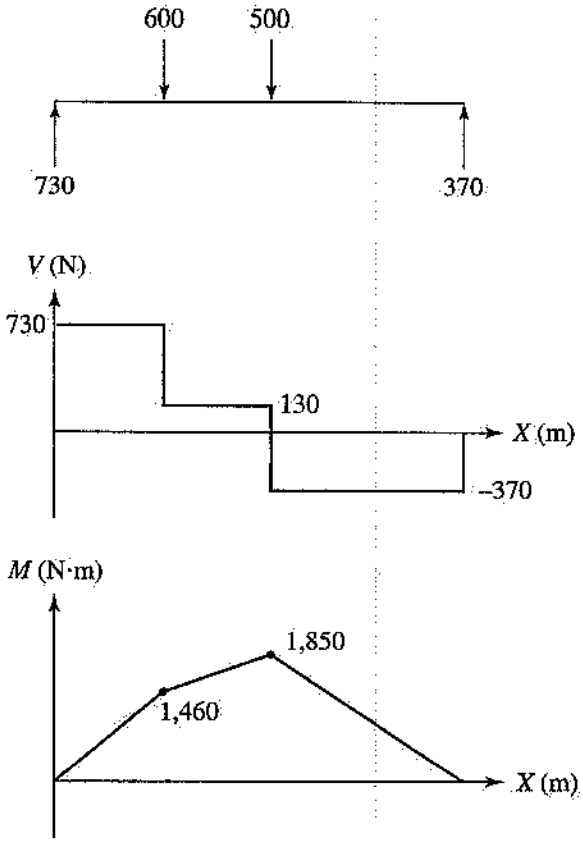


Figure 10.22

Example 10.11

Draw the shear and bending-moment diagrams for the beam shown.

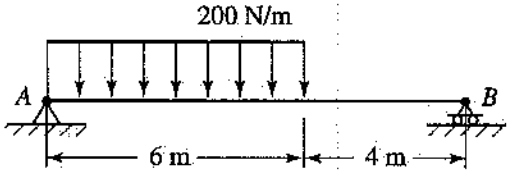


Figure 10.23

Solution:

First, determine the reactions.

$$\Sigma M_A = 0$$
$$B(10) - (200)(6)(3) = 0 \Rightarrow B = 360 \text{ N}$$
$$\Sigma F_y = 0$$
$$A + B - 200(6) = 0$$
$$A = 840 \text{ N}$$

Then draw the diagrams.

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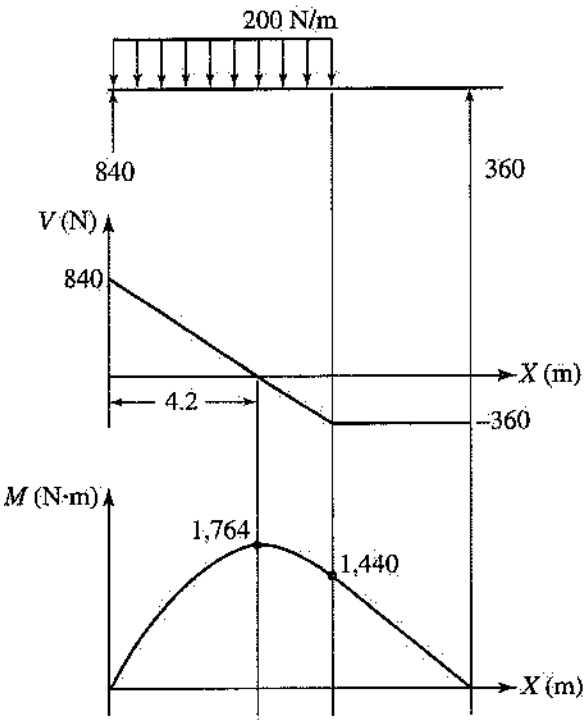


Figure 10.24

Stresses Due to Bending

Consider a prismatic beam that is bent under the action of applied loads, as shown below.

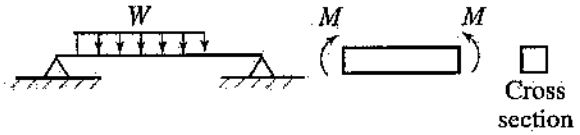


Figure 10.25

The bending stress, also known as the flexural stress, can be determined by using the equation

$$\sigma = \frac{My}{I} \qquad [10.19]$$

where σ = bending (flexural) stress,
M = bending moment at a given point,
y = distance from the centroid (neutral axis) to the point where the stress is to be calculated,
I = area moment of inertia with respect to the axis of bending through the centroid.

The maximum bending stress can be determined by using the equation

$$\sigma_{max} = \frac{Mc}{I} \qquad [10.20]$$

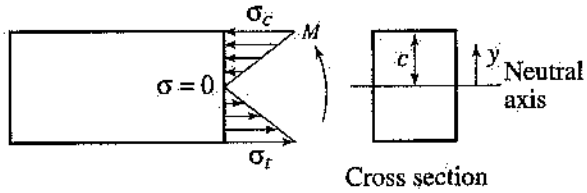


Figure 10.26

where c is the perpendicular distance from the neutral axis to a point farthest away from the neutral axis (top or bottom fiber).

Note that the stress distribution along the cross section is linear and the stress is zero at the centroid (neutral axis).

Example 10.12

Determine the maximum bending stress for the beam and loading of Example 10.10 with the cross section shown below.

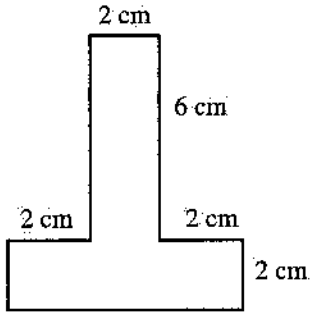


Figure 10.27

Solution:

For the moment diagram of Example 10.10,

$$M_{max} = 1{,}850 \text{ N·m}$$

To determine the maximum bending stress, first determine the area moment of inertia with respect to the neutral axis (refer to Example 11.24 in Chapter 11):

$$I = 100 \text{ cm}^4$$

Then calculate the maximum stress:

$$\sigma_{max} = \frac{M_{max}C}{I} = \frac{(1{,}850)(0.05)}{100 \times 10^{-8}} = 92.5 \text{ MPa}$$

Note that this is the maximum compressive stress that occurs at the top fiber.

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10.11 Transverse Shear Stress

The stress developed in a structure as a result of a shear force, V, is called transverse shear stress and is denoted by τ.

$$\tau = \frac{VQ}{It} \qquad [10.21]$$

where V = shear force at a given point,

Q = first moment of area with respect to the neutral axis = $\Sigma y A = \int y dA$,

I = area moment of inertia with respect to the neutral axis,

t = thickness of beam at position y for which the shear stress is calculated.

Note that the shear stress is zero at the top and bottom fibers (free surfaces) and is maximum at the neutral axis.

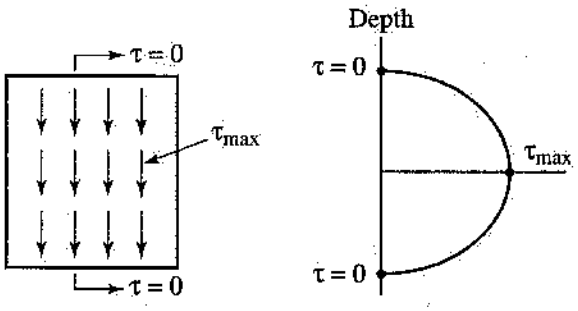


Figure 10.28

For the rectangular cross sections:

$$\tau_{max} = \frac{3V}{2A} \qquad [10.22]$$

where A = cross-sectional area.

For the solid circular cross sections:

$$\tau_{max} = \frac{4V}{3A} \qquad [10.23]$$

Example 10.13

Determine the maximum transverse shear stress for a 40 × 60-cm rectangular beam with the loading of Example 10.11.

Solution:

For the shear diagram of Example 10.11, the maximum shear load V = 840 N.

Since the cross section of the beam is rectangular, equation 10.22 can be used to determine the maximum transverse shear stress:

$$\tau_{max} = \frac{3V}{2A} = \frac{3\,(840)}{2\,(0.24)} = 5250 \text{ Pa}$$

10.12 Combined Loading

Consider a structural member subjected to some combination of axial, bending, and torsional loads.

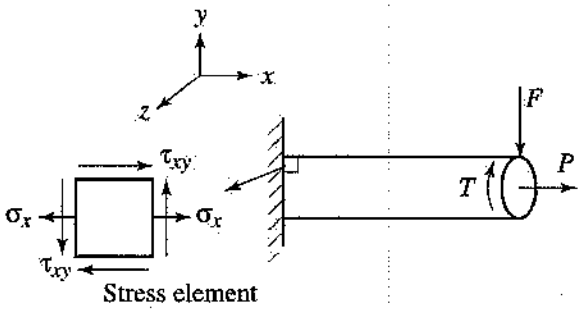


Figure 10.29

To determine the maximum stresses (normal and shear), first locate the point where the stresses are most severe. For a cantilever beam, this point is located at the fixed support. Once the stresses due to different loads are calculated, they can be represented on a very small element (stress element) and then maximum normal and shear stresses can be determined. The maximum normal stresses are called principal stresses, and the planes at which these stresses occur are called principal planes.

Example 10.14

Determine the normal and shearing stresses at points A and B in the figure below.

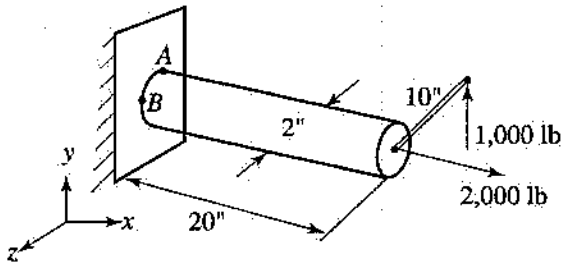


Figure 10.30

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Solution:

$$T = M_x = -1{,}000(10) = -10{,}000 \text{ lb} \cdot \text{in}$$

$$M_z = 1{,}000\,(20) = 20{,}000 \text{ lb} \cdot \text{in}$$

$$V_y = 1{,}000 \text{ lb}$$

$$P_x = 2{,}000 \text{ lb}$$

$$I = \frac{\pi}{4}(1)^4 = 0.7954 \text{ in}^4$$

$$J = 2I = 1.57 \text{ in}^4$$

$$A = \frac{\pi}{4}(2)^2 = 3.14 \text{ in}^2$$

At point A:

$$\sigma_A = -\frac{M_z\, c}{I_z} + \frac{P_x}{A} = -\frac{20{,}000(1)}{0.7854} + \frac{2000}{3.14}$$

$$= -24{,}828 \text{ psi (compressive)}$$

$$\tau_A = \frac{Tc}{J} = \frac{(10{,}000)(1)}{1.57} = 6{,}369 \text{ psi}$$

At point B:

$$\sigma_B = \frac{P_x}{A} = \frac{2000}{3.14} = 637 \text{ psi}$$

$$\tau_B = \frac{Tc}{J} - \frac{4\,V_y}{3A} = \frac{(10{,}000)(1)}{1.57} - \frac{4(1{,}000)}{3(3.14)}$$

$$= 5{,}944 \text{ psi}$$

10.13 Principal and Maximum Shear Stresses

To determine the principal stresses and maximum shear stress and their planes:

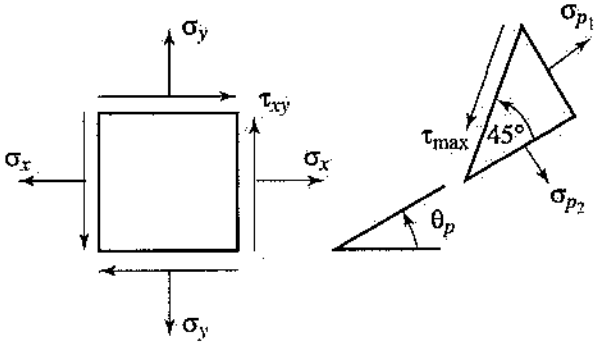


Figure 10.31

use the following equations:

$$\sigma_{p_1,p_2} = \sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \qquad [10.24]$$

$$\tau_{\max} = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

Also note that

$$\theta_p = \theta_s \pm 45 \qquad [10.25]$$

Example 10.15

Determine the principal and maximum shear stresses of Example 10.14 for point *A*.

Solution:

$$\sigma_x = -24{,}828 \text{ psi}, \quad \sigma_y = 0$$

$$\tau_{xy} = 6{,}369 \text{ psi}$$

Using equation 10.24:

$$\sigma_{p_1,p_2} = \frac{-24{,}828}{2} \pm \sqrt{\left(\frac{-24{,}828}{2}\right)^2 + (6{,}369)^2}$$

$$\sigma_1 = 1{,}538 \text{ psi}, \quad \sigma_2 = -26{,}366 \text{ psi}$$

$$\tau_{\max} = \sqrt{\left(\frac{-24{,}828}{2}\right)^2 + (6{,}369)^2} = 13{,}952 \text{ psi}$$

10.14 Thin-Walled Pressure Vessels

Cylindrical and spherical pressure vessels are used in industry for boilers or storage tanks. The term "thin-wall" refers to a tank for which the ratio of inner radius to thickness is equal to or larger than 10 ($r_i/t \geq 10$).

Cylindrical Vessels

For a cylinder of inner radius r and thickness t that is under pressure P, circumferential (hoop) and longitudinal stresses are developed in the vessel.

$$\sigma_{circ} = \sigma_{hoop} = \sigma_1 = \frac{Pr}{t} \qquad [10.26]$$

$$\sigma_{long} = \sigma_2 = \frac{Pr}{2t} \qquad [10.27]$$

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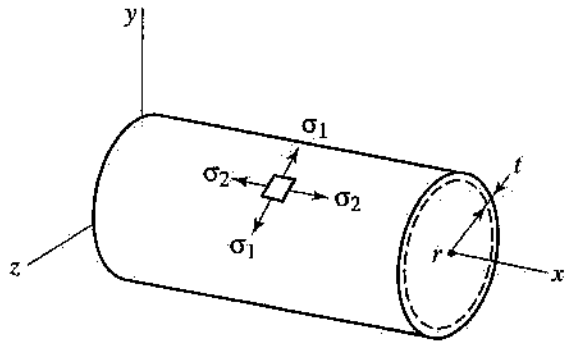


Figure 10.32

Example 10.16

A cylindrical pressure vessel has an inner radius of 2 m and a thickness of 3 cm. Determine the maximum internal pressure this vessel can sustain so that its circumferential stress component will not exceed 120 MPa.

Solution:

$$\sigma_1 = \frac{Pr}{t} \Rightarrow P = \frac{\sigma_1 t}{r}$$

$$P = \frac{(120 \times 10^6)(0.03)}{2} = 1.8 \text{ MPa}$$

Spherical Vessels

For a thin-walled spherical vessel subjected to inside pressure P:

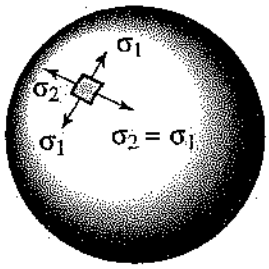


Figure 10.33

the stress at any location is given by the equation

$$\sigma_{long} = \sigma_1 = \sigma_2 = \frac{Pr}{2t} \qquad [10.28]$$

10.15 Columns

A column is defined as a long, slender member that is loaded axially in compression. In designing a column, the following rules must be followed:

1. *The normal stress due to the applied load should not exceed the allowable stress.*
2. *The axial deformation should not exceed the allowable deformation.*
3. *The load applied must be less than the critical load that causes buckling.*

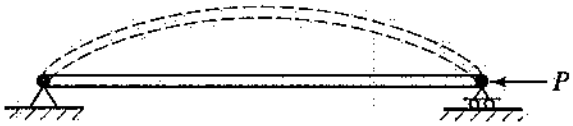


Figure 10.34

If a column is pinned at both ends, the smallest critical load P is given by the equation

$$P_{cr} = \frac{\pi^2 EI}{L^2} \qquad [10.29]$$

Equation 10.29 is known as *Euler's formula*. For other end conditions, the equation

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} \qquad [10.30]$$

can be used. Here, L_e is called the *effective length* and is shown in Figure10.35 for four end conditions.

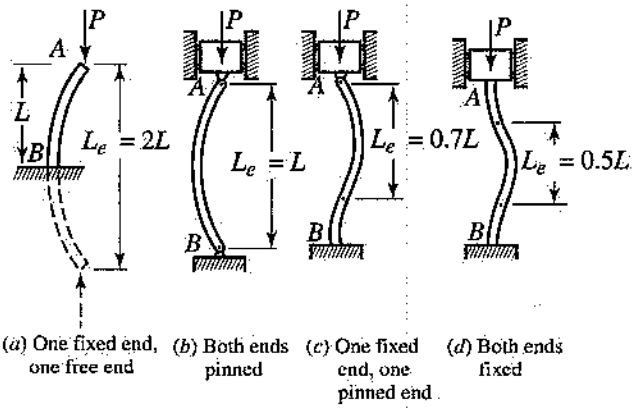


Figure 10.35

Example 10.17

A 2-m-long, pin-ended column of square cross section, for which each side is 10 cm, has $E = 12$ GPa and $\sigma_{all} = 15$ MPa. Determine the maximum allowable load that can be applied.

Solution:

Use Euler's formula.

$$I = \frac{(.1)^4}{12} = 8.3 \times 10^{-6} \text{ m}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{(3.14)^2(12 \times 10^9)(8.3 \times 10^{-6})}{2^2}$$

$$= 246.74 \text{ kN}$$

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Now determine the load based on allowable stress.

$$\sigma = \frac{P}{A} \Rightarrow P = \sigma A$$

$$P = (15 \times 10^6)(0.1)^2 = 150 \text{ kN}$$

The maximum load that can be applied to this column is the smaller of the two loads. Therefore:

$$P = 150 \text{ kN}$$

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PRACTICE PROBLEMS

1. Determine the normal stress in section *AB*.

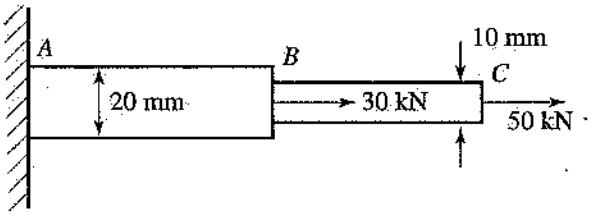


Figure 10.36

(A) 0.095 MPa
(B) 0.255 MPa
(C) 0.159 MPa
(D) 2.6 MPa

2. Determine the normal stress in link *AB* if *AB* has a cross-sectional area of 400 mm^2.

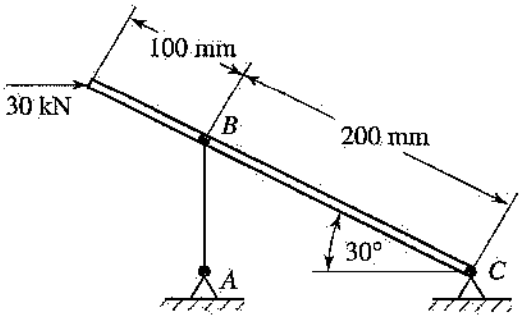


Figure 10.37

(A) 0.065 MPa
(B) 0.075 MPa
(C) 0.113 MPa
(D) 0.05 MPa

3. If the allowable shear stress for the bolts is 10 ksi, determine the minimum required diameter of each bolt.

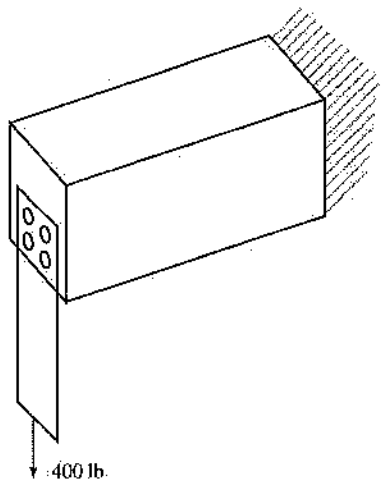


Figure 10.38

(A) 0.2 in
(B) 0.225 in
(C) 0.3in
(D) 0.113 in

4. If member *AB* elongates 10 mm as a result of the applied load, determine the normal strain in *AB*.

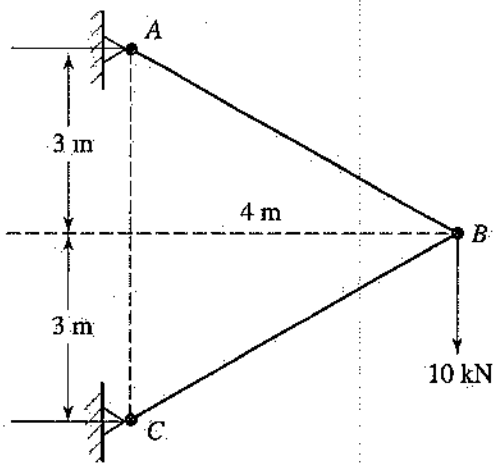


Figure 10.39

(A) 0.0033
(B) 0.002
(C) 0.0025
(D) 0.001

5. Determine the approximate vertical displacement of plate *A*. Each pad has cross-sectional dimensions of 30 × 30 mm and $G = 3$ MPa.

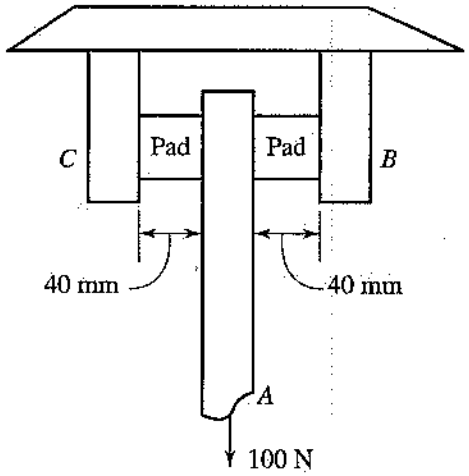


Figure 10.40

(A) 5 mm
(B) 1.48 mm
(C) 1.1 mm
(D) 0.2 mm

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6. Determine the load required to decrease the diameter of the rod by 0.003 mm if $E = 4$ GPa and $\nu = 0.3$.

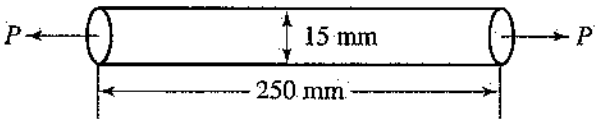
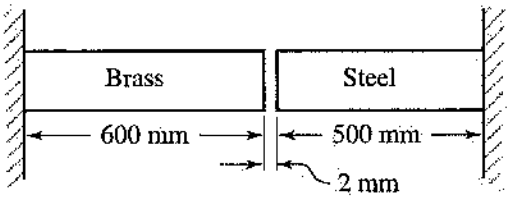


Figure 10.41

(A) 800 N
(B) 205 kN
(C) 253 N
(D) 471 N

7. Determine the change in temperature required to close the 2-mm gap between the sections.



Brass	Steel
$E = 100$ GPa	$E = 200$ GPa
$\alpha = 21 \times 10^{-6}$ 1/°C	$\alpha = 12 \times 10^{-6}$ 1/°C

Figure 10.42

(A) 108 C°
(B) 55 C°
(C) 85 C°
(D) 20 C°

8. Determine diameter d if the deflection of point C is 0.014 in and $E = 30 \times 10^6$ psi for both sections.

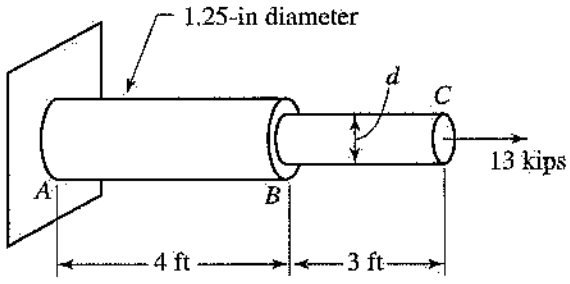


Figure 10.43

(A) 0.25 in
(B) 0.7 in
(C) 0.37 in
(D) 0.625 in

9. If the allowable shear stress is 900 KPa, determine the required diameter of a solid shaft under the action of a 1,500-N · m torque.
(A) 0.209 in
(B) 0.0091 in
(C) 0.121 in
(D) 0.263 in

10. Determine the angle of twist for the hollow shaft if $G = 80$ GPa.

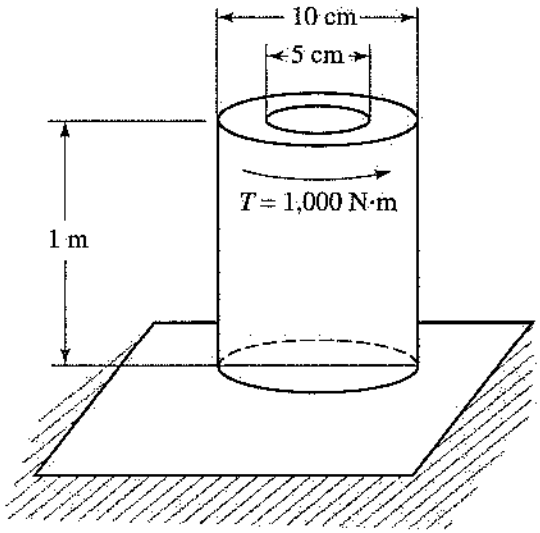


Figure 10.44

(A) 0.0127 rad
(B) 0.00136 rad
(C) 0.001 rad
(D) 0.52 rad

11. Determine the bending moment at the center (point C) of the beam.

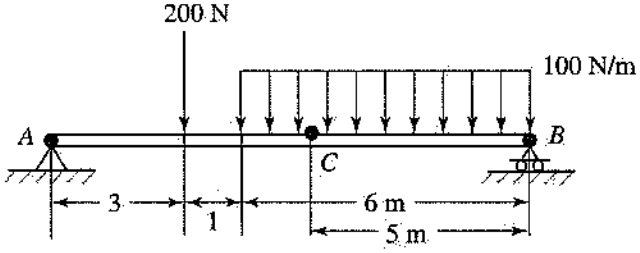


Figure 10.45

(A) 600 N·m
(B) 350 N·m
(C) 1,600 N·m
(D) 480 N·m

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12. Determine the maximum magnitude of the bending moment in the beam. The bending moment at each end of the beam is zero, and there are no concentrated couples along the beam.

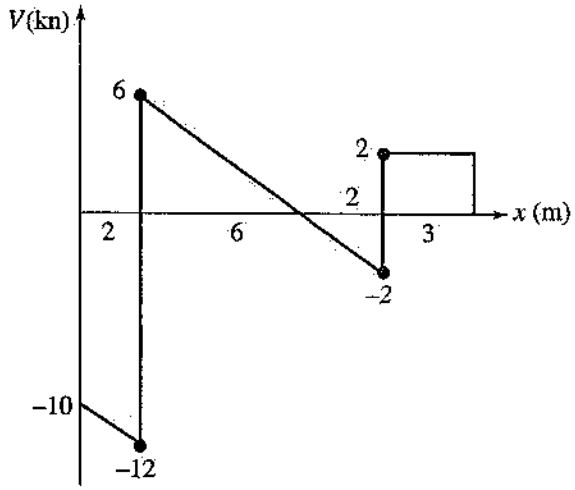


Figure 10.46

(A) 18 kN · m
(B) 6 kN · m
(C) 22 kN · m
(D) 4 kN · m

13. If the allowable bending stress is 5 ksi, determine h.

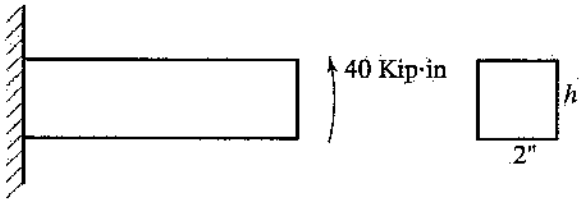


Figure 10.47

(A) 4 in
(B) 4.9 in
(C) 3.2 in
(D) 2 in

14. Determine the maximum bending stress for the beam.

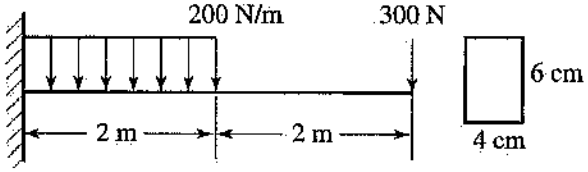


Figure 10.48

(A) 66.7 MPa
(B) 50 MPa
(C) 25.2 MPa
(D) 5.55 Mpa

15. If the allowable bending stress is 10 ksi, determine the diameter of the shaft loaded as shown.

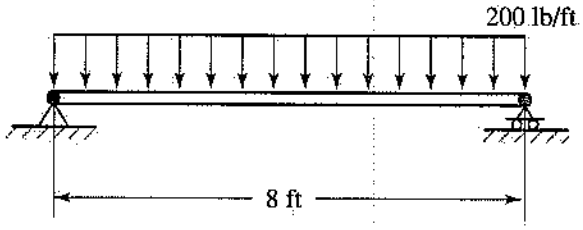


Figure 10.49

(A) 1.17 in
(B) 3.5 in
(C) 2.7 in
(D) 2.35 in

16. Determine the maximum transverse shear stress for the beam.

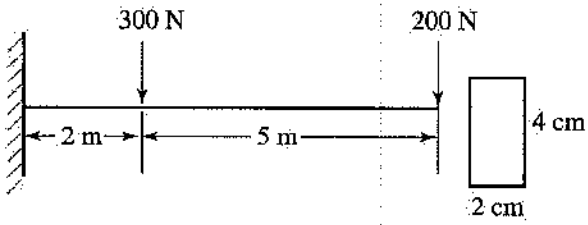


Figure 10.50

(A) 0.94 MPa
(B) 0.625 MPa
(C) 0.375 MPa
(D) 0.833 MPa

17. Determine the normal stress at point A for the shaft.

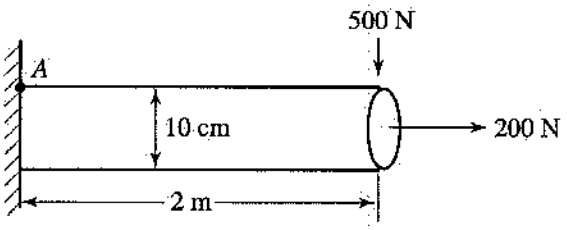


Figure 10.51

(A) 25.46 MPa
(B) 10.18 MPa
(C) 10.16 MPa
(D) 10.21 MPa

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18. Determine the maximum tensile stress for the stress element.

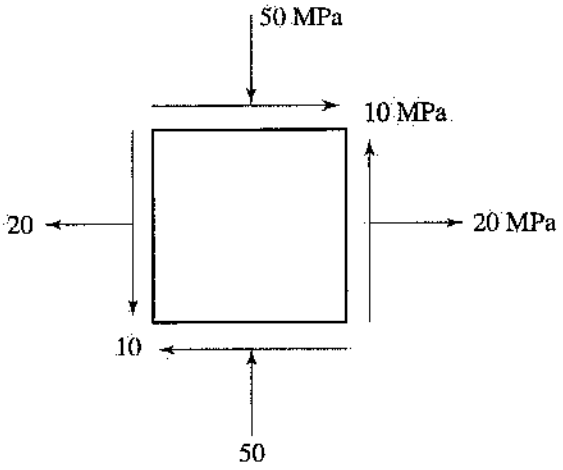


Figure 10.52

(A) 51.4 MPa
(B) 21.4 MPa
(C) 36.4 MPa
(D) 20 MPa

19. Determine the maximum shearing stress for the stress element.

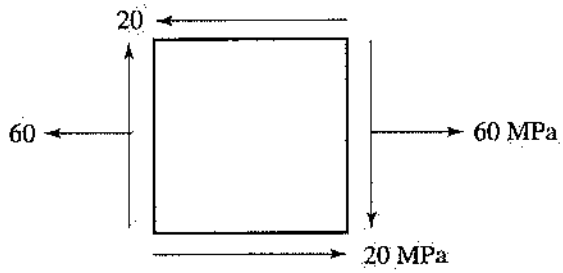


Figure 10.53

(A) 20 MPa
(B) 30 MPa
(C) 36 MPa
(D) 25 MPa

20. Determine the maximum pressure for a 60-cm-diameter cylinder, with thickness of 0.5 cm, if the allowable tensile stress is 150 MPa.
(A) 2.5 MPa
(B) 1.25 MPa
(C) 5 MPa
(D) 1.2 MPa

21. Determine the thickness of a 100-cm-diameter sphere under a pressure of 600 KPa if the allowable stress is 150 MPa.
(A) 2 cm
(B) 1 cm
(C) 1.5 cm
(D) 0.5 cm

22. Determine the effective length of a pin-ended square (20 × 20 cm) column under a 150-kN compressive load if $E = 15$ GPa.
(A) 7.25 m
(B) 39.7 m
(C) 11.5 m
(D) 22.5 m

Answer Key

1. B	7. A	13. B	18. A
2. A	8. C	14. A	19. C
3. D	9. A	15. C	20. A
4. B	10. B	16. A	21. B
5. B	11. B	17. D	22. C
6. D	12. C		

Answers Explained

1. **B** $P_{AB} = 80$ kN in tension, and

$$\sigma_{AB} = \frac{P_{AB}}{A_{AB}} = \frac{80}{\frac{\pi}{4}(0.02)^2} = 254.6 \text{ MPa}$$

2. **A** Draw an FBD of member BC, and take the moment about point C: $\Sigma M_c = 0$. Then $F_{AB} = 25.98$ kN, and

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{25.98}{400 \times 10^{-6}} = 0.065 \text{ MPa}$$

3. **D** Here, $\tau = \frac{V}{A_s}$, and $A_s = \frac{V}{\tau_{all}} = \frac{100}{10{,}000} = 0.01$ in^2. Then:

$$d = \sqrt{\frac{4A_s}{\pi}} = \sqrt{\frac{4(.01)}{\pi}} = 0.1128 \text{ in}$$

4. **B** In this case:

$$\epsilon = \frac{\delta}{l} = \frac{10 \text{ mm}}{5{,}000 \text{ mm}} = 0.002$$

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5. B This is a case of double shear. $V = \frac{100}{2} = 50$ N, and

$$\tau = \frac{V}{A} = \frac{50}{900 \times 10^{-6}} = 0.055 \text{ MPa}$$

Also:

$$\tau = G\gamma$$

$$\gamma = \frac{\tau}{G} = \frac{0.055}{3} = 0.0185 \text{ rad}$$

Then:

$$y = (40 \text{ mm})(0.0185 \text{ rad}) = 0.74 \text{ mm}$$

6. D In this case:

$$\epsilon_{\text{lateral}} = \frac{0.003}{15} = 0.0002 \quad \text{and}$$

$$\epsilon_{\text{axial}} = \frac{\epsilon_{\text{lateral}}}{\nu} = \frac{0.0002}{0.3} = 0.00066$$

Also:

$$\epsilon_{\text{axial}} = \frac{\sigma}{E} = \frac{P}{EA}$$

$$P = EA\epsilon_{\text{axial}} = 471 \text{ N}$$

7. A Here: $\delta = \Sigma \alpha l \, \Delta T$

$$\Delta T = \frac{\delta}{\Sigma \alpha l}$$

$$= \frac{2}{(21 \times 10^{-6})(600) + (12 \times 10^{-6})(500)}$$

$$= 108\text{C}^\circ$$

8. C In this case:

$$\delta = \Sigma \frac{Pl}{EA} \quad \text{and}$$

$$0.04 = \frac{13{,}000}{30 \times 10^6}\left(\frac{48}{\frac{\pi}{4}(1.25)^2} + \frac{36}{\frac{\pi}{4}(d)^2}\right)$$

Then $d = 0.93$ in.

9. A For a solid shaft, $\tau = \frac{16\,T}{\pi d^3}$. Then:

$$d = \sqrt[3]{\frac{16\,T}{\pi\tau}} = \sqrt[3]{\frac{16(1{,}500)}{\pi(840{,}000)}} = 0.209 \text{ in}$$

10. B For the hollow shaft, $J = \frac{\pi}{32}(0.1^4 - 0.05^4) = 9.2 \times 10^{-6}$ m^4. Then:

$$\phi = \frac{TL}{GJ} = \frac{(1{,}000)(1)}{(80 \times 10^9)(9.2 \times 10^{-6})}$$

$$= 0.00136 \text{ rad}$$

11. B First, determine the reactions:

$$B = 480 \text{ N} \qquad A = 320 \text{ N}$$

Cut the beam at point C, and use either the left or the right side of the beam:

$$\Sigma M_C = 0 \quad \text{and} \quad M_C = 1{,}150 \text{ N·m}$$

12. C The area under the shear diagram is equal to the net bending moment, so $M_{\max} = 22$ kN·m

13. B In this case:

$$\sigma_{\max} = \frac{M_{\max} C}{I} \qquad 5{,}000 = \frac{40{,}000\left(\frac{h}{2}\right)}{\frac{1}{12}(2)\,h^3}$$

Then $h = 4.9$ in.

14. A The maximum moment will occur at the support and is equal to 1,600 N·m, so

$$\sigma_{\max} = \frac{M_{\max} C}{I} = \frac{(1{,}600)(0.03)}{\frac{1}{12}(0.04)(0.06)^3}$$

$$= 66.7 \text{ MPa}$$

15. C The maximum moment will occur at the center of the shaft and is equal to 1,600 lb-ft = 19,200 lb-in. Then:

$$\sigma_{\max} = \frac{M_{\max} C}{I} = \frac{(19{,}200)\left(\frac{d}{2}\right)}{\frac{\pi}{64}d^4}$$

and $d = 2.7$ in.

16. A The maximum shear load $V = 500$ N. For a rectangular cross section:

$$\tau = \frac{3V}{2A} = \frac{3(500)}{2(0.02)(0.04)} = 0.94 \text{ MPa}$$

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17. D At point A:

$$\sigma_A = \frac{Mc}{I} + \frac{P}{A} = \frac{(1{,}000)(0.05)}{\frac{\pi}{64}(0.1)^4} + \frac{200}{\frac{\pi}{4}(0.1)^2} = 10.21 \text{ MPa}$$

18. A For the stress element:

$$\sigma_{p_1,p_2} = \frac{20 - 50}{2} \pm \sqrt{\left[\frac{20 - (-50)}{2}\right]^2 + (10)^2} \sigma_{\max} = 51.4 \text{ MPa}$$

19. C Here,

$$\tau_{\max} = \sqrt{\left(\frac{60 - 0}{2}\right)^2 + (-20)^2} = 36 \text{ MPa}$$

20. A For the cylinder:

$$\sigma_1 = \frac{Pr}{t}$$

$$P = \frac{\sigma_1 t}{r} = \frac{(150 \times 10^6)(0.005)}{0.3} = 2.5 \text{ MPa}$$

21. B For the sphere:

$$\sigma = \frac{Pr}{2t}$$

$$t = \frac{Pr}{2\sigma} = \frac{600 \times 10^3 \ (0.5)}{2(15 \times 10^6)} = 0.01 \text{ m} = 1 \text{ cm}$$

22. C Here, $P_{cr} = \frac{\pi^2 EI}{L_e^2}$. Then:

$$L_e = \sqrt{\frac{\pi^2 EI}{P_{cr}}} = 11.47 \text{ m}$$

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CHAPTER

11

Statics

Masoud Olia, Ph.D., PE

Introduction

Statics is the study of rigid bodies at rest. For a body to be at rest, the forces acting on the body have to balance each other. In this chapter, presentation of the concepts of static equilibrium of particles and rigid bodies is followed by discussion of the determination of external and internal forces for structures such as trusses and frames. Moments of inertia, centroids of plane areas, and basic concepts of friction are also discussed. The following is a complete list of this chapter's contents:

11.1 Forces
11.2 Moment
11.3 Equilibrium of Particles
11.4 Equilibrium of Rigid Bodies
11.5 Trusses and Frames
11.6 Friction
11.7 Center of Gravity
11.8 Moments of Inertia

11.1 Forces

A force is considered as a push or a pull exerted by one body on another. Force is a vector quantity, which has both magnitude and direction. Forces are classified as external, internal, concentrated, or distributed, as shown in Figure 11.1.

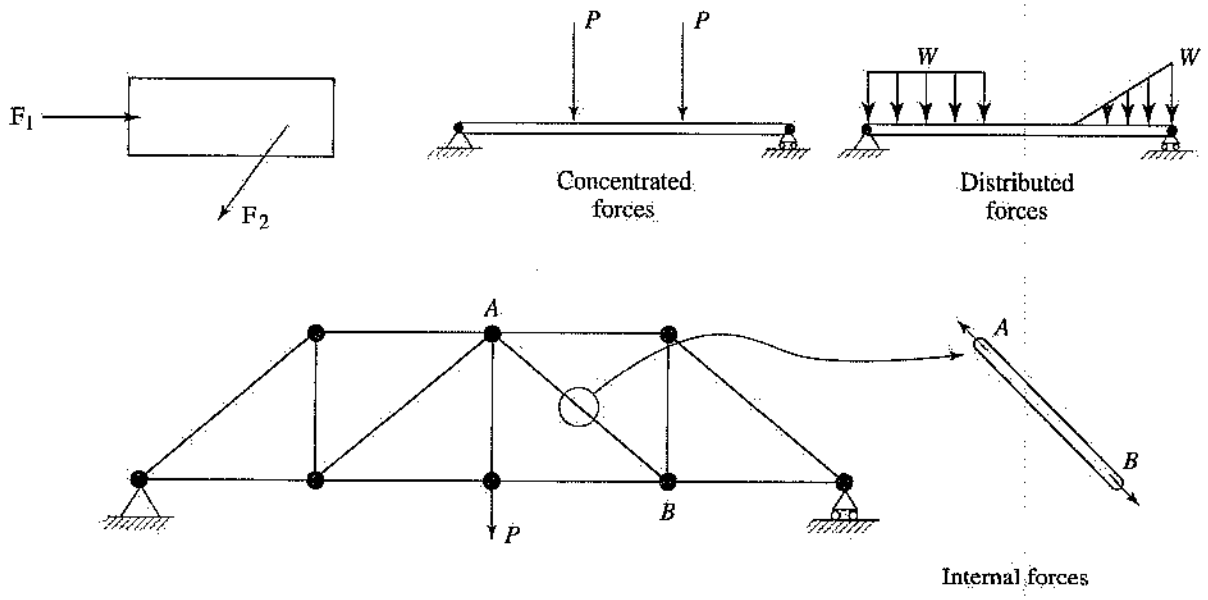
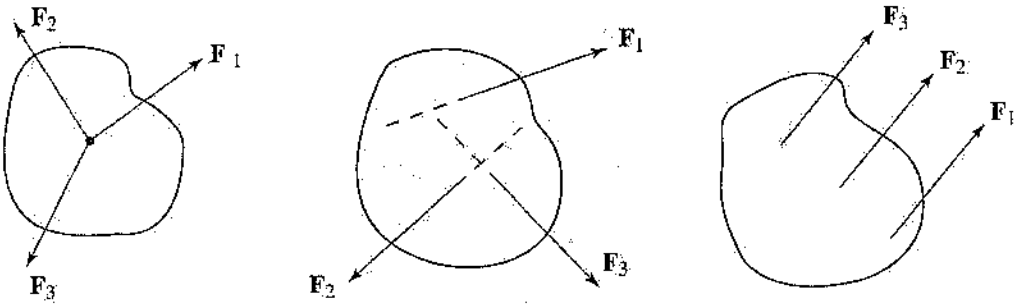


Figure 11.1

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Concurrent forces | Nonconcurrent forces | Parallel forces

Figure 11.2

A force system can be classified as concurrent or nonconcurrent. If all the forces acting on a body pass through the same point, the force system is called concurrent. If all the forces acting on the body do not pass through a common point, the force system is nonconcurrent.

Resolution of a Force (2-D)

A force can be resolved along any two arbitrary axes. It is more convenient and efficient, however, to resolve a force along the x- and y-axes (Cartesian or rectangular coordinates system). The appropriate equations are as follows:

$$F_x = F\cos\theta, \qquad F_y = F\sin\theta \qquad [11.1]$$

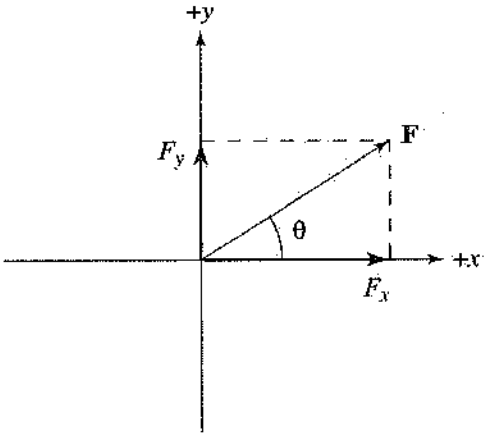


Figure 11.3

Example 11.1

Resolve the forces shown into their rectangular (x and y) components.

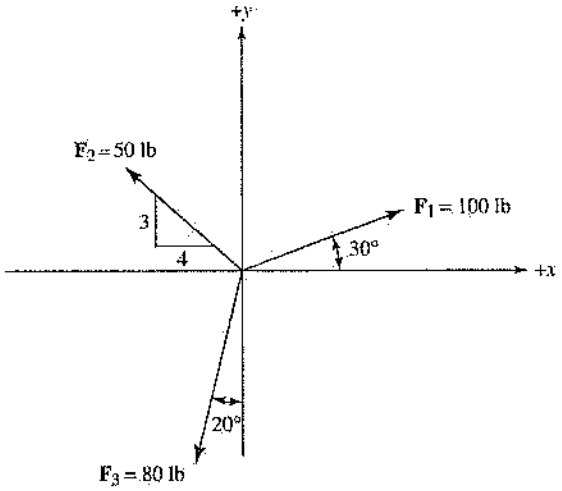


Figure 11.4

Solution:

$F_{1x} = 100\cos 30 = 86.6$ lb, $F_{1y} = 100\sin 30 = 50$ lb

$$F_{2x} = -50\left(\frac{4}{5}\right) = -40 \text{ lb}, \quad F_{2y} = 50\left(\frac{3}{5}\right) = 30 \text{ lb}$$

$F_{3x} = -80\sin 20 = -27.36$ lb, $F_{3y} = -80\cos 20 = -75.17$ lb

Resultant of a Concurrent Force System (2-D)

The resultant force R can be resolved into two rectangular components (R_x and R_y).

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \ldots = \Sigma\mathbf{F}$$

$$R_x = \Sigma F_x, \qquad R_y = \Sigma F_y$$

$$R = \sqrt{R_x^2 + R_y^2}$$

$$\theta = \tan^{-1}\frac{R_y}{R_x} \qquad [11.2]$$

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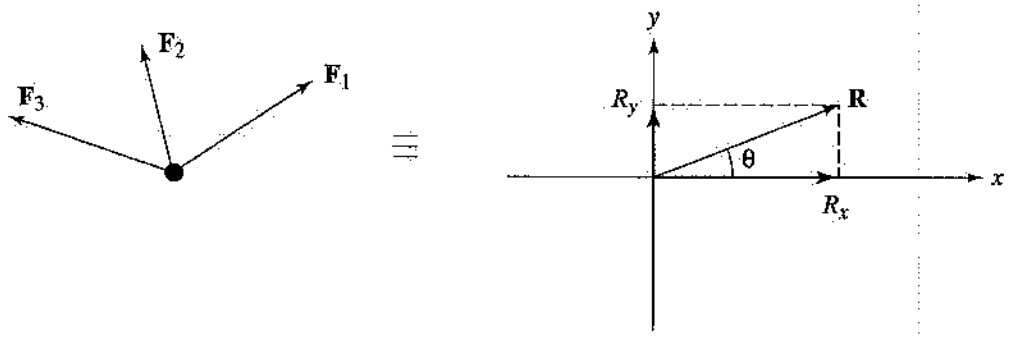


Figure 11.5

Example 11.2

Find the resultant of the forces shown.

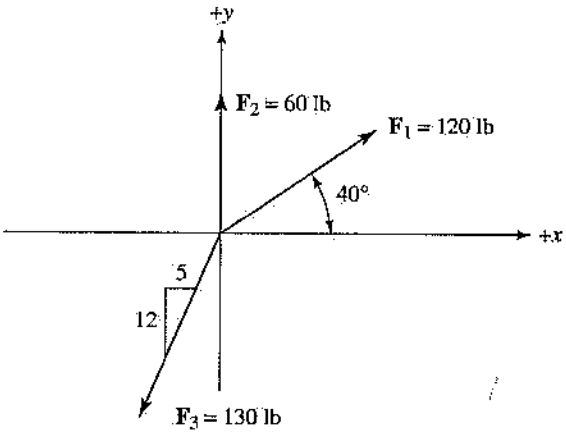


Figure 11.6

Solution:

$$R_x = F_{1x} + F_{2x} + F_{3x} = 120\cos 40 - 130\left(\frac{5}{13}\right) = 41.92 \text{ lb}$$

$$R_y = F_{1y} + F_{2y} + F_{3y} = 120\sin 40 + 60 - 130\left(\frac{12}{13}\right)$$

$$= 17.13 \text{ lb}$$

$$R = \sqrt{41.92^2 + 17.13^2} = 45.28 \text{ lb}$$

$$\theta = \tan^{-1}\frac{17.13}{41.92} = 22.42°$$

Resolution of a Force (3-D)

To resolve a force into its components (F_x, F_y, F_z), two methods are available.

Method 1. If the angles between the force (**F**) and the axes (x, y, z) are known (see Figure 11.7), then

$$F_x = F\cos\theta_x,\ F_y = F\cos\theta_y,\ F_z = F\cos\theta_z \quad [11.3]$$

Also, there is a relation between these angles such that

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1 \quad [11.4]$$

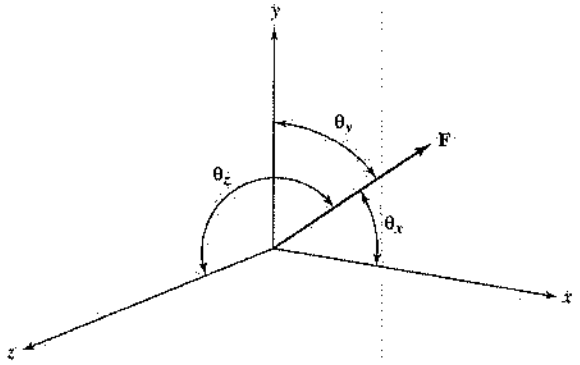


Figure 11.7

Example 11.3

Resolve the force shown into its rectangular components (x, y, z).

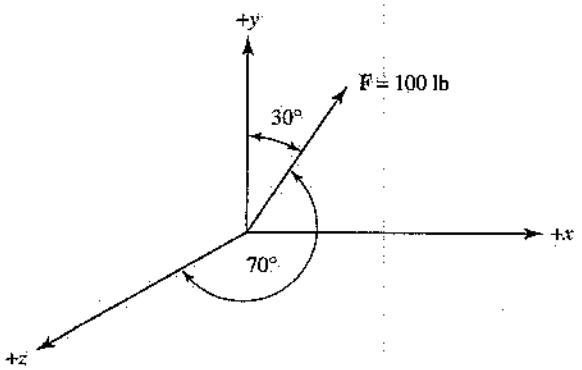


Figure 11.8

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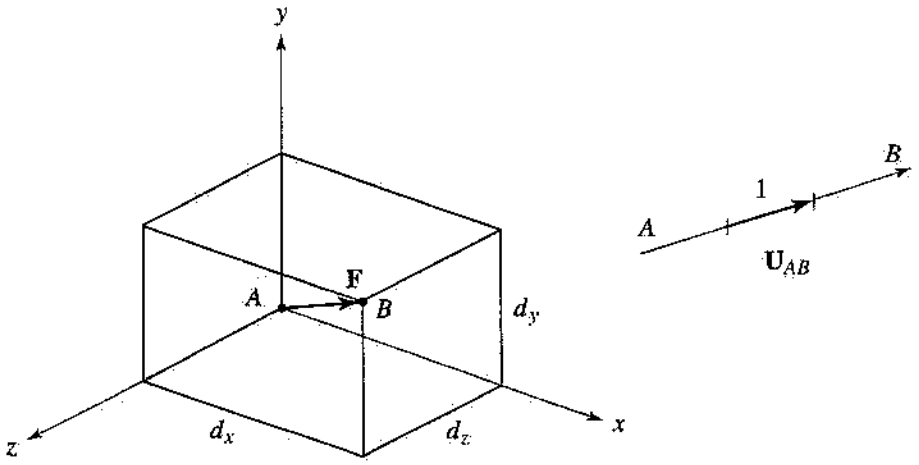


Figure 11.9

Solution:

In this problem, θ_y and θ_z are given, so use equation 11.4 to find θ_x.

$$\cos^2 \theta_x = 1 - \cos^2 \theta_y - \cos^2 \theta_z$$

$$= 1 - \cos^2(30) - \cos^2(70)$$

$$\theta_x = 68.6°$$

Note that another solution for θ_x is $180 - 68.6 = 111.4°$. By inspection, however, the correct angle is 68.6°.

$$F_x = F \cos \theta_x = 36.47 \text{ lb}$$

$$F_y = F \cos \theta_y = 86.6 \text{ lb}$$

$$F_x = F \cos \theta_z = 34.2 \text{ lb}$$

Method 2. If the coordinates of two points along the line of action of the force are given (see Figure 11.9), use the unit vector $\hat{\mathbf{u}}$ to express (resolve) the force into its components.

$$\mathbf{F} = F\hat{\mathbf{u}}_{AB}$$

$$\hat{\mathbf{u}}_{AB} = \frac{d_x\hat{\mathbf{i}} + d_y\hat{\mathbf{j}} + d_z\hat{\mathbf{k}}}{d} \qquad [11.5]$$

$$d = \sqrt{d_x^2 + d_y^2 + d_z^2}$$

where $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are unit vectors along x, y, and z axes, respectively, and d is the distance between two points along the line of action of the force $\mathbf{F}$.

Example 11.4

Resolve the force shown into its rectangular components (x, y, z).

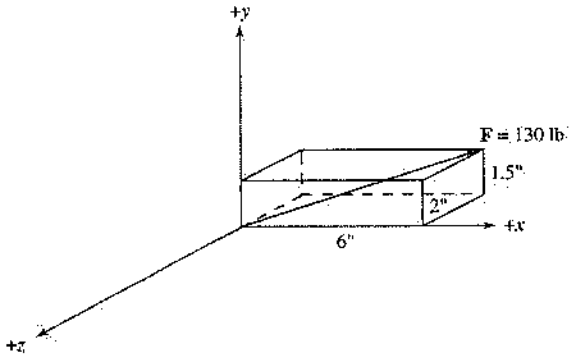


Figure 11.10

Solution:

$$\hat{\mathbf{u}}_{AB} = \frac{6\hat{\mathbf{i}} + 1.5\hat{\mathbf{j}} - 2\hat{\mathbf{k}}}{6.5}$$

$$\mathbf{F} = 130\hat{\mathbf{u}}_{AB}$$

$$= 120\hat{\mathbf{i}} + 30\hat{\mathbf{j}} - 40\hat{\mathbf{k}}$$

where $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are unit vectors along x, y, and z axes, respectively.

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Resultant of a Concurrent Force System (3-D)

This is similar to the two-dimensional case except that there is a third component (R_z). The equations are as follows:

$$R_x = \Sigma F_x, \quad R_y = \Sigma F_y, \quad R_z = \Sigma F_z$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} \qquad [11.6]$$

Example 11.5

Determine the resultant of the forces shown.

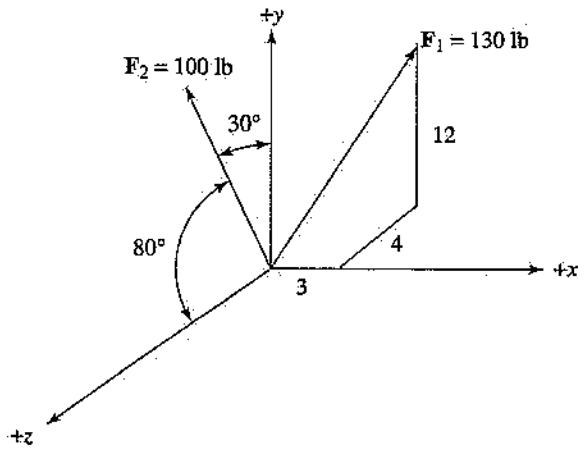


Figure 11.11

Solution:

$$\cos^2\theta_x = 1 - \cos^2\theta_y - \cos^2\theta_z$$
$$= 1 - \cos^2(30) - \cos^2(80)$$

$\theta_x = 62°$ and also $\theta_x = 180° - 62° = 118°$

From the figure, the correct angle is 118°.

$F_{2x} = F_x \cos\theta_x = -46.9$ lb

$F_{2y} = F_2 \cos\theta_y = 86.6$ lb

$F_{2z} = F \cos\theta_z = 34.2$ lb

$$\hat{\mathbf{u}}_{AB} = \frac{3\,\hat{\mathbf{i}} + 12\,\hat{\mathbf{j}} - 4\,\hat{\mathbf{k}}}{13}$$

$$\mathbf{F}_1 = 130\,\hat{\mathbf{u}}_{AB}$$
$$= 30\,\hat{\mathbf{i}} + 120\,\hat{\mathbf{j}} - 40\,\hat{\mathbf{k}}$$

$R_x = \Sigma F_x = 30 - 46.9 = -16.9$ lb

$R_y = \Sigma F_y = 120 + 86.9 = 206.9$ lb

$R_z = \Sigma F_z = -40 + 34.2 = -5.8$ lb

$R = \sqrt{(-16.9)^2 + 206.9^2 + (-5.8)^2} = 207.6$ lb

To specify the direction of the resultant force, determine the angles that the resultant forms with the axes.

$$\cos\theta_x = \frac{R_x}{R} = \frac{-16.9}{207.6} \Rightarrow \theta_x = 94.66°$$

$$\cos\theta_y = \frac{R_y}{R} = \frac{206.9}{207.6} \Rightarrow \theta_y = 4.7°$$

$$\cos\theta_z = \frac{R_z}{R} = \frac{-5.8}{207.6} \Rightarrow \theta_z = 91.6°$$

11.2 Moment

The **moment** of a force is the measure of tendency of a rigid body to rotation about an axis. Moment is divided into two parts, moment of a single force and moment of a couple.

Moment of a Single Force

The moment of a single force can be about (1) a point or (2) an axis.

1. The moment of a force about a point is the product of the force and the perpendicular distance from the point to the line of action of the force. Therefore the magnitude of the moment about point A is expressed by the equation

$$M_A = Fd \qquad [11.7]$$

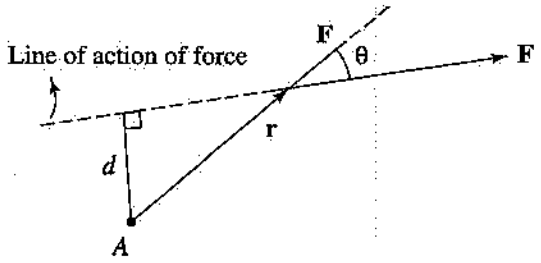


Figure 11.12

Moment can also be determined vectorially, and is given by the equation

$$\mathbf{M}_A = \mathbf{r} \times \mathbf{F} \qquad [11.8]$$

Where $\mathbf{r}$ is the position vector from the point to anywhere along the line of action of the force.

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Example 11.6

Find the moment of the force shown about point A, using both the scalar and the vector approach.

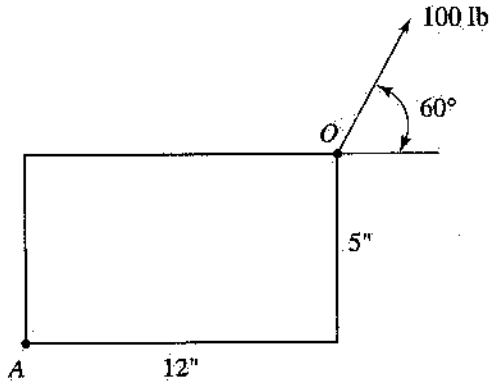


Figure 11.13

Solution:

Scalar approach:

CCW + :

$M_A = -100 \cos 60(5) + 100 \sin 60(12) = 789.2$ lb-in

Therefore, the magnitude of the moment about A is 789.2 lb-in and its direction is counterclockwise.

Vector approach:

$$\mathbf{M}_A = \mathbf{r}_{AO} \times \mathbf{F} = (12\,\hat{i} + 5\,\hat{j}) \times (50\,\hat{i} + 86.6\,\hat{j})$$
$$= 789.2\,\hat{k}$$

2. The moment of a force about an axis is the projection of the moment along that axis, shown in Figure 11.14. For projection along line AB the equations are as follows:

$$M_{AB} = \mathbf{M}_A \cdot \mathbf{u}_{AB}, \quad \mathbf{M}_{AB} = M_{AB}\,\mathbf{u}_{AB} \quad [11.9]$$

Note that $\mathbf{M}_A$ can be determined about any point along line AB, $\mathbf{u}_{AB}$ is the unit vector along line AB, and M_{AB} is the magnitude of the moment about line AB.

Example 11.7

Determine, for the figure shown, the magnitude of the moment of the 140-N force about line AB.

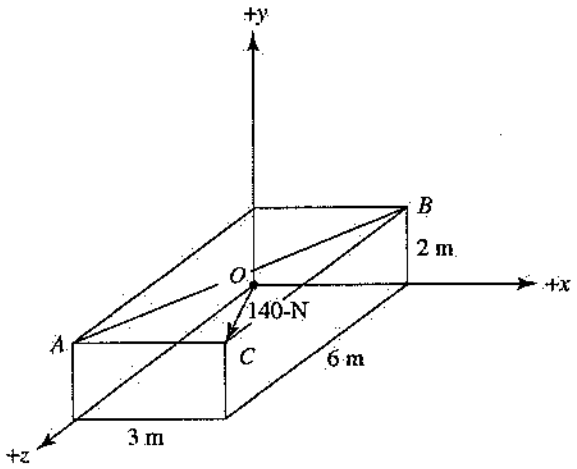


Figure 11.15

Solution:

$$\mathbf{M}_A = \mathbf{r}_{AO} \times \mathbf{F} = (-2\,\hat{j} - 6\,\hat{k}) \times (60\,\hat{i} + 40\,\hat{j} + 250\,\hat{k})$$
$$= -360\,\hat{j} + 120\,\hat{k}$$

$$M_{AB} = \mathbf{M}_A \cdot \mathbf{u}_{AB} = (-360\,\hat{j} + 120\,\hat{k}) \cdot \left(\frac{3\,\hat{i} - 6\,\hat{k}}{\sqrt{45}}\right)$$
$$= -107.33\ N \cdot m$$

Moment of a Couple

Two equal and parallel forces that have noncollinear lines of actions and are opposite in sense form a **couple** (Figure 11.6). The moment of a couple is the product of the force and the shortest distance between the two forces. Note in the equation below that the

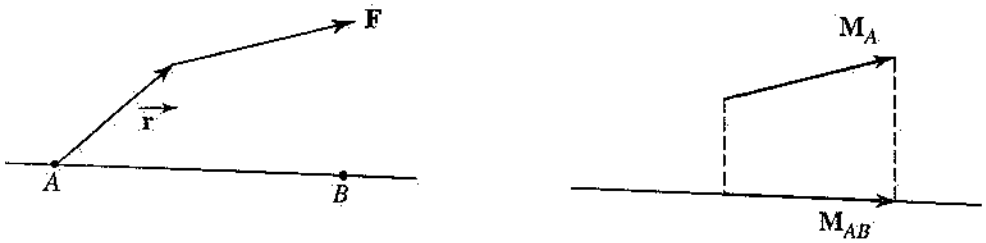


Figure 11.14

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moment of a couple is the same with respect to any point in space.

$$M = Fd$$



Figure 11.16

Example 11.8

Determine the resultant of the couple shown.

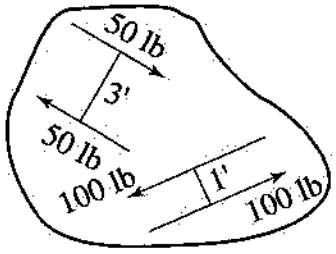


Figure 11.17

Solution:

CCW + :

$$M = (100)(1) - (50)(3) = -50 \text{ lb-ft}$$

Therefore, the resultant moment (couple) is clockwise.

11.3 Equilibrium of Particles

Two-Dimensional Case

For a particle to be in equilibrium (at rest), the resultant of the forces acting on the particle must be equal to zero (i.e., there is no translation).

$$R_x = \Sigma F_x = 0, \quad R_y = \Sigma F_y = 0 \quad [11.10]$$

Example 11.9

Two cables are tied together at C and loaded as shown. Determine the tension in each cable.

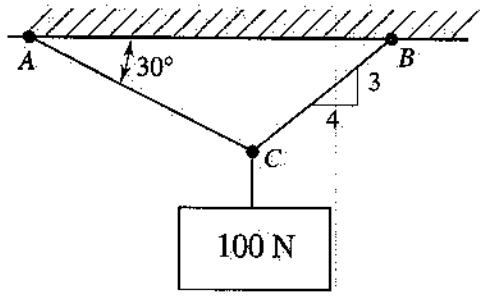


Figure 11.18

Solution:

$$R_x = \Sigma F_x = \frac{4}{5}T_{BC} - T_{AC}\cos 30 = 0$$

$$R_y = \Sigma F_y = \frac{3}{5}T_{BC} + T_{AC}\sin 30 - 100 = 0$$

Solve simultaneously for the tensions:

$$T_{AC} = 87 \text{ N} \quad \text{and} \quad T_{BC} = 94.17 \text{ N}$$

Three-Dimensional Case

This is the same as the two-dimensional case except that there is a component in the z-direction.

$$R_x = \Sigma F_x = 0, \; R_y = \Sigma F_y = 0, \; R_z = \Sigma F_z = 0 \quad [11.11]$$

Example 11.10

A load W is supported by three cables as shown. Determine the magnitude of load W if the tension in cable CD is 900 N.

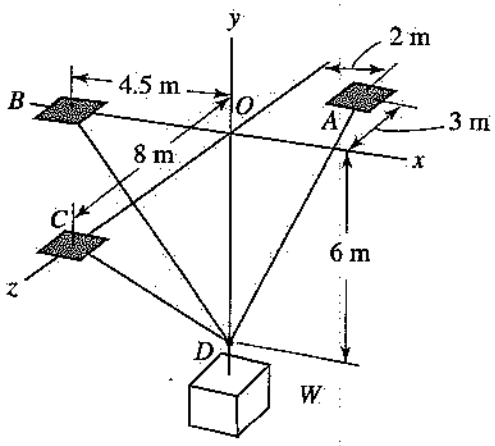


Figure 11.19

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Solution:

It is given that $T_{DC} = 900$ N. Then:

$$\mathbf{T}_{DA} = T_{DA}\left(\frac{2\,\mathbf{i} + 6\,\mathbf{j} - 3\,\mathbf{k}}{7}\right),$$

$$\mathbf{T}_{DB} = T_{DB}\left(\frac{-4.5\,\mathbf{i} + 6\,\mathbf{j}}{7.5}\right),$$

$$\mathbf{T}_{DC} = T_{DC}\left(\frac{6\,\mathbf{j} + 8\,\mathbf{k}}{10}\right) = 540\,\mathbf{j} + 720\,\mathbf{k}$$

$$R_x = \Sigma F_x = 0$$

$$\frac{2}{7}T_{DA} - \frac{4.5}{7.5}T_{DB} = 0$$

$$R_y = \Sigma F_y = 0$$

$$\frac{6}{7}T_{DA} + \frac{6}{7.5}T_{DB} + 540 - W = 0$$

$$R_z = \Sigma F_z = 0$$

$$-\frac{3}{7}T_{DA} + 720 = 0$$

Solve simultaneously for W:

$$W = 2{,}652 \text{ N}$$

11.4 Equilibrium of Rigid Bodies

Two-Force Members

If a member is under the action of only two forces (at different points on the body), the member is called a **two-force member.** For a straight two-force member, the force must act along the axis of the member (tension or compression).

If the force member is not straight, the force must act along the diagonal line connecting the two points where the two forces are acting.

Connections (Supports) and Reactions

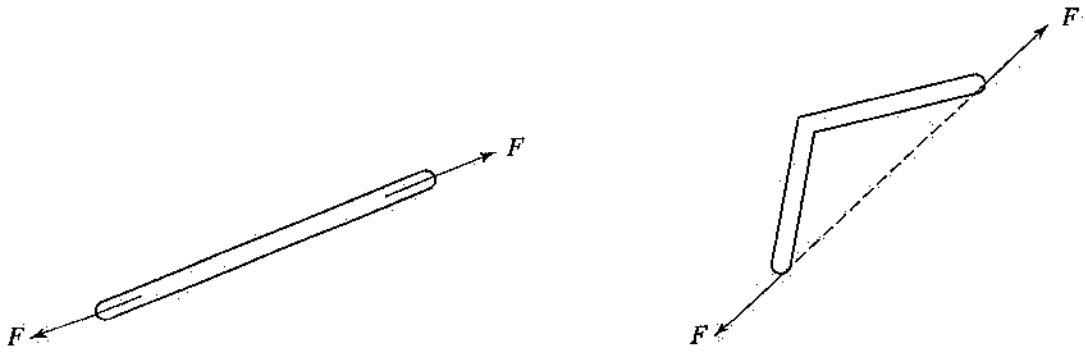
Rigid bodies either are connected to each other or rest on surfaces. In most cases it is necessary to determine the resulting reactions for different supports. To find the number and type of reaction for a given support, determine whether the point of connection can translate or rotate. If the point cannot translate, a **force reaction** along the line of translation (x, y, or z) is required. If the point cannot rotate, a **moment reaction** about that axis is required.

Equilibrium Equations (2-D)

For a rigid body to be in equilibrium, the following equations must be valid.

$$R_x = \Sigma F_x = 0, \quad R_y = \Sigma F_y = 0 \qquad [11.12]$$
$$\Sigma M = 0$$

In equation 11.12 the moment can be taken about any point on the body. Note that for a two-dimensional case there are three equilibrium equations, so there cannot be more than three unknowns for a given free-body diagram (FBD).

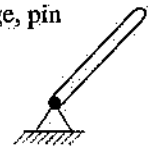
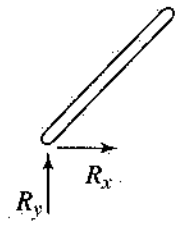
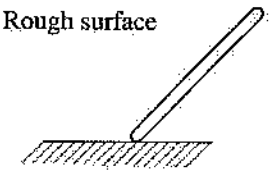
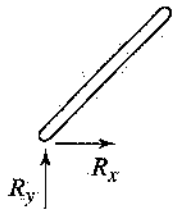
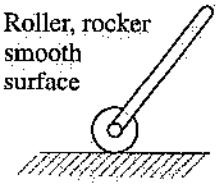
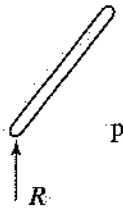
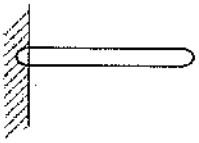
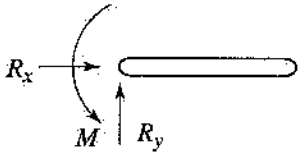
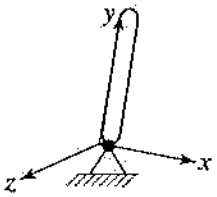
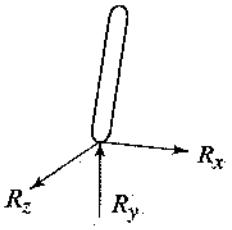
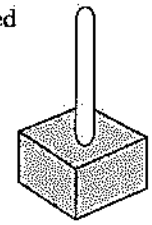
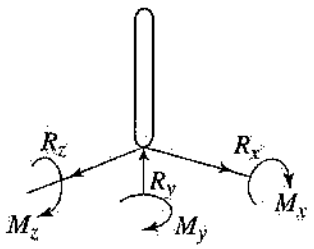


Straight two-force member — Nonstraight two-force member

Figure 11.20

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Table 11.1. Supports for Rigid Bodies

Connections (Supports)	Reactions	Number of Unknowns
Hinge, pin 		2
Rough surface 		2
Roller, rocker smooth surface 		1
Fixed 		3
Ball and socket joint 		3
Fixed 		6

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Example 11.11

For the beam and loading shown, determine the reaction at each support.

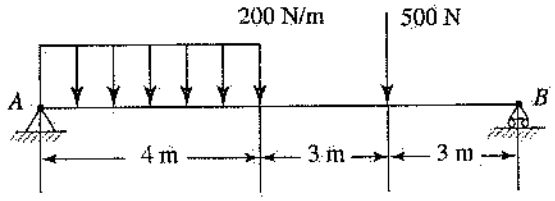


Figure 11.21

Solution:

$\Sigma M_A = 0 \quad CCW+:$

$B(10) - 500(7) - 800(2) = 0$; then $B = 510$ lb

$R_x = \Sigma F_x = 0, \quad A_x = 0$

$R_y = \Sigma F_y = 0,$

$A_y + 510 - 500 - 800 = 0 \Rightarrow A_y = 790$ N

Example 11.12

For the figure shown, determine the tension in cable BC and the reaction at A (neglect the weight of beam AB).

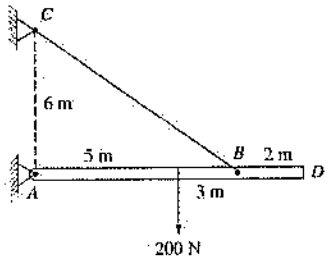


Figure 11.22

Solution:

$\Sigma M_A = 0$

$\frac{3}{5} T_{BC}(8) - 200(5) = 0$

$T_{BC} = 208.33$ N

$R_x = \Sigma F_x = 0$

$A_x - \frac{4}{5}(208.33) = 0$

$A_x = 166.66$ N

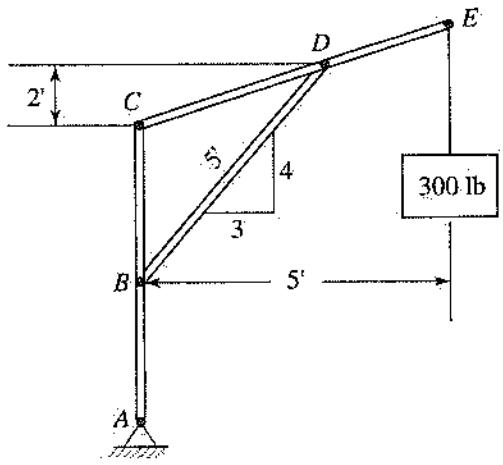
$R_y = \Sigma F_y = 0$

$A_y - 200 + \frac{3}{5}(208.33) = 0$

$A_y = 75$ N

Example 11.13

Determine the force in member BD of the structure shown. Also determine the components of reaction at point C.



Solution:

Use the FBD of member CDE and recognize that member BD is a two-force member. Then:

$\Sigma M_C = 0 \quad CCW+:$

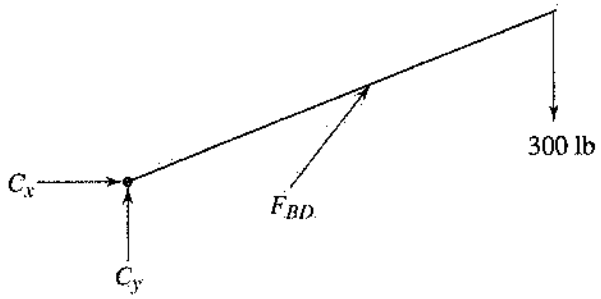


Figure 11.23

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$$-\frac{3}{5}F_{BD}(2)+\frac{4}{5}F_{BD}(3)-300(5)=0$$

$$F_{BD}=1{,}250\text{ lb}$$

$$R_x=\Sigma F_x=0$$

$$C_x+\frac{3}{5}(1{,}250)=0$$

$$C_x=-750\text{ lb}$$

$$R_y=\Sigma F_y=0$$

$$C_y-300+\frac{4}{5}(1{,}250)=0$$

$$C_y=-700\text{ lb}$$

Equilibrium Equations (3-D)

In this case there are six equilibrium equations (three force equations and three moment equations).

$$R_x=\Sigma F_x=0,\quad R_y=\Sigma F_y=0,\quad R_z=\Sigma F_z=0$$

$$\Sigma M_x=0,\quad \Sigma M_y=0,\quad \Sigma M_z=0 \quad [11.13]$$

Example 11.14

In the figure, a plate weighing 135 N is supported by two cables (CD and BE) and a ball and socket joint at A. Determine the tension in each cable.

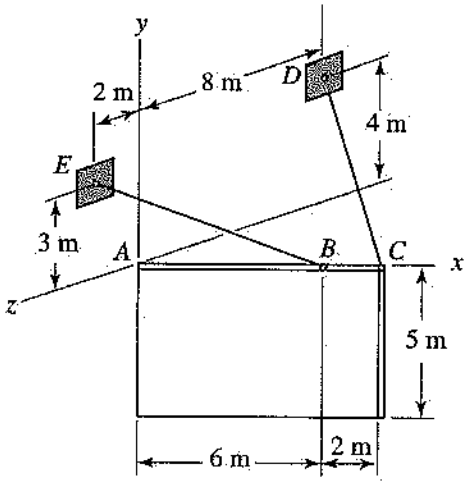


Figure 11.24

Solution:

$$\mathbf{T}_{CD}=T_{CD}\left(\frac{-8\,\mathbf{i}+4\,\mathbf{j}-8\,\mathbf{k}}{12}\right)$$

$$\mathbf{T}_{BE}=T_{BE}\left(\frac{-6\,\mathbf{i}+3\,\mathbf{j}+2\,\mathbf{k}}{7}\right)$$

$$\Sigma M_A=0$$

$$8\,\mathbf{i}\times T_{CD}\left(\frac{-8\,\mathbf{i}+4\,\mathbf{j}-8\,\mathbf{k}}{12}\right)+6\,\mathbf{i}$$

$$\times T_{BE}\left(\frac{-6\,\mathbf{i}+3\,\mathbf{j}+2\,\mathbf{k}}{7}\right)+(4\,\mathbf{i}-2.5\,\mathbf{j})$$

$$\times(-135\,\mathbf{j})=0$$

$$\left(\frac{16}{3}T_{CD}-\frac{12}{7}T_{BE}\right)\mathbf{j}+\left(\frac{8}{3}T_{CD}+\frac{18}{7}T_{BE}-540\right)\mathbf{k}=0$$

Set each parenthesis equal to zero, and solve for the tensions:

$$T_{BE}=157.5\text{ N},\qquad T_{CD}=50.63\text{ N}$$

11.5 Trusses and Frames

Trusses

A collection of rigid bodies that are pin connected to each other at their ends to form a new rigid body is known as a *truss*. Each member of a truss is a straight two-force body. The main objective is to determine the internal forces in some or all members of a truss. Each member can be in tension (T) or compression (C), depending on the loading or configuration of the truss.

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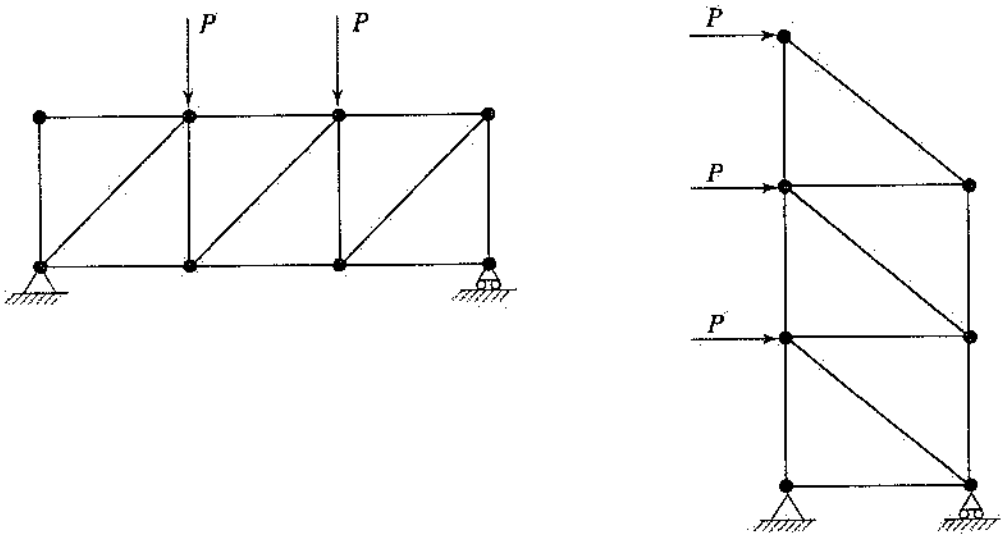


Figure 11.25

Generally two methods are available to determine internal forces in a given truss, the method of joints and the method of sections.

Method of Joints

To use this method, draw the FBD of the joint and apply the two-dimensional equilibrium equations for particles. Note that for a given joint there cannot be more than two unknowns.

$$R_x = \Sigma F_x = 0, \quad R_y = \Sigma F_y = 0$$

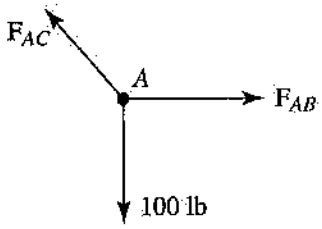
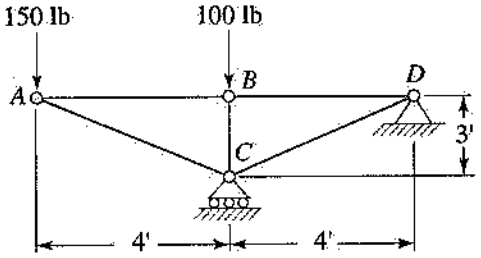


Figure 11.26

Example 11.15

Using the method of joints, determine the force in each member of the truss shown.



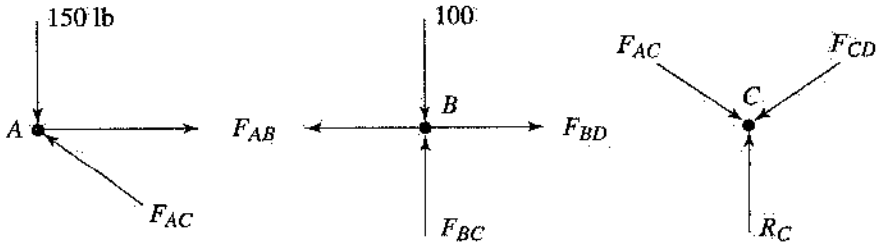


Figure 11.27

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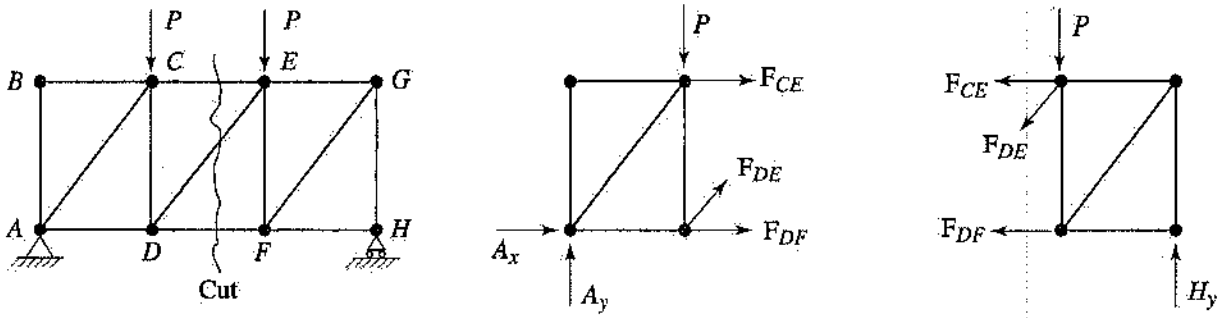


Figure 11.28

Solution:

Using Figure 11.27:

Joint *A*:

$$\Sigma F_y = 0 \Rightarrow -150 - \frac{3}{5}F_{AC} = 0$$

$$F_{AC} = 250 \text{ lb C}$$

$$\Sigma F_x = 0 \Rightarrow F_{AB} - \frac{4}{5}(250) = 0$$

$$F_{AB} = 200 \text{ lb T}$$

Joint *B*:

$$\Sigma F_x = 0 \Rightarrow F_{BD} = 200 \text{ lb T}$$

$$\Sigma F_x = 0 \Rightarrow F_{BC} = 100 \text{ lb T}$$

Joint *C*:

$$\Sigma F_x = 0 \Rightarrow F_{CD} = 250 \text{ lb C}$$

Method of Sections

This method is used when the forces for a few members of a large truss are to be determined. Cut the truss in a section where the forces are to be determined (see Figure 11.28). Then consider the section to the left or right (up or down in other cases), draw the FBD, and apply the two-dimensional equilibrium equations for rigid bodies. ***Generally the section considered should not pass or cut more than three members at a time (since only three equilibrium equations are available).***

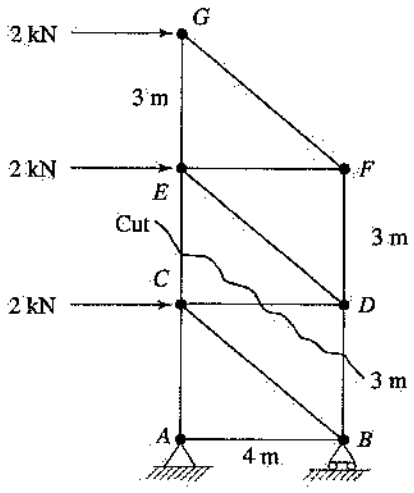
$$R_x = \Sigma F_x = 0, \quad R_y = \Sigma F_y = 0$$

$$\Sigma M = 0$$

Note that, for some trusses, it is necessary to determine the external forces (reactions) before solving for the internal forces.

Example 11.16

Determine the forces in members *CE*, *CD*, and *BD* of the truss shown.



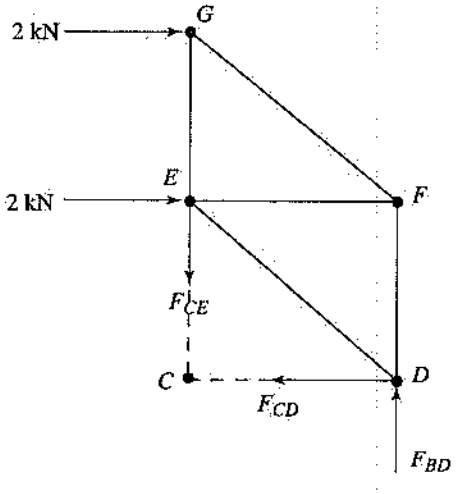


Figure 11.29

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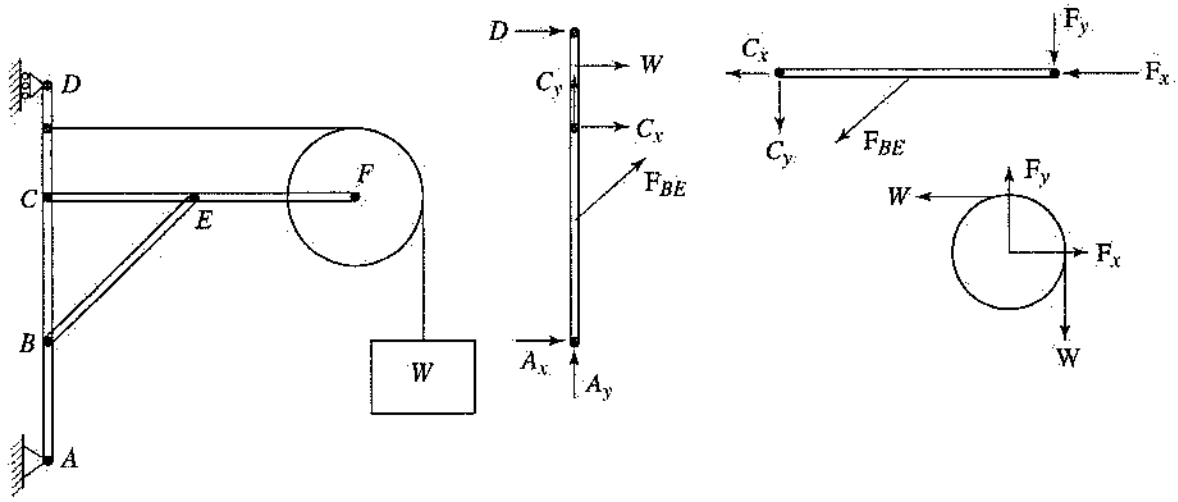


Figure 11.30

Solution:

$$\Sigma M_D = 0 \quad \text{CCW} +:$$

$$F_{CE}(4) - 2(3) - 2(6) = 0$$

$$F_{CE} = 4.5 \text{ kN T}$$

$$\Sigma F_x = 0 \Rightarrow F_{CD} = 4 \text{ kN T}$$

$$\Sigma F_y = 0 \Rightarrow F_{BD} = 4.5 \text{ kN C}$$

Frames

Frames are structures containing multiforce members and designed to support applied loads. Frames are usually stationary. The analysis is done by disassembling the frame, shown in Figure 11.30, drawing the FBD for each member, and applying the equilibrium equations (note that some members of a frame can be two-force members).

$$R_x = \Sigma F_x = 0, \quad R_y = \Sigma F_y = 0$$

$$\Sigma M = 0$$

Example 11.17

Determine the components of all forces acting on member AC of the frame shown.

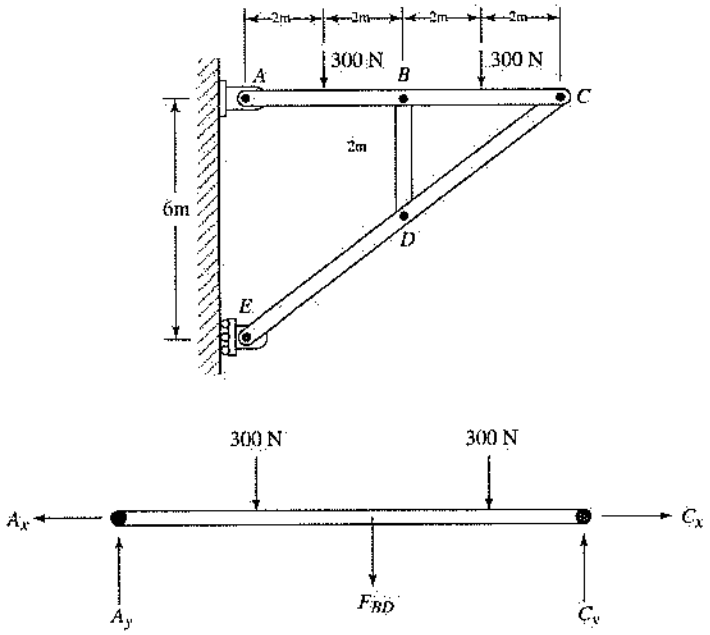


Figure 11.31

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Solution:

First determine the reactions without disassembling the frame.

$$\Sigma M_A = 0$$

$$E(6) - 300(2) - 300(6) = 0$$

$$E = 400 \text{ N}$$

$$\Sigma F_x = 0 \Rightarrow A_x = 400 \text{ N} \leftarrow$$

$$\Sigma F_y = 0 \Rightarrow A_y = 600 \text{ N} \uparrow$$

Now use the FBD of member AC, noting that BD is a two-force member.

$$\Sigma M_C = 0$$

$$F_{BD}(4) + 300(2) + 300(6) - 600(8) = 0$$

$$F_{BD} = 600 \text{ N} \downarrow$$

$$\Sigma F_x = 0 \Rightarrow C_x = 400 \text{ N} \rightarrow$$

$$\Sigma F_y = 0 \Rightarrow C_y = 600 \text{ N} \uparrow$$

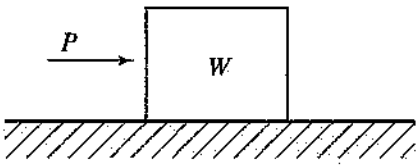
11.6 Friction

When two surfaces are in contact, the force tangent to the surface that opposes the direction of tendency to motion is called the **friction.** There are two types of friction: dry friction and fluid friction.

To understand the laws of dry friction consider the figure shown below.

The maximum value of friction force (F_{max}) is related to the normal force (N) and the coefficient of static friction (μ_s):

$$F_{max} = \mu_s N \qquad [11.14]$$



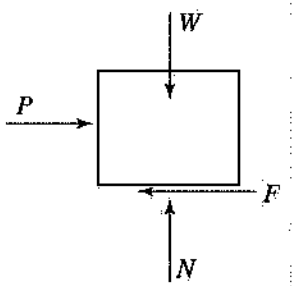


Figure 11.32

Two Types of Friction Problems

There are two distinct types of friction problems:

1. The impending motion exists (the object is about to move). In this case friction has reached its maximum value, and $F = F_{max} = \mu_s N$. Apply the equilibrium equations for the particle or rigid body, and solve for the unknowns.
2. The impending motion is not known (you don't know whether the object is moving or not). In this case assume that the system is in equilibrium, and apply the appropriate equilibrium equations (particle or rigid body) to solve for the friction force. Two outcomes are possible.
   a. If $F < \mu_s N$, then the system is in equilibrium and the friction force calculated is correct.
   b. If $F > \mu_s N$, then the system is in motion and for ***particle problems only*** the correct value of friction force is $F = \mu_k N$, where μ_k is called the coefficient of kinetic friction. In general, μ_k is about 25% smaller than μ_s.

For rigid body problems, a dynamical analysis is used to determine the magnitude of the friction force.

Example 11.18

Determine the value of the friction force.

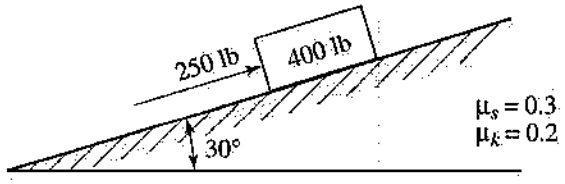


Figure 11.33

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Solution:

This is case 2; that is, the impending motion is not known.

$\nwarrow \Sigma F_y = 0$

$N - 400 \cos 30 = 0 \Rightarrow N = 346.4$ lb

$\nearrow \Sigma F_x = 0$

$-F + 250 - 400 \sin 30 = 0 \Rightarrow F = 50$ lb

Now check: $\mu_s N = 0.3(364.4) = 103.92$ since $F < \mu_s N$. Then the friction force calculated is correct: $F = 50$ lb.

Example 11.19

If the bar weighs 50 N, determine the minimum value of load P that will initiate the motion (neglect friction at A).

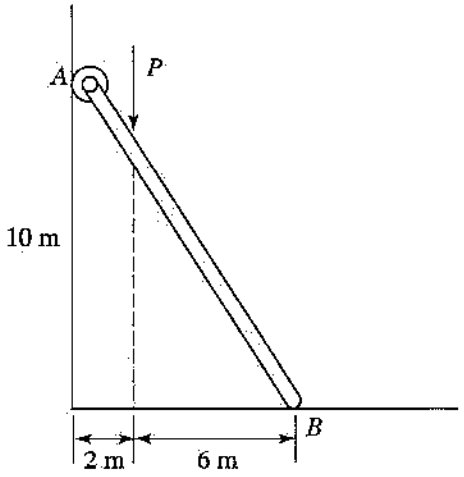


Figure 11.34

Solution:

This is case 1; that is, friction has reached its maximum value at B. Set $F = \mu_s N$.

$\Sigma M_A = 0 \quad \text{CCW} +:$

$N(8) - 0.5N(10) - 50(4) - P(2) = 0$

$3N - 2P = 200$

$\Sigma F_y = 0$

$N - 50 - P = 0$

Solve for P: $P = 50$ N

Belt Friction

Consider a belt that is pressed against a rough, curved surface and is pulled on both ends. If friction is considered, the tensions (T) on both ends are not the same. It can be shown that

$$T_2 = T_1 e^{\mu_s \theta} \qquad [11.15]$$

where θ is the angle of contact between the belt and the drum.

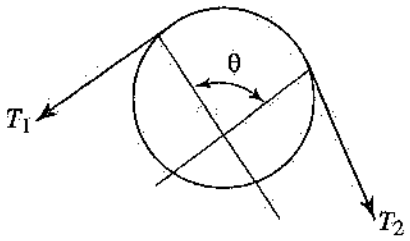


Figure 11.35

Example 11.20

A flat belt transmits a 30 lb-ft torque. Determine the minimum values of tension in both parts of the belt that will prevent slippage. The diameter of the drum is 6 in and $\mu_s = 0.3$.

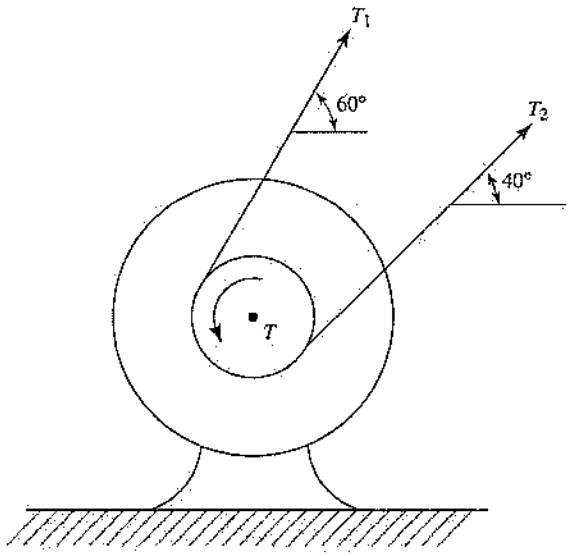


Figure 11.36

Solution:

$\Sigma M_O = 0$

$T_1(3) - T_2(3) + 360 = 0$

$T_1 - T_2 = -120$

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The angle of contact between belt and drum is 180°.

$$\theta = 180° = \pi = 3.14 \text{ rad}$$

$$\frac{T_2}{T_1} = e^{0.3(3.14)} = e^{0.942} = 2.565$$

$$T_2 = 2.565T_1$$

$$T_1 = 76.7 \text{ lb}, \quad T_2 = 196.7 \text{ lb}$$

11.7 Center of Gravity

Center of gravity is defined as the point of a body where the weight of the body will pass through. For a homogeneous body the center of gravity is the same as the **centroid.**

In general, to determine the location of centroid of a plane area use the following equations:

$$\bar{X}_c = \frac{\int x_c dA}{A}, \quad \bar{Y}_c = \frac{\int y_c dA}{A} \qquad [11.16]$$

where $\int x_c\, dA$ and $\int y_c\, dA$ are called the first moments of the area with respect to the y-axis and x-axis, respectively.

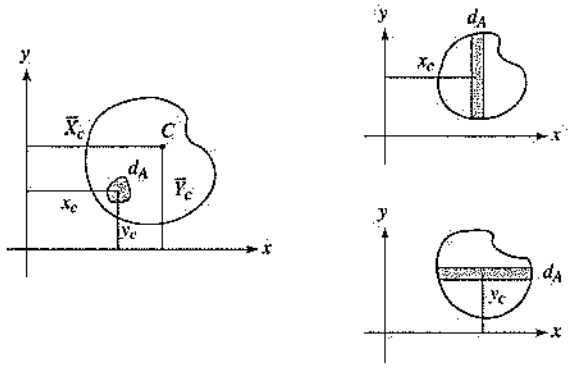


Figure 11.37

If the plane area can be divided into common shapes, as shown in Fig 11.38 (i.e., rectangles, semicircles, triangles), then the discrete form of the equations can be used:

$$\bar{X}_c = \frac{\Sigma x_i A_i}{A}, \quad \bar{Y}_c = \frac{\Sigma y_i A_i}{A} \qquad [11.17]$$

Example 11.21

Determine the center of gravity of the shaded plane area shown.

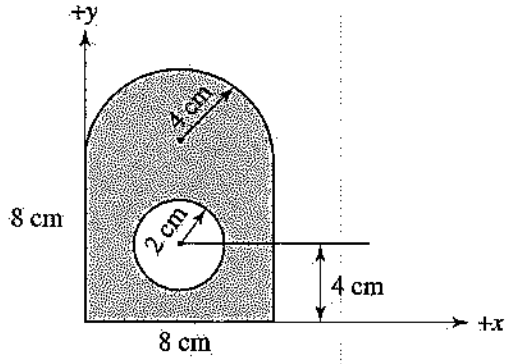


Figure 11.39

Solution:

$$\bar{X}_c = \frac{\Sigma x_i A_i}{A}$$

$$= \frac{(4)(64) + (4)\left[\frac{1}{2}\pi(4)^2\right] - (4)\left[\pi(2)^2\right]}{64 + \frac{1}{2}\pi(4)^2 - \pi(2)^2} = 4 \text{ cm}$$

$$\bar{Y}_c = \frac{\Sigma y_i A_i}{A}$$

$$= \frac{(4)(64) + (9.697)\left[\frac{1}{2}\pi(4)^2\right] - (4)\left[\pi(2)^2\right]}{64 + \frac{1}{2}\pi(4)^2 - \pi(2)^2}$$

$$= 5.87 \text{ cm}$$

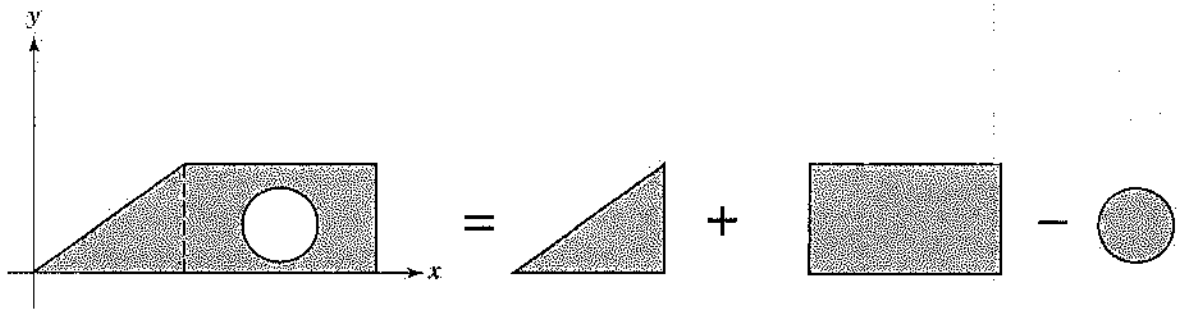


Figure 11.38

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To find the centroid of lines (wires) and volumes, use the following similar equations.

For lines:

$$\bar{X}_c = \frac{\Sigma x_i L_i}{L}, \quad \bar{Y}_c = \frac{\Sigma y_i L_i}{L} \qquad [11.18]$$

For volumes:

$$\bar{X}_c = \frac{\Sigma x_i V_i}{V}, \quad \bar{Y}_c = \frac{\Sigma y_i V_i}{V} \qquad [11.19]$$

Example 11.22

Determine the centroid of the composite wire shown.

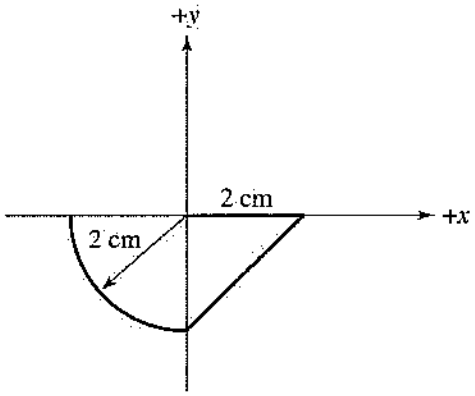


Figure 11.40

Solution:

$$\bar{X}_c = \frac{\Sigma x_i L_i}{L} = \frac{(1)(2) + (1)(2.83) + (-1.273)(3.14)}{2 + 2.83 + 3.14}$$

$$= 0.1045 \text{ cm}$$

$$\bar{Y}_c = \frac{\Sigma y_i L_i}{L} = \frac{(0)(2) + (-1)(2.83) + (-1.273)(3.14)}{2 + 2.83 + 3.14}$$

$$= -0.856 \text{ cm}$$

11.8 Moments of Inertia

Area Moment of Inertia

Area moment of inertia (I) is defined as the second moment of area and is determined as follows:

$$I_x = \int y^2 \, dA, \quad I_y = \int x^2 \, dA \qquad [11.20]$$

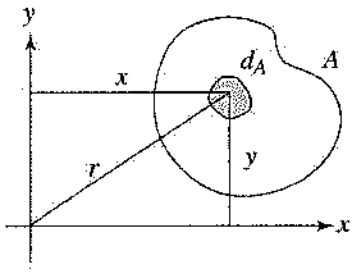


Figure 11.41

Area moment of inertia is a measure of resistance to bending and is needed to calculate stresses due to bending.

Polar Moment of Inertia

Polar moment of inertia (J) is the second moment of area with respect to the z-axis. It represents a resistance to twisting and is needed to calculate shear stresses due to twisting.

$$J = I_z = \int r^2 \, dA = \int x^2 \, dA + \int y^2 \, dA$$
$$= I_x + I_y \qquad [11.21]$$

Mass Moment of Inertia

Mass moment of inertia (I) is similar to area moment of inertia, except that area (A) is replaced in equation 11.20 by mass (m).

$$I = \int r^2 \, dm \qquad [11.22]$$

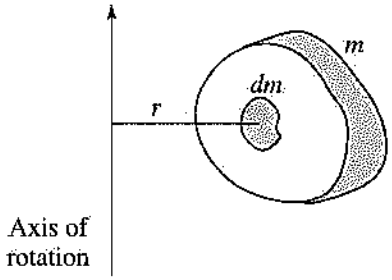
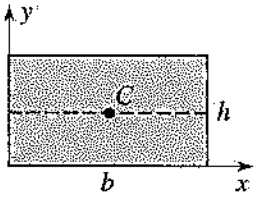
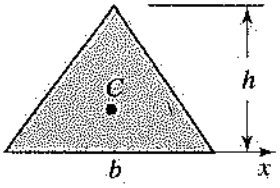
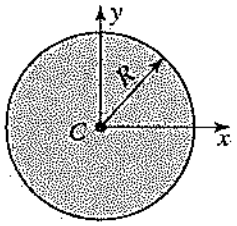
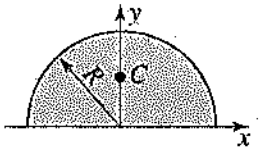
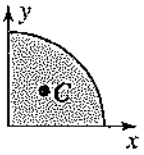
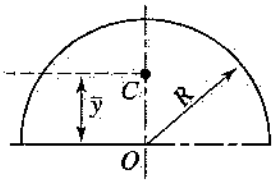
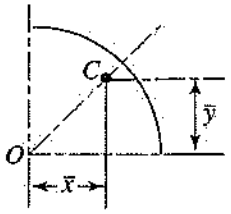


Figure 11.42

Mass moment of inertia is a measure of resistance of the body to rotational acceleration and is needed in dynamic problems to calculate forces acting on a rigid body in rotation.

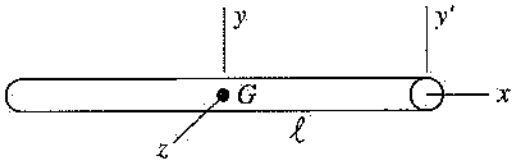
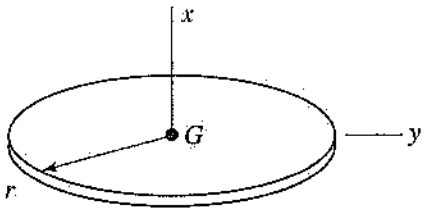
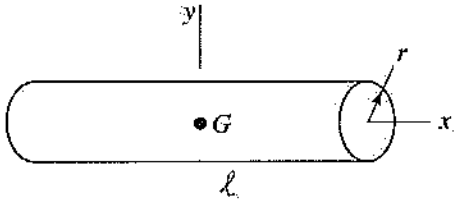
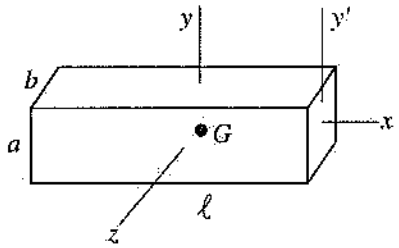
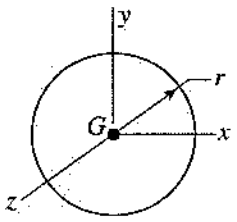
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Table 11.2. Centroids and Area Moments of Inertia of Common Shapes of Areas and Lines

Shape		Area	Centroid	Area Moment of Inertia
Rectangle		bh	$\bar{x} = \frac{b}{2}$ $\bar{y} = \frac{h}{2}$	$\bar{I}_c = \frac{1}{12}bh^3$ $I_x = \frac{1}{3}bh^3$ $I_y = \frac{1}{3}b^3h$
Triangle		$\frac{1}{2}bh$	$\bar{y} = \frac{h}{3}$	$\bar{I}_c = \frac{1}{36}bh^3$ $I_x = \frac{1}{12}bh^3$
Circle		πR^2	$\bar{x} = 0$ $\bar{y} = 0$	$\bar{I}_x = I_y = \frac{\pi R^4}{4}$ $I_z = J = \frac{\pi}{2}R^4$
Semicircle		$\frac{\pi R^2}{2}$	$\bar{y} = \frac{4R}{3\pi}$	$\bar{I}_x = I_y = \frac{1}{8}\pi R^4$ $I_z = J = \frac{1}{4}\pi R^4$
Quarter circle		$\frac{\pi R^2}{4}$	$\bar{x} = \frac{4R}{3\pi}$ $\bar{y} = \frac{4R}{3\pi}$	$I_x = I_y = \frac{1}{16}\pi R^4$ $I_z = J = \frac{1}{8}\pi R^4$
Semicircular arc			$\bar{x} = 0$ $\bar{y} = \frac{2R}{\pi}$	
Quarter-circular arc			$\bar{x} = \frac{2R}{\pi}$ $\bar{y} = \frac{2R}{\pi}$	

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Table 11.3. Mass Moments of Inertia of Common Shapes

Shape		Mass Moment of Inertia
Slender rod		$\bar{I}_y = I_z = \frac{1}{12} m\ell^2$ $I_{y'} = \frac{1}{3} m\ell^2$ $I_x = 0$
Thin disk		$\bar{I}_x = \frac{1}{2} mr^2$ $\bar{I}_y = \frac{1}{4} mr^2$
Cylinder		$\bar{I}_x = \frac{1}{2} mr^2$ $\bar{I}_y = \frac{1}{12} m(\ell^2 + 3r^2)$
Rectangular prism		$\bar{I}_x = \frac{1}{12} m(a^2 + b^2)$ $\bar{I}_y = \frac{1}{12} m(\ell^2 + b^2)$ $\bar{I}_z = \frac{1}{12} m(\ell^2 + a^2)$ $\bar{I}_{y'} = \frac{1}{12} m(4\ell^2 + a^2)$
Sphere		$\bar{I}_x = \bar{I}_y = \bar{I}_z = \frac{2}{5} mr^2$

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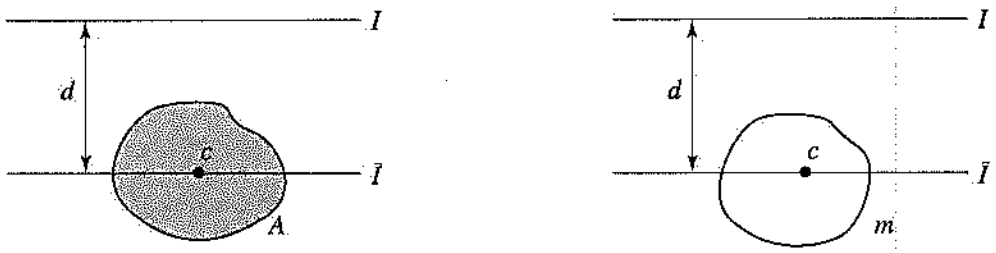


Figure 11.43

Parallel Axis Equation

In many situations the cross-sectional areas are not symmetric. To calculate moment of inertia with respect to an axis that is parallel to the centroidal axis (see Figure 11.43), the following equations can be used:

$$I = \bar{I} + Ad^2, \quad I = \bar{I} + md^2 \qquad [11.23]$$

Example 11.23

For the figure shown, determine the area moment of inertia of the shaded area with respect to the x-axis.

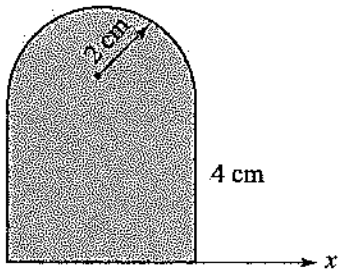


Figure 11.44

Solution:

$$I_{\text{shape}} = I_{\text{rect}} + I_{\text{semicircle}}$$

For the rectangle:

$$I_x = \frac{1}{3}(4)(4)^3 = 85.33 \text{ cm}^4$$

For the semicircle use the parallel axis equation:

$$I_x = \bar{I} + Ad^2$$

$$= 0.007(4)^4 + \frac{\pi}{2}(2)^2\left[4 + \frac{4(2)}{3\pi}\right]^2 = 149.52 \text{ cm}^4$$

$$I_{x\,\text{total}} = 85.33 + 149.52 = 234.85 \text{ cm}^4$$

Example 11.24

For the figure shown, determine the area moment of inertia with respect to the horizontal centroidal axis.

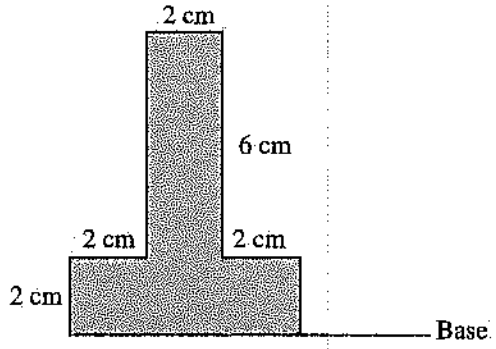


Figure 11.45

Solution:

First locate the centroid of the shape.

$$\bar{Y}_c = \frac{\Sigma y_i A_i}{A} = \frac{(6)(2)(1) + (6)(2)(5)}{(6)(2) + (6)(2)} = 3 \text{ cm from the base}$$

$$I = I_{\text{flange}} + I_{\text{web}}$$

$$I_{\text{flange}} = \frac{1}{12}(6)(2)^3 + (6)(2)(2)^2 = 52 \text{ cm}^4$$

$$I_{\text{web}} = \frac{1}{12}(2)(6)^3 + (6)(2)(2)^2 = 84 \text{ cm}^4$$

$$I = 52 + 84 = 136 \text{ cm}^4$$

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Example 11.25

For the figure shown, determine the mass moment of inertia with respect to the x-axis. Each bar has mass equal to m.

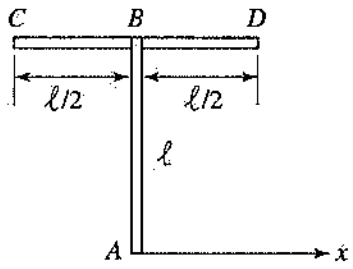


Figure 11.46

Solution:

$$I_{AB} = \frac{1}{12}m\ell^2 + m\left(\frac{\ell}{2}\right)^2 = \frac{1}{3}m\ell^2$$

$$I_{CD} = 0 + m\ell^2 = m\ell^2$$

$$I_{x\ \text{total}} = \frac{1}{3}m\ell^2 + m\ell^2 = \frac{4}{3}m\ell^2$$

Radius of Gyration

Radius of gyration (k) represents the distance from the axis at which the area or mass must be concentrated if the moment of inertia is to remain the same. The following equations are used to calculate radius of gyration:

$$k = \sqrt{\frac{I}{A}}, \quad k = \sqrt{\frac{I}{m}} \qquad [11.24]$$

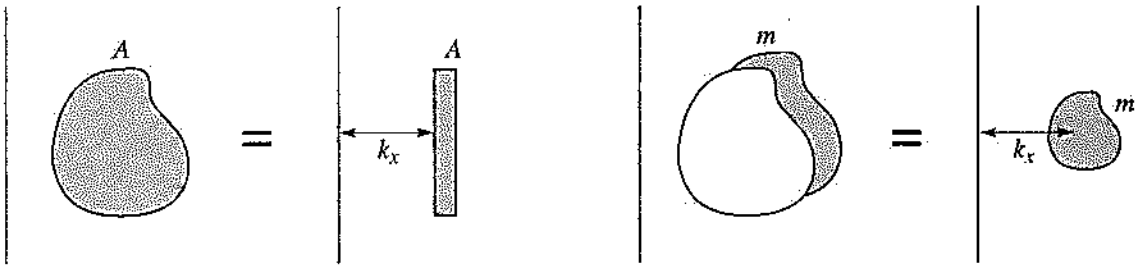


Figure 11.47

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PRACTICE PROBLEMS

1. Determine the resultant of the forces.
   (A) 38 N
   (B) 121 N
   (C) 78 N
   (D) 525 N

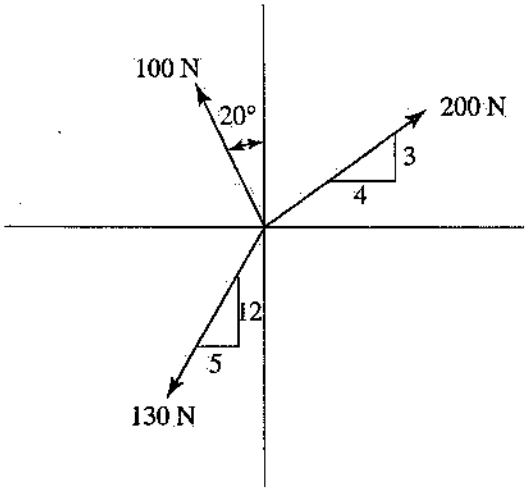


Figure 11.48

2. Determine the resultant of the forces exerted at point B if the tension in each cable is 450 N.
   (A) 735 N
   (B) 472 N
   (C) 375 N
   (D) 250 N

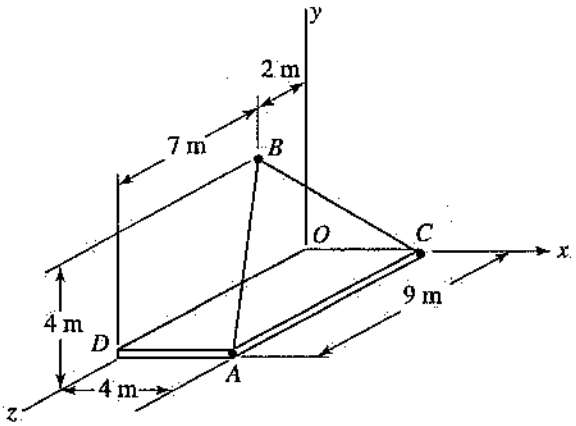


Figure 11.49

3. Determine the moment of force F about point A.
   (A) 30.4 N · m
   (B) 3.8 N · m
   (C) 1.6 N · m
   (D) 20 N · m

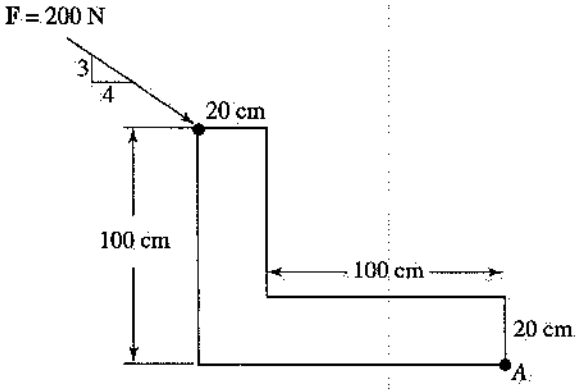


Figure 11.50

4. Determine the moment of force $P = 50$ N about line OC.
   (A) 3,600 N · m
   (B) 424 N · m
   (C) 1,600 N · m
   (D) 0

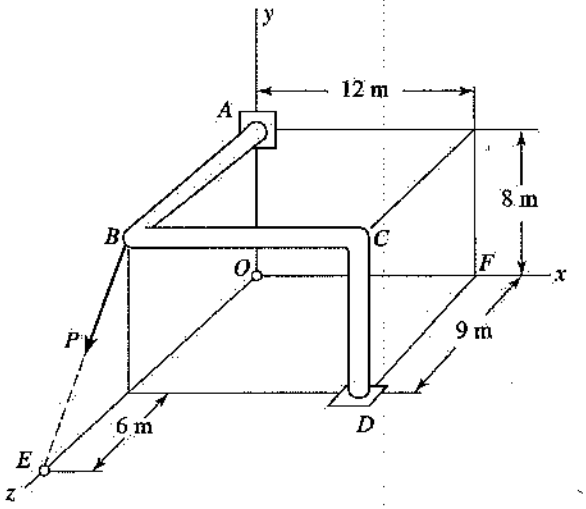


Figure 11.51

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5. Determine the resultant of the couples.
   (A) 250 N · m
   (B) 550 N · m
   (C) 1,100 N · m
   (D) 500 N · m

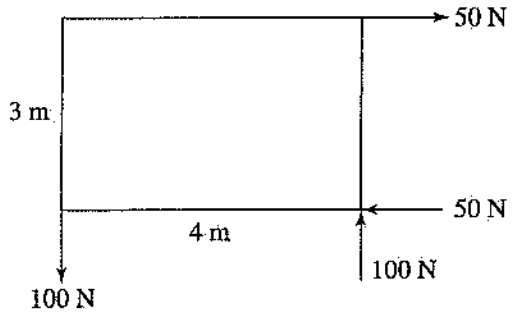


Figure 11.52

6. Determine the tension in cable *AC*.
   (A) 640 N
   (B) 863 N
   (C) 1,200 N
   (D) 550 N

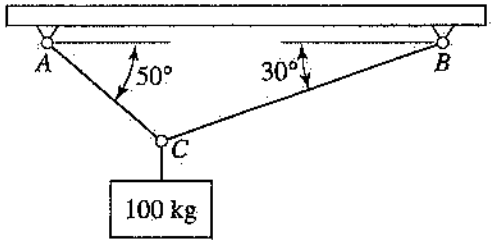


Figure 11.53

7. Determine the tension in cable *DB* if load *W* is 444 lb.
   (A) 150 lb
   (B) 450 lb
   (C) 210 lb
   (D) 156 lb

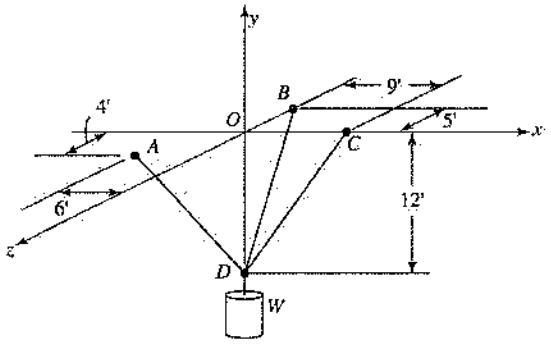


Figure 11.54

8. Determine the reaction at roller support *B*.
   (A) 365 N
   (B) 310 N
   (C) 435 N
   (D) 400 N

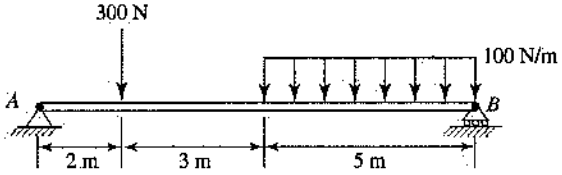


Figure 11.55

9. Determine the tension in cable *CE*.
   (A) 2,000 N
   (B) 2,253 N
   (C) 939 N
   (D) 2,080 N

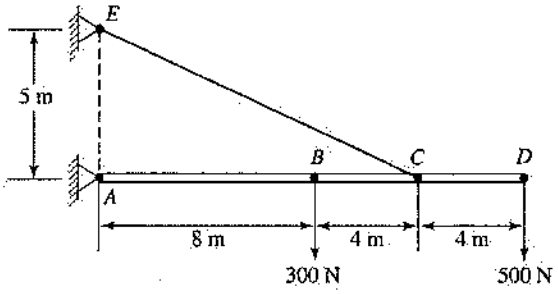


Figure 11.56

10. Determine the force in hydraulic cylinder *BD*.
    (A) 850 N
    (B) 608 N
    (C) 250 N
    (D) 869 N

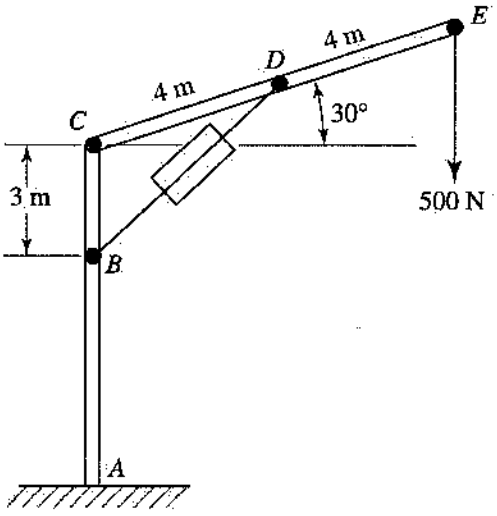


Figure 11.57

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11. Determine the tension in cable *BE*.
(A) 2,600 N
(B) 1,310 N
(C) 2,000 N
(D) 917 N

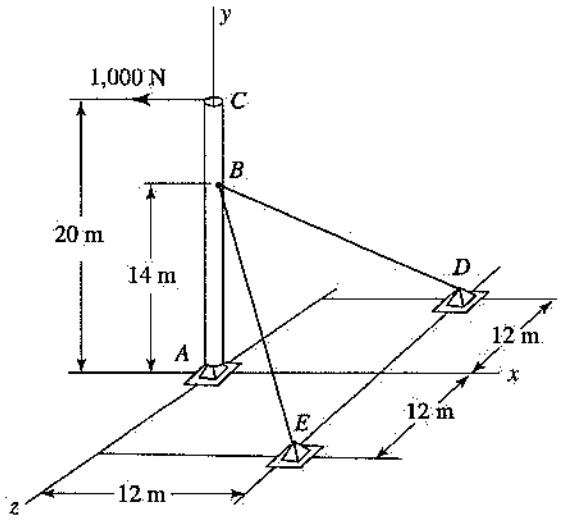


Figure 11.58

12. Determine the force in member *CE* of the truss.
(A) 3 kN
(B) 5 kN
(C) 8 kN
(D) 0

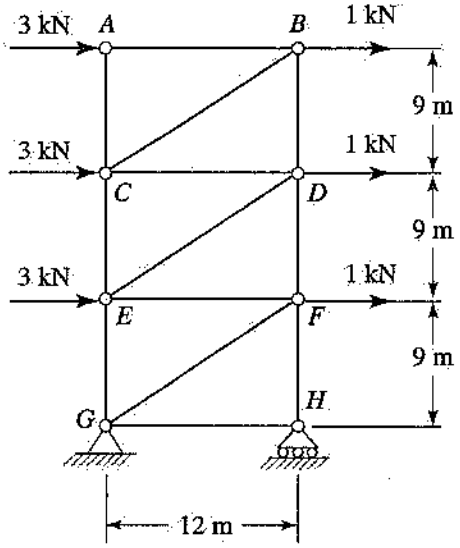


Figure 11.59

13. Determine the force in member *CF* of the truss.
(A) 30 kips
(B) 24 kips
(C) 48 kips
(D) 18 kips

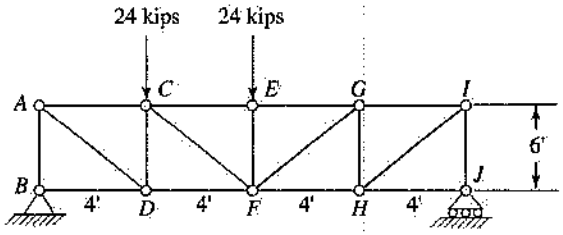


Figure 11.60

14. Determine the resultant reaction at *B* for the frame.
(A) 300 N
(B) 450 N
(C) 200 N
(D) 150 N

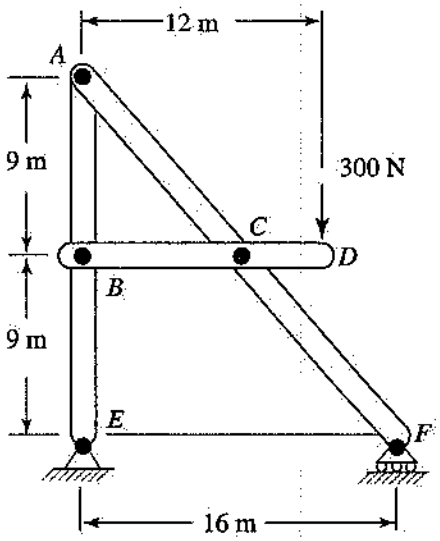


Figure 11.61

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15. Determine the friction force if $\mu_s = 0.3$ and $\mu_k = 0.2$.
(A) 170 N
(B) 191 N
(C) 850 N
(D) 294 N

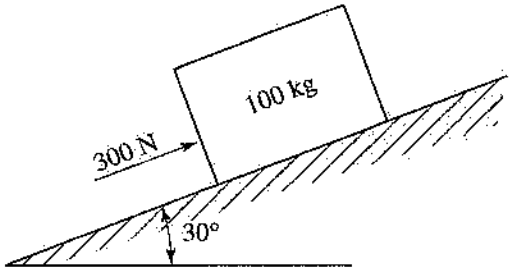


Figure 11.62

16. Determine the force required to move the cabinet. The casters at A and B are locked, the cabinet weighs 100 lb, $h = 30$ in, and $\mu_s = 0.3$.
(A) 30 lb
(B) 60 lb
(C) 100 lb
(D) 50 lb

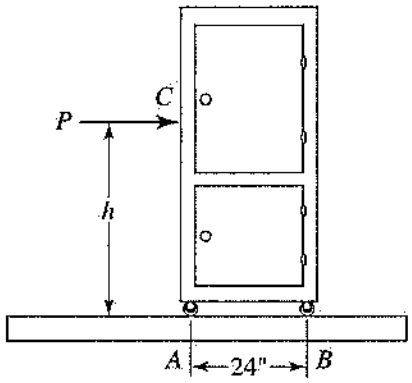


Figure 11.63

17. Determine the torque that must be applied to the flywheel in order to keep it rotating at a constant speed if $P = 8$ lb.
(A) 45 ft-lb
(B) 120 ft-lb
(C) 80 ft-lb
(D) 90 ft-lb

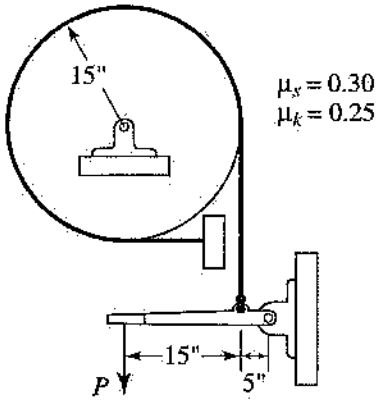


Figure 11.64

18. Determine the x-coordinate of the centroid of the plane area shown.
(A) 2 cm
(B) 6.25 cm
(C) 3.34 cm
(D) 0

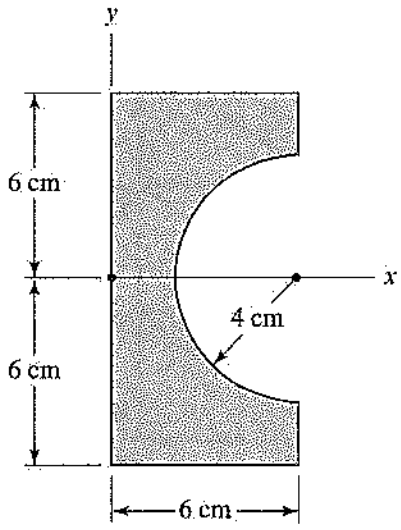


Figure 11.65

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19. Determine the area moment of inertia with respect to the x-axis.
(A) 435 cm^4
(B) 525 cm^4
(C) 429 cm^4
(D) 125 cm^4

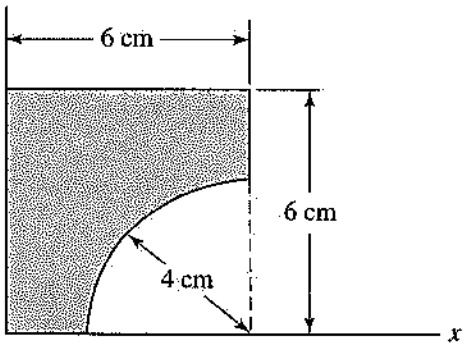


Figure 11.66

20. Determine the centroidal area moment of inertia with respect to the horizontal axis.
(A) 36 cm^4
(B) 257 cm^4
(C) 293 cm^4
(D) 220 cm^4

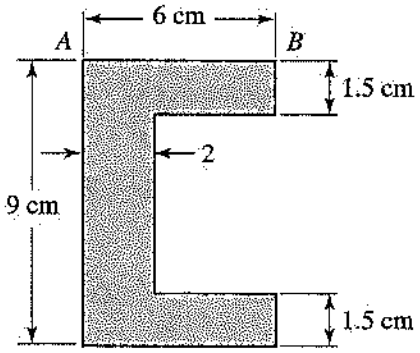


Figure 11.67

21. Determine the mass moment of inertia with respect to the x-axis. The sphere and the slender rod have the same mass, m.
(A) 19.4 ma^2
(B) 24 ma^2
(C) .33 ma^2
(D) 16 ma^2

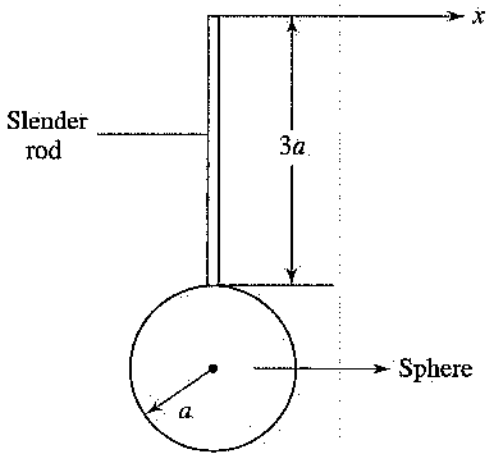


Figure 11.68

Answer Key

1. B	**8.** C	**15.** B
2. A	**9.** B	**16.** A
3. C	**10.** D	**17.** D
4. B	**11.** B	**18.** C
5. A	**12.** A	**19.** C
6. B	**13.** B	**20.** B
7. D	**14.** D	**21.** A

Answers Explained

1. B
$$R_x = 200\left(\frac{4}{5}\right) - 100\sin 20 - 130\left(\frac{5}{13}\right)$$
$$= 75.79\text{ N}$$
$$R_y = 200\left(\frac{3}{5}\right) + 100\cos 20 - 130\left(\frac{12}{13}\right)$$
$$= 93.97\text{ N}$$
$$R = \sqrt{(75.79)^2 + (93.97)^2} = 120.7\text{ N}$$

2. A
$$\mathbf{T}_{BA} = 450\frac{4\mathbf{i} - 4\mathbf{j} + 7\mathbf{k}}{9}$$
$$= 200\mathbf{i} - 200\mathbf{j} + 350\mathbf{k}$$
$$\mathbf{T}_{BC} = 450\frac{4\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}}{6}$$
$$= 300\mathbf{i} - 300\mathbf{j} - 150\mathbf{k}$$

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Also, $R_x = 500$ N, $R_y = -500$ N, $R_z = 200$ N.

$$R = \sqrt{(500)^2 + (-500)^2 + (200)^2} = 734.8 \text{ N}$$

3. C $M_A = 160(0.1) - 120(0.12) = 1.6$ N·m clockwise

4. B $\mathbf{P} = 50\dfrac{-8\,\mathbf{j} + 6\,\mathbf{k}}{10} = -40\,\mathbf{j} + 60\,\mathbf{k}$

$$M_{OC} = \mathbf{M}_O \cdot \mathbf{u}_{OC} = [15\mathbf{k} \times (-40\mathbf{j} + 60\mathbf{k})] \cdot \left(\frac{12\mathbf{i} + 8\mathbf{j} + 9\mathbf{k}}{17}\right) = 423.53 \text{ N·m}$$

5. A $(100)(4) - (50)(3) = 250$ N·m counterclockwise

6. B $R_x = \Sigma F_x = T_{BC}\cos 30 - T_{AC}\cos 50 = 0$
$R_y = \Sigma F_y = T_{BC}\sin 30 + T_{AC}\sin 50 - 981 = 0$

Solve simultaneously: $T_{AC} = 863$ N.

7. D $\mathbf{T}_{DA} = T_{DA}\left(\dfrac{-6\mathbf{i} + 12\mathbf{j} + 4\mathbf{k}}{14}\right)$

$$\mathbf{T}_{DB} = T_{DB}\left(\frac{12\mathbf{j} - 5\mathbf{k}}{13}\right)$$

$$\mathbf{T}_{DC} = T_{DC}\left(\frac{9\mathbf{i} + 12\mathbf{j}}{15}\right)$$

Then,

$$R_x = 0 \Rightarrow -\frac{3}{7}T_{DA} + \frac{9}{15}T_{DC} = 0$$

$$R_y = 0 \Rightarrow \frac{6}{7}T_{DA} + \frac{12}{13}T_{DB} - 444 = 0$$

$$R_z = 0 \Rightarrow \frac{2}{7}T_{DA} - \frac{5}{13}T_{DB} = 0$$

Solve simultaneously: $T_{DB} = 156$ lb.

8. C $\Sigma M_A = 0$,

$$B(10) - 500(7.5) - 300(2) = 0,$$

$B = 435$ N

9. B $\Sigma M_A = 0$,

$$\left(\frac{5}{13}T_{CE}\right)(12) - (500)(16) - (300)(8) = 0,$$

$T_{CE} = 2{,}253$ N

10. D Use the FBD of member *CDE*. Then:

$$\Sigma M_C = 0, \qquad F_{BD} = 869 \text{ N}$$

11. B $\Sigma M_A = 0$, $\quad 2\left(\dfrac{12}{22}T_{BE}\right)(14) - 1{,}000(20) = 0$,

$T_{BE} = 1{,}309.5$ N

12. A Cut through members *CE*, *DE*, and *DF* and use the FBD of the upper section of the truss. Then:

$\Sigma M_D = 0$, $\quad F_{CE}(12) - 3(9) - 1(9) = 0$,
$F_{CE} = 3$ kN T

13. B First, determine the reactions at *B*. $B = 30$ N. Cut through members *CE*, *CF*, and *DF*, and use the FBD of the left section of the truss. Then:

$\Sigma M_F = 0$, $\quad F_{CE}(6) + 30(8) - 24(4) = 0$,
$F_{CE} = 24$ kips C

14. D First, determine the reactions at *E*. $E_x = 0$ and $E_y = 75$ N. Use the FBD of member *ABE*.

$\Sigma M_A = 0$, $\quad B_x = 0$ and
$\Sigma F_y = 0$, $\quad B_y = 150$ N

Then $B = 150$ N.

15. B $\Sigma F_y = 0$, $\quad N = 849.5$ N
$\Sigma F_x = 0$, $\quad F = 190.5$ N

Since $F < \mu_s N$, the block is in equilibrium and the friction force is 190.5 N.

16. A $\Sigma F_y = 0$, $\quad N_A + N_B = 100$
$\Sigma F_x = 0$, $\quad P = 30$ lb

17. D Use the FBD of the lever, and take the moment about the pin:

$$\Sigma M = 0, \qquad T_1 = 32 \text{ lb}$$

Use equation 11.15 with $\mu_k = 0.25$ and $\beta = 270° = 3\pi/2$. Then $T_2 = 103.94$ lb.

Use the FBD of the drum, and take the moment about its center:

$$\Sigma M = 0, \qquad M = 89.925 \text{ ft-lb}$$

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18. **C** $\bar{X}_c = \dfrac{\Sigma x_i A_i}{A}$

$$= \frac{(3)(72) + \left[6 - \dfrac{4(4)}{3\pi}\right]\left[\dfrac{1}{2}\pi(4)^2\right]}{72 + \dfrac{1}{2}\pi(4)^2} = 3.34 \text{ cm}$$

19. **C** $I_x = \dfrac{6(6)^3}{3} - \dfrac{\pi(4)^2}{16} = 428.86 \text{ cm}^4$

20. **B** $I = 2I_{\text{flange}} + I_{\text{web}}$

$$I_{\text{flange}} = \frac{1}{12}(6)(1.5)^3 + 9(3.75)^2 = 128.25 \text{ cm}^4$$

$$I_{\text{web}} = \frac{1}{12}(2)(6)^3 = 36 \text{ cm}^4$$

$$I = 2(128.25) + 36 = 292.5 \text{ cm}^4$$

21. **A** $I_x = I_{\text{rod}} + I_{\text{sphere}}$

$$= \frac{1}{3}m(3a)^2 + \left[\frac{2}{5}ma^2 + m(4a)^2\right]$$

$$= 19.4\, ma^2$$

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CHAPTER

12

Thermodynamics

Mohamad Metghalchi, Sc.D.

Robert M. Scott

Introduction

Thermodynamics is the study of the properties of matter—in particular, the relationships among properties and the changes in the values that either occur spontaneously or result through interaction with other objects. The ultimate goal of thermodynamics is to understand the operation of thermal systems, to be able to design these systems, and to recognize opportunities for improving the performance of existing systems.

This presentation is divided into four major sections:

- Definitions of terms: The vocabulary of thermodynamics is developed to provide a framework for the explanation of thermodynamic principles.
- Laws and Principles: The basic laws and principles that describe the relationships among the properties of matter are presented.
- Material Systems: Several types of materials that can form the contents of thermodynamic systems are introduced. For certain special cases, the general expressions for some of the basic laws and principles are simplified, and some new relationships are introduced.
- Applications: The basic principles are applied to systems frequently encountered, and specific methods for analyzing these systems are presented.

12.1 Definitions of Terms
12.2 Laws and Principles
12.3 Material Systems
12.4 Applications

12.1 Definitions of Terms

System: A system is specified by a description of the material it contains, and the boundaries that contain it. There are two main classifications of systems.

Open System: Material may enter into or exit from the system. Many sources call this type of system a *control volume.*

Closed System: Material may not enter into or exit from the system. Many sources call this type of system a *control mass.*

Property: A property is an attribute of a system to which a value can be assigned to describe the system. A necessary feature of a property is that the value can be stated at a given time independently of its value at any other time, and independently of the process by which the system arrived at its state.

Intensive properties are independent of the mass of the system; examples are temperature and pressure.

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110. A flowchart:
- **(A)** is a sequence of instructions to perform operations
- **(B)** is alphabetic code to instruct the computer what to do
- **(C)** defines basic operations and decisions required
- **(D)** is a series of human language statements and common math symbols

111. A European country is experiencing an inflation rate of 25% per month. What is the effective annual rate of interest?
- **(A)** 300%
- **(B)** 25%
- **(C)** 1,355%
- **(D)** 1,455%

112. A company can manufacture parts for a manufacturing cost of $4.00 per unit plus $4,000 for tooling. A semiautomated system has been developed as an alternative. The costs for the proposed method are $40,000 for equipment and $1.00 per unit manufacturing cost per unit. What is the break-even number of units per year?
- **(A)** 12,000
- **(B)** 11,000
- **(C)** 4,000
- **(D)** 15,000

113. An engineer borrows $18,000 at 12% per annum compounded annually. The first payment will be made 3 years from now. The remaining balance at that time will be:
- **(A)** $24,050
- **(B)** $21,554
- **(C)** $25,288
- **(D)** $23,080

114. The annual payment into a fund is $800, and payments will be made for 15 years. Each payment is made at the end of the year. How much will the fund be worth when it matures if interest is 10% per year?
- **(A)** $31,215
- **(B)** $25,418
- **(C)** $35,600
- **(D)** $26,400

115. For an asset costing $25,000, what will be the depreciation charge for the 4th year? (Use the MACRS system.)
- **(A)** $1,680
- **(B)** $1,790
- **(C)** $1,264
- **(D)** $2,150

116. What body has jurisdiction over an alleged ethics violation by a Registered Professional Engineer?
- **(A)** National Society of Professional Engineers
- **(B)** State Board of Registration for Engineers
- **(C)** National professional society (e.g., ASME, IEEE) of the engineer involved
- **(D)** Local district court

117. Snow and ice heavily damaged a building that was designed by Registered Engineers and Architects. The owner was not able to recover damages because of:
- **(A)** the insolvency of the bond holder
- **(B)** the statute of limitations
- **(C)** a hold-harmless clause between architect and engineer
- **(D)** an Act of God clause in the insurance policy

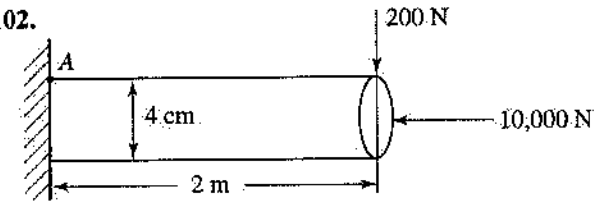
118. An engineer working for a consulting firm has decided to go into private practice. One of her first steps should be to:
- **(A)** submit her letter of resignation
- **(B)** tell colleagues about her plans
- **(C)** send e-mail to the firm regarding her plans
- **(D)** discuss her plans with her supervisor

119. As an engineer, your ethical responsibility, in order from highest to lowest importance, is to:
- **(A)** client, society, yourself, profession
- **(B)** yourself, profession, client, society
- **(C)** profession, yourself, society, client
- **(D)** society, client, profession, yourself

120. After leaving a company and setting up your own engineering practice, a former competitor contacts you to solve a problem similar to one you solved while working at the company. Your previous design seems to be a solution to this former competitor's problem. Can you accept this assignment?
- **(A)** No. Information from the original design would be required.
- **(B)** Yes. Knowledge transfer happens.
- **(C)** No. One should not accept money twice for the same job.
- **(D)** Yes. No stated nondisclosure statement was signed.

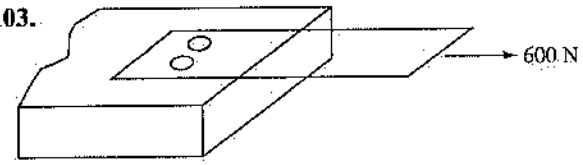
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102.



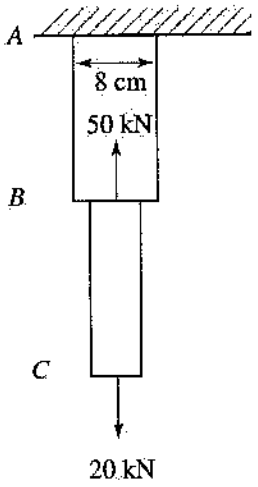
In the above diagram, the normal stress at point *A* is most nearly:
(A) 71.6 MPa
(B) 55.71 MPa
(C) 7.95 MPa
(D) 63.6 MPa

103.



The average shearing stress for each 3-centimeter-diameter bolt shown above is most nearly:
(A) 0.424 MPa
(B) 1.2 MPa
(C) 0.848 MPa
(D) 0.56 MPa

104.



The normal stress in section *AB* of the above diagram is most nearly:
(A) 9.95 MPa
(B) 3.97 MPa
(C) 13.92 MPa
(D) 5.97 MPa

105. What is the base-10 equivalent of the binary whole number 00100111_2?
(A) 12
(B) 24
(C) 39
(D) 56

106. For the base-10 number 47, the binary equivalent is:
(A) 011011_2
(B) 101111_2
(C) 101011_2
(D) 101100_2

107. How many times will the line labeled "START" execute in the following program?

```
        I=4
        J=2
START   J=J+I
        I=J^2
        IF J<100 THEN GO TO START
        ELSE GO TO FINISH
FINISH  PRINT J
```

(A) 3
(B) 4
(C) 5
(D) 6

Questions 108 and 109

The following is a segment of a spreadsheet. Use the data in the cells to answer questions 108 and 109.

	A	B	C	D
1	23	24	25	26
2	7	A2*2	B2*A$1	
3	8	A3*2	B3*B$1	
4	9	A4*2	B4*C$1	
5	10	A5*2	B5*D$1	

108. What will be the top to bottom values in Column B?
(A) 12,14,16,18
(B) 16,18,20,24
(C) 14,16,18,20
(D) 49,64,81,100

109. What will be the top to bottom values in Column C?
(A) 1127,1536,2025,2600
(B) 340,390,460,530
(C) 70,96,126,160
(D) 160,240,320,450

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92. A tensile specimen with a diameter of 10 millimeters is subjected to a load of 500 newtons. What is the engineering stress?
(A) 50 N/mm
(B) 30 ksi
(C) 5.2 N/mm^2
(D) 6.36 MN/m^2

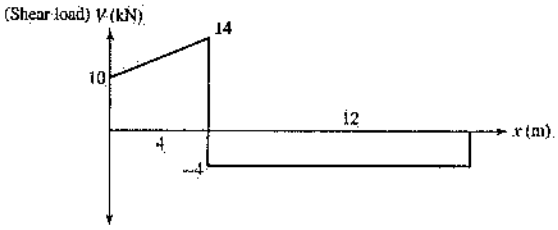
93. In a normalized plain carbon steel containing 0.5 weight percent carbon, approximately what percentage of the alloy is pearlite?
(A) 37%
(B) 63%
(C) 91%
(D) 7.4%

94. At room temperature a plain carbon steel containing 0.2 weight percent carbon is:
(A) hypereutectoid
(B) ferromagnetic
(C) brittle
(D) highly alloyed

95. A bar is rolled to a reduction in area of 30 percent. What is the true strain?
(A) 0.357
(B) 1.43
(C) 0.154
(D) 0.30

96. A material whose electrons cannot be excited across the band gap is considered to be:
(A) a metal
(B) a semiconductor
(C) an insulator
(D) a superconductor

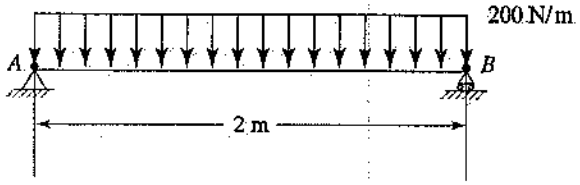
97.



What is the maximum magnitude of the bending moment in the beam shown in the above diagram? The bending moment at each end of the beam is 0, and there are no concentrated couples along the beam.
(A) 96 kN·m
(B) 48 kN·m
(C) 24 kN·m
(D) 40 kN·m

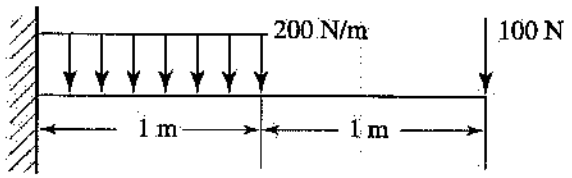
98. If the allowable shearing stress is 900 kilopascals, the torque transmitted by a 150-millimeter-diameter solid shaft is most nearly:
(A) 1,193 N·m
(B) 298 N·m
(C) 596 N·m
(D) 1,590 N·m

99.



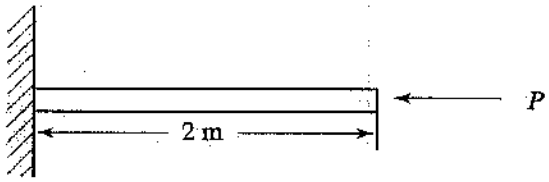
The maximum bending stress for the 4 × 6 centimeter beam shown in the diagram above is most nearly:
(A) 4.17 MPa
(B) 2.08 MPa
(C) 16.7 MPa
(D) 100 MPa

100.



The maximum transverse shearing stress for the 10-centimeter-diameter shaft beam shown in the above diagram is most nearly:
(A) 57.3 kPa
(B) 38.2 kPa
(C) 25.5 kPa
(D) 50.9 kPa

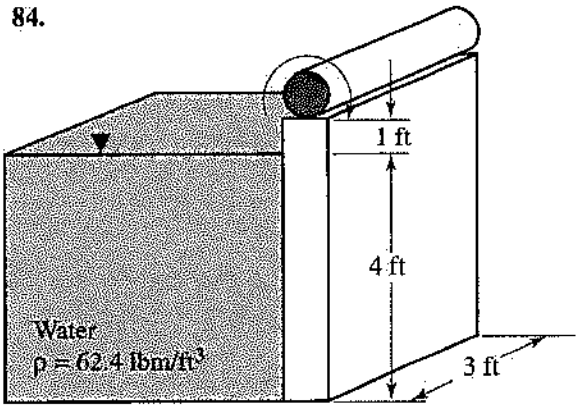
101.



The critical load, P, for the 2-meter long cantilever beam shown in the diagram is most nearly:
(A) 2.468 EI
(B) 1.234 EI
(C) 0.617 EI
(D) 0.5 EI

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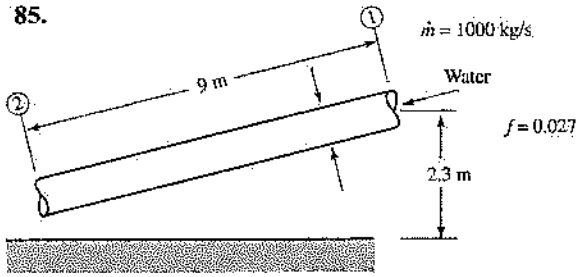
84.



In the above diagram, what is the approximate moment required at the hinge to keep the gate closed?

(A) 3,000 ft-lbf
(B) 4,500 ft-lbf
(C) 5,500 ft-lbf
(D) 4,000 ft-lbf

85.



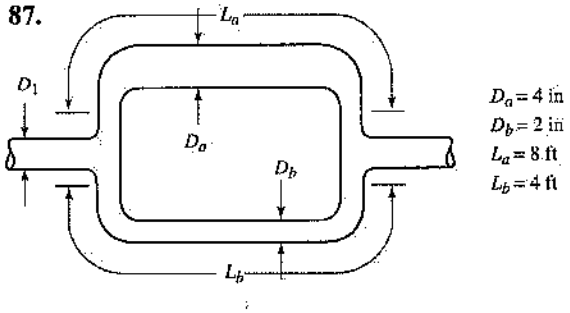
For the pipe shown above, what is the pressure drop ($p_1 - p_2$)?

(A) 2 kPa
(B) −4 kPa
(C) 2.06 kPa
(D) −2 kPa

86. The Reynolds number is:

(A) a measure of the rate of fluid flow
(B) the ratio of inertial forces to viscous forces
(C) the ratio of pressure forces to viscous forces
(D) rarely important in fluid flow analysis

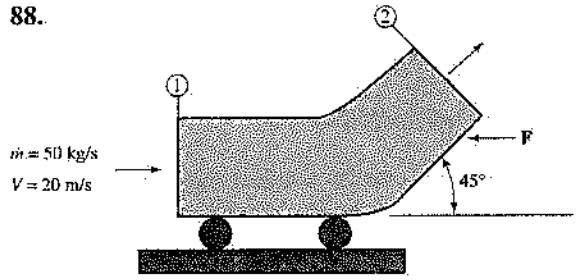
87.



What is the approximate volumetric flow rate through tube *a* in the above diagram?

(A) 40 ft³/sec
(B) 20 ft³/sec
(C) 50 ft³/sec
(D) 30 ft³/sec

88.



What force, **F**, is required to keep the cart shown above from moving? (Assume in the cross-sectional area that $\text{Area}_1 = \text{Area}_2$)

(A) 1,707 N
(B) 720 N
(C) 707 N
(D) 293 N

89. A copper hook is attached to a plain carbon steel chain below the surface in a saltwater bay. If galvanic corrosion occurs, what will be the anodic reaction?

(A) $Fe \rightleftarrows Fe^{2+} + 2e^-$
(B) $Cu \rightleftarrows Cu^{2+} + 2e^-$
(C) $Cu^{2+} + 2e^- \rightleftarrows Cu$
(D) $NaCl \rightleftarrows Na^+ + Cl^-$

90. A solution is required to be:

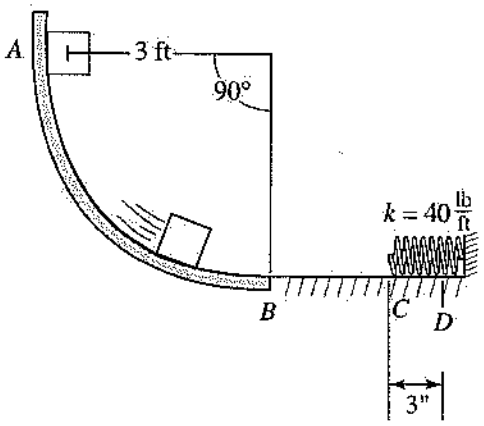
(A) a single phase
(B) a liquid composed of two or more components
(C) homogeneous
(D) comprised partly of water

91. Which of the following is the ground-state electronic configuration of a copper atom?

(A) $1s^2 2s^2 2p^6 3s^2 3p^6 4s^1 3d^{10}$
(B) $1s^2 1p^6 2s^2 2p^6 3s^2\ 3p^6 3d^5$
(C) $1s^2 1p^2 2s^2 2p^6 3s^2\ 3p^6 4s^2 4p^6 4d^1$
(D) $1s^2 2s^2 2p^6 3s^2 3p^6 3d^6 4s^2 4p^3$

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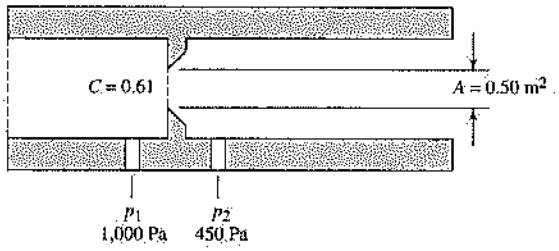
80.



The 5.0-pound block illustrated above is released from rest at A and slides along the frictionless circular surface AB. It then continues to move across the horizontal surface without friction and proceeds to strike a spring, C, having a stiffness of 40 pounds per foot. Assume that the spring is initially undeformed. What will be the velocity of the block at the instant when the spring is compressed 3.0 inches?

(A) 3 ft/sec
(B) 4 ft/sec
(C) 13.3 ft/sec
(D) 5.5 ft/sec

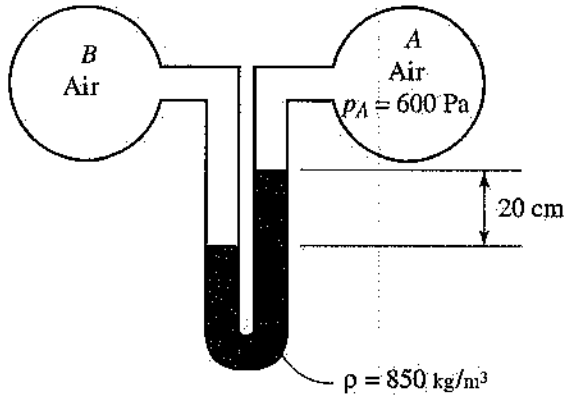
81.



In the above diagram, where C is the discharge coefficient, what is the volumetric flow rate through the orifice?

(A) .00542 m^3/s
(B) 0.28 m^3/s
(C) 1.28 m^3/s
(D) 0.65 m^3/s

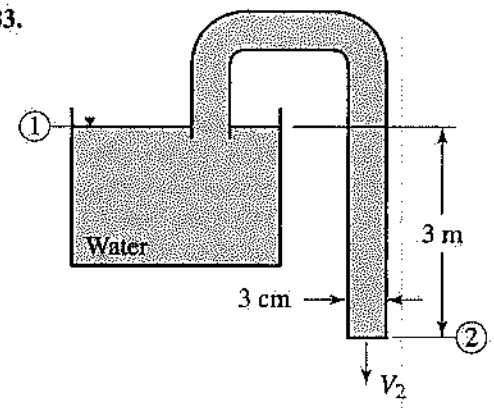
82.



The pressure in vessel B shown in the diagram above is most nearly:

(A) 2,268 Pa
(B) 684 Pa
(C) 433 Pa
(D) 767 Pa

83.

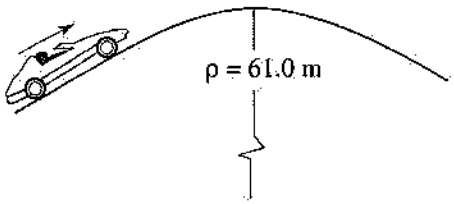


What is the volumetric flow rate of water leaving the siphon in the above diagram? (Assume that the diameter of the tank is much larger than the diameter of the siphon tube.)

(A) 0.32 m^3/s
(B) .00542 m^3/s
(C) 1.28 m^3/s
(D) 0.65 m^3/s

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75.



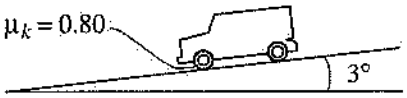
A 1,455-kilogram automobile (including the mass of the driver) is traveling along a hill that has a parabolic shape. The section of the hill near the crest has a radius of curvature of 61.0 meters, as shown in the above diagram. Wind resistance is considered negligible. If the speed of the automobile is 18.3 meters per second at the crest of the hill, the apparent weight of the system at this location should be:

(A) 7,988 N
(B) 22,261 N
(C) 6,285 N
(D) 641 N

76. A 3.0-pound-mass rigid body, **I**, initially moving at 20.0 feet per second along a straight line, collides with a 7.0-pound-mass body, **II**, traveling at 5.0 feet per second in the opposite direction. Body **I** has a postcollision linear momentum of 34.5 pounds mass-feet per second in a direction opposite to its initial motion. The coefficient of restitution for this impact must be:

(A) 0.57
(B) 0.65
(C) 0.80
(D) 0.93

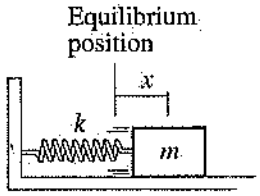
77.



A police officer measures skid marks left by all four wheels of a 1,300-kilogram car to be 20.0 meters along a road surface that has a 3° slope from the horizontal as illustrated in the above diagram. It is known from prevailing conditions that the coefficient of kinetic friction between the tires and the pavement was approximately 0.80 and that the car had a speed of 40.0 kilometers per hour at the end of the skid. The amount of energy dissipated by friction due to the entire skid is:

(A) 203,760 J
(B) 13,350 J
(C) 1,380 J
(D) 101,880 J

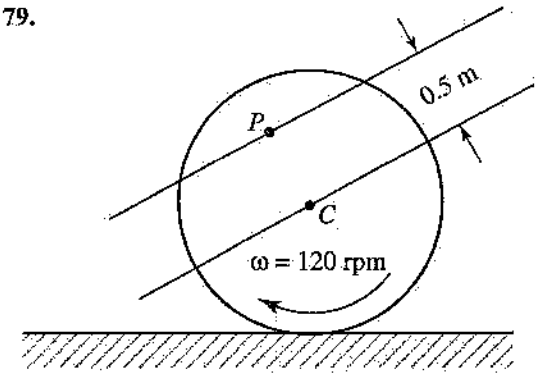
78.



The spring-mass system shown in the above diagram consists of a 1.8-kilogram block and a linear spring with a stiffness of 0.26 kilonewton per meter. The initial displacement of the block is 2.5 centimeters to the right of the equilibrium position, and the block is released from rest. Friction is considered negligible between the block and the horizontal surface. The period of this vibration is necessarily:

(A) 1.81 s
(B) 6.28 s
(C) 0.08 s
(D) 0.52 s

79.



A tractor wheel has a diameter of 1.5 meters and rolls with a constant angular speed of 120 rpm, as illustrated above. The speed of point *P*, measured with respect to the center *C* of the wheel, is:

(A) 9.4 m/s
(B) 60.0 m/s
(C) 6.3 m/s
(D) 18.8 m/s

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67. One kilogram of air undergoes an isometric process, starting from 200°C and 600 kilopascals, in which the temperature drops to 110°C. The final pressure is:
(A) 1,091 kPa
(B) 741 kPa
(C) 330 kPa
(D) 486 kPa

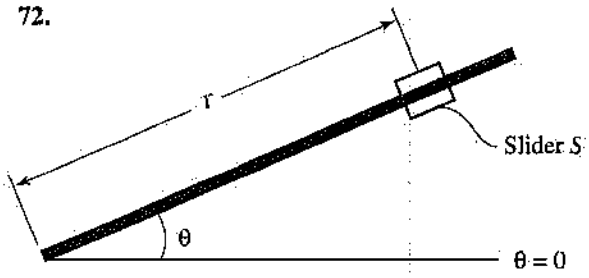
68. An ideal gas ($k = 1.4$), with an initial specific volume of 30 cubic meters per kilogram, experiences an isothermal compression from 200 kilopascals to 800 kilopascals. What is the final specific volume of the gas?
(A) 7.5 m^3/kg
(B) 11.14 m^3/kg
(C) 120 m^3/kg
(D) 75 m^3/kg

69. What is the specific internal energy of a liquid water/water vapor mixture with a quality of 58% at 40°C?
(A) 167.56 kJ/kg
(B) 2,262.6 kJ/kg
(C) 2,430.1 kJ/kg
(D) 1,479.8 kJ/kg

70. What is the power output of a turbine operating where 15 kilograms per second of air enters at 750 kilopascals and 280°C, and exits at 370 kilopascals and 37°C?
(A) 2,617 kW
(B) 3,645 kW
(C) 61 kW
(D) 44 kW

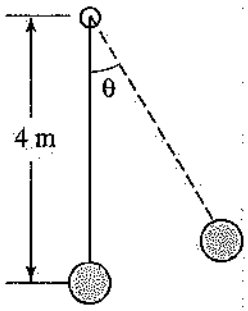
71. A rigid body experiences rectilinear motion (x-direction) and has an initial position $X_0 = 7.0$ feet at starting time $t_0 = 0$. The velocity of the body, in feet per second, is known according to the function $V(t) = t^2 - 6t + 9$, where t represents time in seconds. The distance traveled during the first 25.0 seconds of motion is:
(A) 169.9 ft
(B) −36.0 ft
(C) 635.0 ft
(D) 3,558.3 ft

72.



A rod/slider block mechanism rotates in a horizontal plane as shown in the diagram above. The motion is governed by the equations $r(t) = 2.0t^3 - 0.5t^2$ and $\theta(t) = t^2 + 8t$, where the units for r, t, and θ are meters, seconds, and radians, respectively. The mass of the slider block is 2.0 kilograms, and there is negligible friction between the rod and the slider block. The magnitude of the slider block's angular momentum when angular speed $\omega = 20.0$ radians per second is:
(A) 6.85×10^6 kg·m^2/s
(B) 1.20×10^3 kg·m^2/s
(C) 5.18×10^4 kg·m^2/s
(D) 4.06×10^5 kg·m^2/s

73.



The 15.0-kilogram ball shown in the diagram above is attached to an inextensible cord. The ball swings in a vertical plane, and air resistance is negligible. When $\theta = 30°$, the speed of the ball is known to be 3.0 meters per second. What is the tension in the cord at this instant?
(A) 161.2 N
(B) 3.8 N
(C) 46.7 N
(D) 127.5 N

74. A 6.0-pound-mass rectangular block slides down a 40° incline (from the horizontal), and the kinetic friction coefficient is 0.25. What is the magnitude of the block's total acceleration?
(A) 19.5 ft/sec^2
(B) 0.61 ft/sec^2
(C) 26.0 ft/sec^2
(D) 14.5 ft/sec^2

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Questions 57 and 58

Ethane gas burns according to this reaction:

$$2C_2H_6 + 7O_2 \rightarrow 4CO_2 + 6H_2O$$

57. How many grams of CO_2 gas are produced from 300 grams of C_2H_6?
(A) 300 g
(B) 440 g
(C) 600 g
(D) 880 g

58. How many liters of CO_2 gas are produced, assuming ideal gas behavior, at STP?
(A) 224 L
(B) 448 L
(C) 660 L
(D) 880 L

59. How many liters of a 0.25 M HCl solution are required to neutralize 20 grams of NaOH dissolved in enough water to make 1.0 liter of solution?
(A) 1.0 L
(B) 2.0 L
(C) 4.0 L
(D) 5.0 L

60. Which of the following is an intensive property?
(A) Pressure
(B) Entropy
(C) Internal energy
(D) Enthalpy

61. Which of the following statements is NOT true regarding the state of a system?
(A) It remains unchanged for a steady-state process.
(B) It is determined by the values of any two independent properties in a macroscopically homogeneous substance.
(C) It remains unchanged for a work interaction with a closed system.
(D) It may in some instances change spontaneously until the entropy of the system reaches a maximum value.

62. Which of the following devices is possible?
(A) A cyclic machine that will experience no other interaction than to produce energy through a work interaction, while transferring energy from a high-temperature reservoir to a low-temperature reservoir through heat interactions
(B) A cyclic machine that will experience no other interaction than to transfer to a thermal reservoir an amount of energy equal to the amount of energy it receives from a work interaction
(C) A device that will change the thermodynamic state of a material from one equilibrium state to another without experiencing a change in the amount of energy contained in the material, in the amount of material, or in the external forces placed on the material
(D) A cyclic machine that will experience no other interaction than to accept from a heat interaction with a high-temperature reservoir an amount of energy equal to the amount of energy it receives from a work interaction

63. Energy is added in the amount of 50 kilojoules in a heat interaction to a closed system while 30 kilojoules of work is done by the system. The change in the internal energy of the system is:
(A) 80 kJ
(B) 20 kJ
(C) −20 kJ
(D) −80 kJ

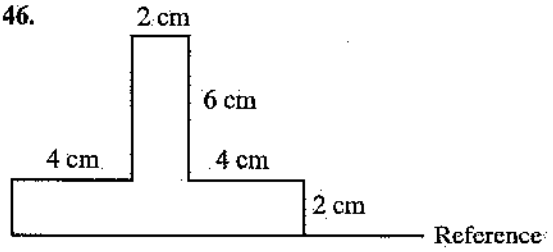
64. If a sample experiencing a change in temperature from 23°C to 46°C also experiences a change in specific enthalpy of 120 kilojoules per kilogram, of what material is the sample most likely to be composed? (Base your answer on the specific heat data provided in Chapter 12.)
(A) Water
(B) Helium
(C) Ammonia
(D) Hydrogen

65. How much energy must be transferred through a heat interaction to raise the temperature of a 4-kilogram sample of methane in a closed system from 15°C to 35°C?
(A) 34.8 kJ
(B) 45 kJ
(C) 139 kJ
(D) 180 kJ

66. For an isentropic process of an ideal gas ($k = 1.40$), with an initial pressure of 50 pounds per square inch absolute, an initial specific volume of 8.2 cubic feet per pound mass, and a final pressure of 120 psia, what is the final value of the specific volume?
(A) 8.2 ft^3/lbm
(B) 3.42 ft^3/lbm
(C) 19.7 ft^3/lbm
(D) 4.39 ft^3/lbm

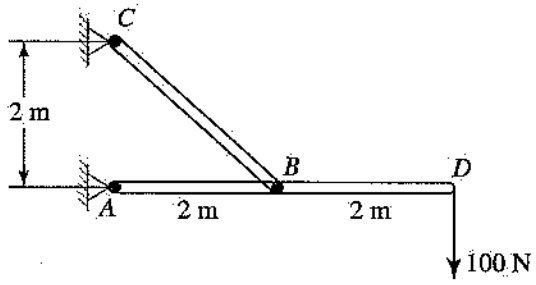
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46.



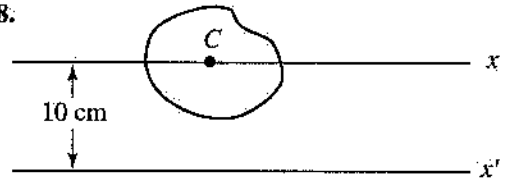
The y-coordinate of the centroid of the plane area shown above is most nearly:
(A) 2 cm
(B) 1.75 cm
(C) 4 cm
(D) 2.5 cm

47.



The resultant pin reaction at point B in the above diagram (neglecting the weights of BC and AD) is most nearly:
(A) 283 N
(B) 141.5 N
(C) 100 N
(D) 200 N

48.



If, in the above diagram, the area moment of inertia with respect to the x-axis (centroidal axis) is 2,000 (centimeters)^4 and with respect to x'-axis is 7,000 cm^4, the area is most nearly:
(A) 100 cm^2
(B) 20 cm^2
(C) 50 cm^2
(D) 90 cm^2

49. What is the valence or oxidation state of sulfur in sodium thiosulfate, $Na_2S_2O_3$?
(A) +1
(B) +2
(C) +3
(D) −2

50. The element most likely to form a cation is:
(A) Li
(B) O
(C) He
(D) F

51. Ten moles of $Ca(NO_3)_2$, in grams, is:
(A) 1,020 g
(B) 1,100 g
(C) 1,640 g
(D) 1,800 g

52. The correct formula for the compound iron(III) hydroxide is:
(A) Fe_3OH
(B) Fe_2O_3
(C) $Fe(OH)_3$
(D) $Fe_3(OH)_3$

53. What is the approximate freezing point of an aqueous solution of 111 grams of $CaCl_2$ dissolved in 1 kilogram of H_2O? [$K_f(H_2O) = 1.86$]
(A) −5.5°C
(B) −3.7°C
(C) −1.8°C
(D) +5.5°C

54. What is the correct expression for the equilibrium constant K_c for the chemical reaction shown below?

$$N_2(g) + 3H_2(g) \Leftrightarrow 2NH_3(g)$$

(A) $K_c = \dfrac{[NH_3]}{[H_2][N_2]}$

(B) $K_c = \dfrac{2[NH_3]}{3[H_2][N_2]}$

(C) $K_c = \dfrac{[NH_3]^2}{[N_2][H_2]^3}$

(D) $K_c = \dfrac{[N_2][H_2]^3}{[NH_3]^2}$

55. The number of oxygen atoms in 100 grams of calcium carbonate, $CaCO_3$, is:
(A) 300
(B) 6.022×10^{23}
(C) 18.066×10^{23}
(D) 30,000,000

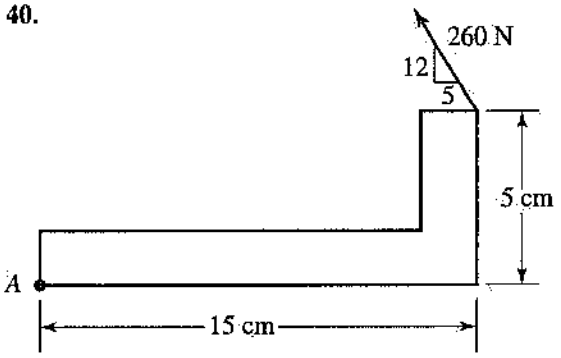
56. In the following reaction:

$$CuSO_4(aq) + Zn(s) \rightarrow Cu(s) + ZnSO_4(aq)$$

the oxidizing agent is:
(A) Zn
(B) Cu
(C) $CuSO_4$
(D) $ZnSO_4$

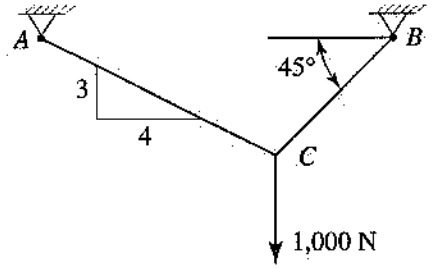
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40.



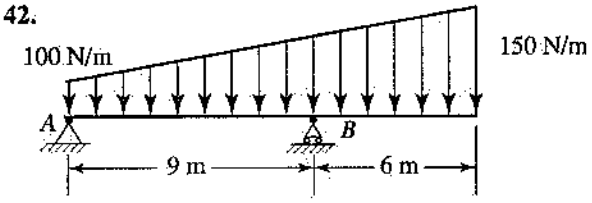
The moment of the 260-newton force about point A in the diagram above is most nearly:
(A) 27 N·m
(B) 31 N·m
(C) 18.4 N·m
(D) 41 N·m

41.



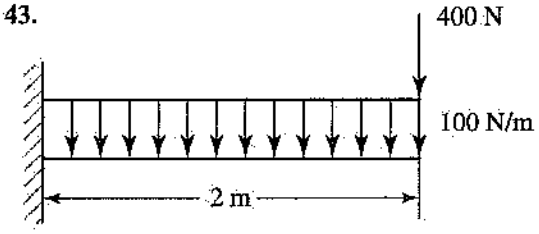
The tension in cable AC, shown in the diagram above, is most nearly:
(A) 806.5 N
(B) 714 N
(C) 1414 N
(D) 600 N

42.



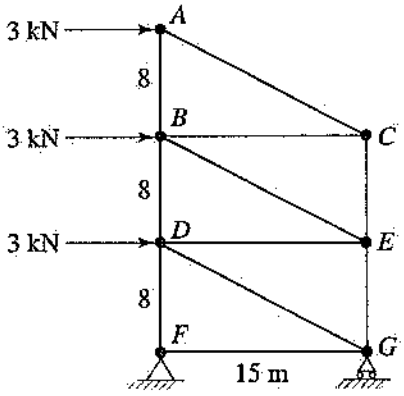
The reaction at point A in the diagram is most nearly:
(A) 208 N
(B) 1,625 N
(C) 500 N
(D) 937 N

43.



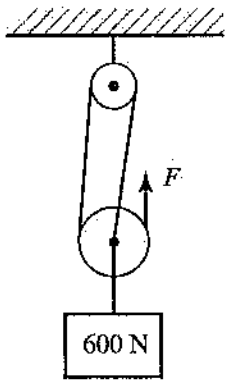
The maximum moment in the beam shown in the diagram is most nearly:
(A) 800 N·m
(B) 1,000 N·m
(C) 200 N·m
(D) 500 N·m

44.



In the above diagram, the force in member DE is most nearly:
(A) 9 kN
(B) 24 kN
(C) 6 kN
(D) 9.6 kN

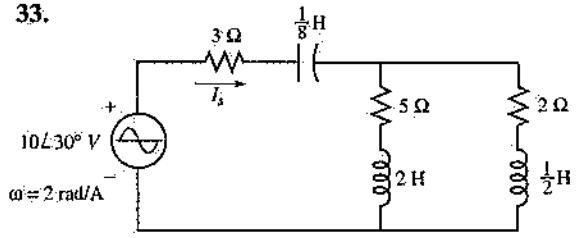
45.



In the above diagram, the force needed for equilibrium is most nearly:
(A) 600 N
(B) 200 N
(C) 300 N
(D) 150 N

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33.



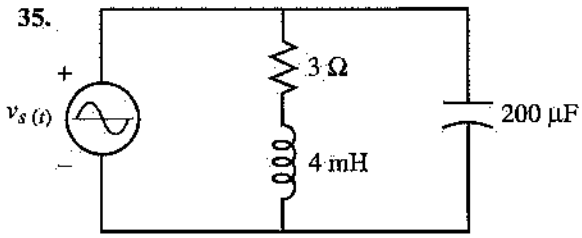
What is the source current, I_s, in the circuit shown above?

(A) $1.84\angle 66°$ A
(B) $2.56\angle -42°$ A
(C) $0.56\angle 28°$ A
(D) $3.31\angle -25°$ A

34. A series-tuned circuit is to be designed to have a resonant frequency of 31.8 kilohertz and a bandwidth of 2 kilohertz. If the inductor value is 10 millihenrys, the values of the capacitor and the resistor are:

(A) 7.5 μF, 5 Ω
(B) 5.5 μF, 2.5 Ω
(C) 2.5 μF, 125.7 Ω
(D) 4.5 μF, 20 Ω

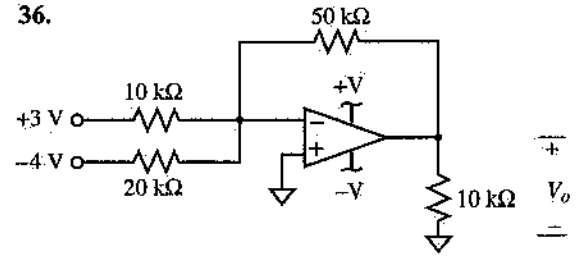
35.



If the source voltage in the circuit shown above is given by the equation $v_s(t) = 100\sqrt{2}\cos 1000t$ volts, what is the complex power delivered to the load?

(A) $565\angle 52°$ VA
(B) $12.65\angle 18.4°$ VA
(C) $720\angle -22.5°$ VA
(D) $975\angle -65°$ VA

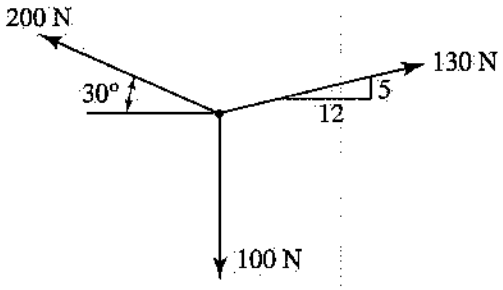
36.



The output voltage, V_o, for the operational amplifier circuit shown above is:

(A) 3.33 V
(B) −2.5 V
(C) 2.5 V
(D) −5.0 V

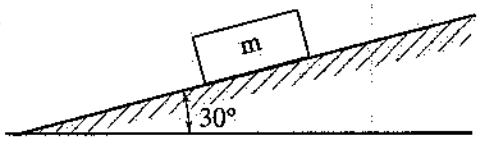
37.



The magnitude of the resultant of the forces shown in the above diagram is most nearly:

(A) 456.6 N
(B) 225.6 N
(C) 73 N
(D) 230 N

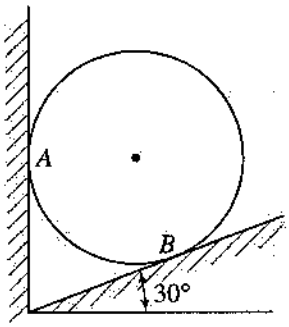
38.



The minimum coefficient of static friction for the block shown in the diagram to be in equilibrium is most nearly:

(A) 0.58
(B) 0.5
(C) 1
(D) 0.87

39.

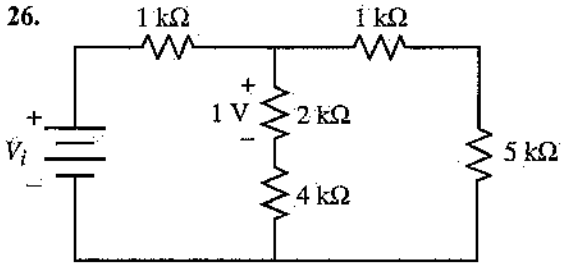


A cylinder weighing 100 newtons rests between two smooth walls, as shown in the diagram. The reaction at point B is most nearly:

(A) 100 N
(B) 115.5 N
(C) 50 N
(D) 86.6 N

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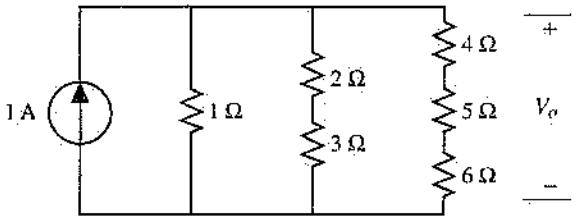
26.



The value of the source voltage, V_i, for the circuit shown above is:

(A) 4 V **(C)** 5.5 V
(B) 7 V **(D)** 9.5 V

Questions 27 and 28 are based on the diagram shown below.



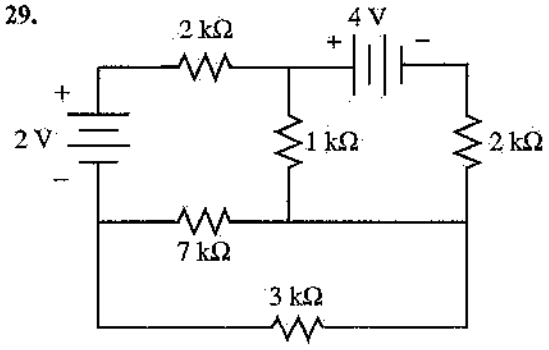
27. What is the value of the output voltage, V_o in this circuit?

(A) 0.789 V **(C)** 4.321 V
(B) 1.256 V **(D)** 0.543 V

28. What is the power dissipated by the 2-kiloohm resistor in this circuit?

(A) 1.22 W **(C)** 0.24 W
(B) 0.31 W **(D)** 0.05 W

29.



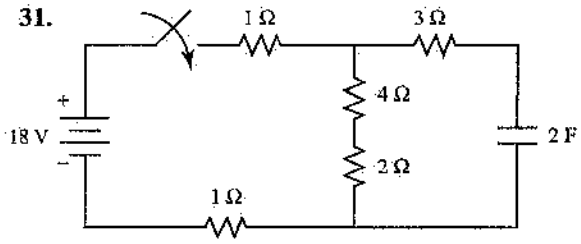
What is the voltage drop across the 1-kiloohm resistor shown in the diagram above?

(A) 0.294 V **(C)** 3.24 V
(B) 1.43 V **(D)** 4.56 V

30. The voltage between the terminals of a 0.01microfarad capacitor is 50 volts. What is the charge on the plates of the capacitor?

(A) 50 μC
(B) 5.0 μC
(C) 0.5 μC
(D) 10 μC

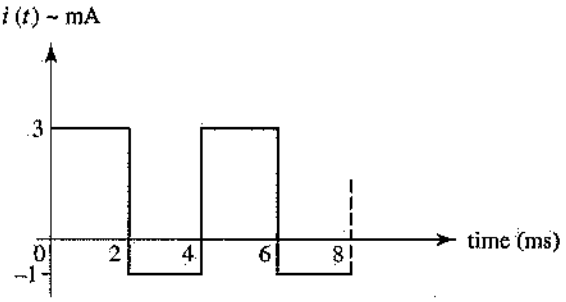
31.



The initial capacitor voltage in the circuit shown above is 0 volt. What voltage will be measured across the capacitor 2 seconds after the switch is closed?

(A) 8.65 V
(B) 1.23 V
(C) 10.64 V
(D) 2.18 V

32.



The waveform shown in the above diagram repeats every 4 milliseconds. The average and rms values of the waveform are:

(A) 2.0 V, 2.82 V
(B) 1.0 V, 2.24 V
(C) 0.5 V, 1.0 V
(D) 1.25 V, 2.82 V

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12. The area of Quadrant I that is bounded by $y = x^2$, $x = 3$, and the x-axis is:
(A) 9 square units
(B) 27 square units
(C) 6 square units
(D) 10 square units

13. What is the maximum value of the function f defined by $f(x) = x^3 - 3x$?
(A) -2
(B) 2
(C) 0
(D) 9

14. The slope of the line that passes through points (a,b) and (b,a) is:
(A) 0
(B) 1
(C) -1
(D) undefined

15. The value of $\log_3 50$ is a number between:
(A) 16 and 17
(B) 3 and 4
(C) 0 and 2
(D) 5 and 6

16. What is the inverse of the matrix $\begin{bmatrix} 3 & 1 \\ -5 & -2 \end{bmatrix}$?
(A) $\begin{bmatrix} 2 & 1 \\ -5 & -3 \end{bmatrix}$
(B) $\frac{1}{11}\begin{bmatrix} 2 & -1 \\ 5 & 3 \end{bmatrix}$
(C) $\frac{1}{11}\begin{bmatrix} -2 & 1 \\ 5 & 3 \end{bmatrix}$
(D) undefined

17. The complex number $3 - 3i$ is equivalent to:
(A) $\sqrt{2}e^{(\pi/4)i}$
(B) $3\sqrt{2}e^{(\pi/4)i}$
(C) $3\sqrt{2}e^{(5\pi/4)i}$
(D) $3\sqrt{2}e^{(7\pi/4)i}$

18. The measure of the smallest angle in a triangle with sides that are 4, 5, and 8 meters in length is:
(A) 30.8°
(B) 24.1°
(C) 22.0°
(D) 29.7°

19. What are the coordinates of the center of a sphere whose equation is $x^2 + y^2 + z^2 - 4y + 6z = 0$?
(A) $(0,-2,3)$
(B) $(0,2,-3)$
(C) $(1,4,9)$
(D) $(1,-2,-3)$

20. What is the sum of the infinite geometric series $\sum_{i=1}^{\infty} (0.3)^i$?
(A) $\frac{3}{10}$
(B) $\frac{1}{2}$
(C) $\frac{2}{5}$
(D) $\frac{3}{7}$

21. The value of the definite integral $\int_2^3 \frac{1}{x-1}dx$ is:
(A) ln 2
(B) ln 3
(C) 1
(D) 0

22. The equation of the tangent to the curve $x^2 + y^2 = 25$ at point (3,4) is:
(A) $4x - 3y = 0$
(B) $3x - 4y = -7$
(C) $4x + 3y = 24$
(D) $3x + 4y = 25$

23. The volume of the solid generated by revolving the area bounded by $y = \sqrt{x}$, $y = 2$, and the y-axis about the y-axis is:
(A) 8π cubic units
(B) $\frac{16\pi}{3}$ cubic units
(C) $\frac{32\pi}{5}$ cubic units
(D) $\frac{8\pi}{3}$ cubic units

24. For $0 < x < \frac{\pi}{2}$, the trigonometric expression $\tan x \csc x \cos x$ is equal to:
(A) 1
(B) 0
(C) $\sin x$
(D) $\cos 2x$

25. If a 100-foot spool of wire has a resistance between its ends of 5 milliohms, what is the resistivity of the wire? The cross-sectional area of the wire is 0.2 sq. in.
(A) $1.666 \times 10^{-6}\ \Omega \cdot m$
(B) $5.242 \times 10^{-6}\ \Omega \cdot m$
(C) $4.233 \times 10^{-7}\ \Omega \cdot m$
(D) $2.115 \times 10^{-8}\ \Omega \cdot m$

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Practice Examination

Morning Session

120 Questions

Directions: For each incomplete statement or question, select the answer that best completes the sentence or answers the question. Then fill in the corresponding space on the answer sheet.

1. The partial derivative $\frac{\partial z}{\partial x}$ of $z = 2x^2y - 2(x + 3y)$ is:
   (A) $2x^2 + 3$
   (B) $4xy - 2$
   (C) $2x^2y$
   (D) $4xy + 3$

2. A bag contains 20 balls numbered 1 to 20. If one ball is removed from the bag, what is the probability that the ball is even or is less than 5?
   (A) .5
   (B) .75
   (C) .65
   (D) .6

3. The value of the determinant $\begin{vmatrix} -1 & 4 & -2 \\ 6 & 3 & -4 \\ 0 & -2 & 1 \end{vmatrix}$ is:
   (A) 5
   (B) 0
   (C) -1
   (D) 6

4. The graph of the general quadratic equation $3x^2 - 10xy + 3y^2 + 8 = 0$ is:
   (A) an ellipse
   (B) a parabola
   (C) a hyperbola
   (D) a circle

5. In statistics, the measure of the spread or dispersion of the sample data is usually given by the:
   (A) median
   (B) standard deviation
   (C) mean
   (D) mode

6. A geometric interpretation of the first derivative of $y = f(x)$ is:
   (A) the area under the curve $y = f(x)$
   (B) the volume found by rotating the area under the curve $y = f(x)$ about the x-axis
   (C) the slope of the normal to the curve $y = f(x)$
   (D) the slope of a tangent to the curve $y = f(x)$

7. The general solution of the differential equation $\frac{dy}{dx} + 3y = 0$ with $y(0) = 1$ is:
   (A) $y = e^{-3x}$
   (B) $y = e^{3x}$
   (C) $y = \ln 3x$
   (D) $y = xe^{3x}$

8. The general solution of the differential equation $y'' + 4y' + 4 = 0$ is:
   (A) $y = c_1e^{2x}$
   (B) $y = e^{-2x}(c_1 + c_2x)$
   (C) $y = c_1e^{2x} + c_2xe^{2x}$
   (D) $y = c_1e^{-2x} + c_2$

9. A particle travels in a straight line in such a way that its distance s in meters from a given point on the line after time t in seconds is given by the equation $s = 10t^3 - 2t^5$. What is the instantaneous velocity of the particle at $t = 1$ second?
   (A) -2 m/s
   (B) 15 m/s
   (C) 50 m/s
   (D) 20 m/s

10. A unit vector perpendicular to the vector $\mathbf{A} = 3\mathbf{i} - 4\mathbf{j}$ is:
    (A) $4\mathbf{i} - 3\mathbf{j}$
    (B) $\mathbf{i}$
    (C) $\frac{3}{5}\mathbf{i} - \frac{4}{5}\mathbf{j}$
    (D) $\frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{j}$

11. A physical interpretation of the definite integral $\int_a^b f(x)dx$ is:
    (A) the instantaneous velocity of an object along the curve $y = f(x)$ from a to b
    (B) the acceleration of an object along the curve $y = f(x)$ from a to b
    (C) the work done by a variable force $y = f(x)$ in moving an object from a to b
    (D) both (A) and (C)

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ANSWER SHEET: Morning Session

1. Ⓐ Ⓑ Ⓒ Ⓓ	25. Ⓐ Ⓑ Ⓒ Ⓓ	49. Ⓐ Ⓑ Ⓒ Ⓓ	73. Ⓐ Ⓑ Ⓒ Ⓓ	97. Ⓐ Ⓑ Ⓒ Ⓓ
2. Ⓐ Ⓑ Ⓒ Ⓓ	26. Ⓐ Ⓑ Ⓒ Ⓓ	50. Ⓐ Ⓑ Ⓒ Ⓓ	74. Ⓐ Ⓑ Ⓒ Ⓓ	98. Ⓐ Ⓑ Ⓒ Ⓓ
3. Ⓐ Ⓑ Ⓒ Ⓓ	27. Ⓐ Ⓑ Ⓒ Ⓓ	51. Ⓐ Ⓑ Ⓒ Ⓓ	75. Ⓐ Ⓑ Ⓒ Ⓓ	99. Ⓐ Ⓑ Ⓒ Ⓓ
4. Ⓐ Ⓑ Ⓒ Ⓓ	28. Ⓐ Ⓑ Ⓒ Ⓓ	52. Ⓐ Ⓑ Ⓒ Ⓓ	76. Ⓐ Ⓑ Ⓒ Ⓓ	100. Ⓐ Ⓑ Ⓒ Ⓓ
5. Ⓐ Ⓑ Ⓒ Ⓓ	29. Ⓐ Ⓑ Ⓒ Ⓓ	53. Ⓐ Ⓑ Ⓒ Ⓓ	77. Ⓐ Ⓑ Ⓒ Ⓓ	101. Ⓐ Ⓑ Ⓒ Ⓓ
6. Ⓐ Ⓑ Ⓒ Ⓓ	30. Ⓐ Ⓑ Ⓒ Ⓓ	54. Ⓐ Ⓑ Ⓒ Ⓓ	78. Ⓐ Ⓑ Ⓒ Ⓓ	102. Ⓐ Ⓑ Ⓒ Ⓓ
7. Ⓐ Ⓑ Ⓒ Ⓓ	31. Ⓐ Ⓑ Ⓒ Ⓓ	55. Ⓐ Ⓑ Ⓒ Ⓓ	79. Ⓐ Ⓑ Ⓒ Ⓓ	103. Ⓐ Ⓑ Ⓒ Ⓓ
8. Ⓐ Ⓑ Ⓒ Ⓓ	32. Ⓐ Ⓑ Ⓒ Ⓓ	56. Ⓐ Ⓑ Ⓒ Ⓓ	80. Ⓐ Ⓑ Ⓒ Ⓓ	104. Ⓐ Ⓑ Ⓒ Ⓓ
9. Ⓐ Ⓑ Ⓒ Ⓓ	33. Ⓐ Ⓑ Ⓒ Ⓓ	57. Ⓐ Ⓑ Ⓒ Ⓓ	81. Ⓐ Ⓑ Ⓒ Ⓓ	105. Ⓐ Ⓑ Ⓒ Ⓓ
10. Ⓐ Ⓑ Ⓒ Ⓓ	34. Ⓐ Ⓑ Ⓒ Ⓓ	58. Ⓐ Ⓑ Ⓒ Ⓓ	82. Ⓐ Ⓑ Ⓒ Ⓓ	106. Ⓐ Ⓑ Ⓒ Ⓓ
11. Ⓐ Ⓑ Ⓒ Ⓓ	35. Ⓐ Ⓑ Ⓒ Ⓓ	59. Ⓐ Ⓑ Ⓒ Ⓓ	83. Ⓐ Ⓑ Ⓒ Ⓓ	107. Ⓐ Ⓑ Ⓒ Ⓓ
12. Ⓐ Ⓑ Ⓒ Ⓓ	36. Ⓐ Ⓑ Ⓒ Ⓓ	60. Ⓐ Ⓑ Ⓒ Ⓓ	84. Ⓐ Ⓑ Ⓒ Ⓓ	108. Ⓐ Ⓑ Ⓒ Ⓓ
13. Ⓐ Ⓑ Ⓒ Ⓓ	37. Ⓐ Ⓑ Ⓒ Ⓓ	61. Ⓐ Ⓑ Ⓒ Ⓓ	85. Ⓐ Ⓑ Ⓒ Ⓓ	109. Ⓐ Ⓑ Ⓒ Ⓓ
14. Ⓐ Ⓑ Ⓒ Ⓓ	38. Ⓐ Ⓑ Ⓒ Ⓓ	62. Ⓐ Ⓑ Ⓒ Ⓓ	86. Ⓐ Ⓑ Ⓒ Ⓓ	110. Ⓐ Ⓑ Ⓒ Ⓓ
15. Ⓐ Ⓑ Ⓒ Ⓓ	39. Ⓐ Ⓑ Ⓒ Ⓓ	63. Ⓐ Ⓑ Ⓒ Ⓓ	87. Ⓐ Ⓑ Ⓒ Ⓓ	111. Ⓐ Ⓑ Ⓒ Ⓓ
16. Ⓐ Ⓑ Ⓒ Ⓓ	40. Ⓐ Ⓑ Ⓒ Ⓓ	64. Ⓐ Ⓑ Ⓒ Ⓓ	88. Ⓐ Ⓑ Ⓒ Ⓓ	112. Ⓐ Ⓑ Ⓒ Ⓓ
17. Ⓐ Ⓑ Ⓒ Ⓓ	41. Ⓐ Ⓑ Ⓒ Ⓓ	65. Ⓐ Ⓑ Ⓒ Ⓓ	89. Ⓐ Ⓑ Ⓒ Ⓓ	113. Ⓐ Ⓑ Ⓒ Ⓓ
18. Ⓐ Ⓑ Ⓒ Ⓓ	42. Ⓐ Ⓑ Ⓒ Ⓓ	66. Ⓐ Ⓑ Ⓒ Ⓓ	90. Ⓐ Ⓑ Ⓒ Ⓓ	114. Ⓐ Ⓑ Ⓒ Ⓓ
19. Ⓐ Ⓑ Ⓒ Ⓓ	43. Ⓐ Ⓑ Ⓒ Ⓓ	67. Ⓐ Ⓑ Ⓒ Ⓓ	91. Ⓐ Ⓑ Ⓒ Ⓓ	115. Ⓐ Ⓑ Ⓒ Ⓓ
20. Ⓐ Ⓑ Ⓒ Ⓓ	44. Ⓐ Ⓑ Ⓒ Ⓓ	68. Ⓐ Ⓑ Ⓒ Ⓓ	92. Ⓐ Ⓑ Ⓒ Ⓓ	116. Ⓐ Ⓑ Ⓒ Ⓓ
21. Ⓐ Ⓑ Ⓒ Ⓓ	45. Ⓐ Ⓑ Ⓒ Ⓓ	69. Ⓐ Ⓑ Ⓒ Ⓓ	93. Ⓐ Ⓑ Ⓒ Ⓓ	117. Ⓐ Ⓑ Ⓒ Ⓓ
22. Ⓐ Ⓑ Ⓒ Ⓓ	46. Ⓐ Ⓑ Ⓒ Ⓓ	70. Ⓐ Ⓑ Ⓒ Ⓓ	94. Ⓐ Ⓑ Ⓒ Ⓓ	118. Ⓐ Ⓑ Ⓒ Ⓓ
23. Ⓐ Ⓑ Ⓒ Ⓓ	47. Ⓐ Ⓑ Ⓒ Ⓓ	71. Ⓐ Ⓑ Ⓒ Ⓓ	95. Ⓐ Ⓑ Ⓒ Ⓓ	119. Ⓐ Ⓑ Ⓒ Ⓓ
24. Ⓐ Ⓑ Ⓒ Ⓓ	48. Ⓐ Ⓑ Ⓒ Ⓓ	72. Ⓐ Ⓑ Ⓒ Ⓓ	96. Ⓐ Ⓑ Ⓒ Ⓓ	120. Ⓐ Ⓑ Ⓒ Ⓓ

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General

Taking the practice exams included in this book is an important part of your preparation for the FE/EIT exam. Take the practice exams only after you have thoroughly reviewed the concepts and worked the sample problems presented in this book.

Included are two full-length practice exams. The first practice exam covers the material presented in the Morning Session exam. As on the actual exam, you have four hours to complete the practice test. For examinees not electing to take a discipline-specific afternoon exam, the second practice exam covers the material in the afternoon General Exam option. Again, as on the actual exam, you have four hours to complete the practice test.

Training is the best way to increase your test-taking staminal For this reason it is strongly recommended that you take these practice exams in two back-to-back, four-hour sessions. Simulate test conditions; use only your calculator and your copy of the "NCEES FE Reference Handbook" while taking the practice exams. Read each question carefully, study the diagrams, and eliminate obviously incorrect answers. Since there is no penalty for wrong answers, make your best guess when you do not know the answer to a question.

Allow enough time in your study program not only to take the practice exams but also to review any problem areas that become evident when you analyze your incorrect answers. Fill in the charts at the end of each exam as you score your results. An analysis of incorrect answers is a useful self-teaching tool and will help you to fine-tune your final study sessions.

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Answer Key

Morning Session

1. B	25. D	49. B	73. A	97. B
2. D	26. A	50. A	74. D	98. C
3. A	27. A	51. C	75. C	99. A
4. C	28. D	52. C	76. C	100. D
5. B	29. B	53. A	77. A	101. C
6. D	30. C	54. C	78. D	102. B
7. A	31. D	55. C	79. C	103. A
8. B	32. B	56. C	80. C	104. D
9. D	33. A	57. D	81. A	105. C
10. D	34. C	58. B	82. A	106. B
11. C	35. B	59. B	83. B	107. A
12. A	36. D	60. A	84. C	108. D
13. B	37. C	61. C	85. C	109. A
14. C	38. A	62. A	86. B	110. C
15. B	39. B	63. B	87. A	111. C
16. A	40. D	64. B	88. D	112. A
17. D	41. B	65. C	89. A	113. C
18. B	42. A	66. D	90. A	114. B
19. B	43. B	67. D	91. A	115. C
20. D	44. C	68. A	92. D	116. B
21. A	45. B	69. D	93. B	117. C
22. D	46. D	70. B	94. B	118. D
23. C	47. A	71. D	95. A	119. D
24. A	48. C	72. A	96. C	120. A

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Diagnostic Chart

Morning Session Practice Exam

		Reason for Incorrect Answer					
Subject Area	*Total Number of Questions*	*Number Correct*	*Number Incorrect*	*Lack of Knowledge*	*Misread Problem*	*Careless or Mathematical Error*	*Wrong Guess*
Mathematics (1–24)	24						
Electric Circuits (25–36)	12						
Statics (37–48)	12						
Chemistry (49–60)	12						
Thermodynamics (61–70)	10						
Dynamics (71–80)	10						
Fluid Mechanics (81–88)	8						
Materials Science (89–96)	8						
Mechanics of Materials (97–104)	8						
Computers (105–110)	6						
Ethics (111–115)	5						
Economics (116–120)	5						
Total	120						

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Answers Explained

1. B Treat y as a constant to obtain $\frac{\partial z}{\partial z} = 4xy - 2$.

2. D $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$= \frac{10}{20} + \frac{4}{20} - \frac{2}{20} = \frac{12}{20}$ or .6.

3. A Expand about the third row to obtain

$$2\begin{vmatrix} -1 & -2 \\ 6 & -4 \end{vmatrix} + 1\begin{vmatrix} -1 & 4 \\ 6 & 3 \end{vmatrix} = 2(16) + 1(-27) = 5$$

4. C The discriminant

$$B^2 - 4AC = (-10)^2 - 4(3)(3) = 64 > 0$$

Hence, the graph is a hyperbola.

5. B This defines the standard deviation. The median is the value of the middle term. The mean is the average of the terms. The mode is the most frequently occurring value. These are basic definitions that should be memorized.

6. D The first derivative is equal to the slope of the tangent to the curve, $y = f(x)$. The area under the curve is equal to the integral of the function $f(x)$. The other responses do not mean anything.

7. A The equation is linear in y with integrating factor $e^{\int 3dx} = e^{3x}$. Hence, the general solution is $ye^{3x} = c$, or $y = ce^{-3x}$. For $y(0) = 1$, $c = 1$. Therefore, $y = e^{-3x}$.

8. B The solution of the auxiliary equation $m^2 + 4m + 4 = 0$ is the repeated root $m = -2$. Hence, the general solution is $y = c_1e^{-2x} + c_2xe^{-2x}$ or $y = e^{-2x}(c_1 + c_2x)$.

9. D $v = \frac{ds}{dt} = 30t^2 - 10t^4$. Therefore, at $t = 1$ s, $v = 20$ m/s.

10. D $\mathbf{B} = 4\mathbf{i} + 3\mathbf{j}$ is perpendicular to $\mathbf{A} = 3\mathbf{i} - 4\mathbf{j}$, since $\mathbf{A} \cdot \mathbf{B} = 0$. Since $|\mathbf{B}| = 5$, a unit vector perpendicular to A is $\frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{j}$.

11. C The work done by a variable force in moving an object is equal to the definite integral $\int_a^b f(x)dx$. The instantaneous velocity is equal to the first derivative of $f(x)$, and instantaneous acceleration is equal to the second derivative of $f(x)$.

12. A Area $= \int_0^3 x^2\,dx = \frac{x^3}{3}\Big|_0^3 = 9$ square units.

13. B $f'(x) = 3x^2 - 3$ and $f''(x) = 9x$. Solving $f'(x) = 0$ yields $x = \pm 1$, with $f(1) = -2$, and $f(-1) = 2$. Since $f''(-1) = -9 < 0$, the maximum value of f is 2.

14. C Slope $m = \frac{a - b}{b - a} = -1$.

15. B Since $3^3 = 27$ and $3^4 = 81$, the value of $\log_3 50$ must be a number between 3 and 4.

16. A $|A| = \begin{vmatrix} 3 & 1 \\ -5 & -2 \end{vmatrix} = -1$, $A^T = \begin{bmatrix} 3 & -5 \\ 1 & -2 \end{bmatrix}$ implies $\text{adj}(A) = \begin{bmatrix} -2 & -1 \\ 5 & 3 \end{bmatrix}$. Hence:

$$A^{-1} = \frac{\text{adj}(A)}{|A|} = \begin{bmatrix} 2 & 1 \\ -5 & -3 \end{bmatrix}$$

17. D $r = \sqrt{3^2 + (-3)^2} = 3\sqrt{2}$.
Also, $\theta = \tan^{-1}(-1) = -\pi/4$, which is equivalent to $7\pi/4$. Hence, $3 - 3i = 3\sqrt{2}e^{(7\pi/4)i}$.

18. B The smallest angle is opposite the smallest side. Thus, by the law of cosines, $4^2 = 5^2 + 8^2 - 2(5)(8)\cos\theta$, which implies that $\theta = 24.1°$.

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19. B Complete the squares, and rewrite the equation as $(x-0)^2+(y-2)^2+(z+3)^2=13$. Hence, the center of the sphere is at (0,2,−3).

20. D $a_1 = 0.3$ and $r = 0.3$. Hence,

$$S = \frac{0.3}{1-0.3} = \frac{3}{7}$$

21. A $\int_2^3 \frac{1}{x-1}dx = \ln|x-1|\Big|_2^3 = \ln 2 - \ln 1$
$= \ln 2.$

22. D Differentiate implicitly to obtain $2x + 2y\frac{dy}{dx} = 0$, which implies $\frac{dy}{dx} = -\frac{x}{y}$. Now, at (3,4), slope $m = -3/4$. Hence, the equation of the tangent is $(y-4) = -\frac{3}{4}(x-3)$ or $3x + 4y = 25$.

23. C Volume $= \pi\int_0^2 (y^2)^2 \, dy = \pi\left(\frac{y^5}{5}\right)\Big|_0^2 = \frac{32\pi}{2}$ cubic units.

24. A Rewrite the expression as $\frac{\sin x}{\cos x}\cdot\frac{1}{\sin x}\cdot\frac{\cos x}{1}$, which, if $\sin x \neq \cos x \neq 0$, is equal to 1.

25. D Rearrange eq. 4.4:

$$\rho = \frac{RA}{l}$$
$$= \frac{(5\times10^{-3}\ \Omega)(0.2\ \text{in}^2)(2.54\ \text{cm/in})(1\ \text{m}/100\ \text{cm})}{100\ \text{ft}\times 12\ \text{in/ft}}$$
$$= 2.115\times10^{-8}\ \Omega\cdot\text{m}$$

26. A Solve for the current through the center branch:

$$I_2 = \frac{1\ \text{V}}{2\ \text{k}\Omega} = 0.5\ \text{mA}$$

The voltage across the 2-kΩ and 4-kΩ resistors is:

$$V_2 = (0.5\ \text{mA})(2\ \text{k}\Omega + 4\ \text{k}\Omega) = 3\ \text{V}$$

The voltage across the 1-kΩ and 5-kΩ resistors is also 3 V. Therefore the current through this combination is:

$$I_3 = \frac{3\ \text{V}}{1\ \text{k}\Omega + 5\ \text{k}\Omega} = 0.5\ \text{mA}$$

The total current is

$$I_t = I_2 + I_3 = 0.5\ \text{mA} + 0.5\ \text{mA} = 1.0\ \text{mA}$$

The voltage across R_1 (the first 1-kΩ resistor) is

$$V_1 = (1.0\ \text{mA})(1\ \text{k}\Omega) = 1\ \text{V}$$

Apply Kirchhoff's voltage law around the input loop:

$$V_t = V_1 + V_2 = 1\ \text{V} + 3\ \text{V} = 4\ \text{V}$$

27. A Determine the total equivalent resistance "seen" by the current source. First, find the equivalent resistance of each branch.

Branch 1: 1 Ω
Branch 2: 2 Ω + 3 Ω = 5 Ω
Branch 3: 4 Ω + 5 Ω + 6 Ω = 15 Ω
Apply eq. 4.17(b):

$$R_T = \frac{1}{\frac{1}{1\ \Omega}+\frac{1}{5\ \Omega}+\frac{1}{15\ \Omega}} = 0.789\ \Omega$$

Voltage $V_o = (1\ \text{A})(0.789\ \Omega) = 0.789\ \text{V}$

28. D From Problem 27, the voltage across each branch is known to be 0.789 V. Therefore the current through the center branch is:

$$I = \frac{0.789\ \text{V}}{2\ \Omega + 3\ \Omega} = 0.1578\ \text{A}$$

Apply eq. 4.9:

$$P = I^2 \times R = (0.1578\ \text{A})^2(2\ \Omega) = 0.05\ \text{W}$$

29. B Select the loop currents so that only one (I_A for this problem) flows through the 1-kΩ resistor. Therefore, only loop current I_A will have to be calculated before applying Ohm's law to solve for the voltage.

Loop *A* path includes the 2-V source, resistors 2 kΩ, 1 kΩ, and 7 kΩ.
Loop *B* path includes the 2-V and 4-V sources, resistors 2 kΩ, 2 kΩ, and 3 kΩ.
Loop *C* path includes resistors 7 kΩ and 3 kΩ.

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Assume all loop currents are chosen in a clockwise direction.

Loop A: $10\ \text{k}\Omega + 2\ \text{k}\Omega - 7\ \text{k}\Omega = 2\ \text{V}$

Loop B: $2\ \text{k}\Omega + 7\ \text{k}\Omega + 3\ \text{k}\Omega = -4\ \text{V} + 2\ \text{V}$

Loop C: $-7\ \text{k}\Omega + 3\ \text{k}\Omega + 10\ \text{k}\Omega = 0$

Solve for I_A (note the answer will be in mA):

$$I_A = \frac{\begin{vmatrix} 2 & 2 & -7 \\ -2 & 7 & 3 \\ 0 & 3 & 10 \end{vmatrix}}{\begin{vmatrix} 10 & 2 & -7 \\ 2 & 7 & 3 \\ -7 & 3 & 10 \end{vmatrix}} = \frac{204}{143} = 1.43\ \text{mA}$$

$$V = (1.43\ \text{mA})(1\ \text{k}\Omega) = 1.43\ \text{V}$$

30. C Apply eq. 4.26:

$$Q = C \times V = (0.01 \times 10^{-6}\ \text{F}) \times 50\ \text{V} = 0.5\ \mu\text{C}$$

31. D To determine the maximum voltage the capacitor can charge to and the time constant, first obtain the Thévenin equivalent circuit. The Thévenin equivalent resistance is found by replacing the voltage source with 0 Ω; then the 1-Ω and 2-Ω resistors are in series, and this series combination is in parallel with the series combination of 4 Ω and 2 Ω.

Let $R_a = 1\ \Omega + 2\ \Omega = 3\ \Omega$

and $R_b = 4\ \Omega + 2\ \Omega = 6\ \Omega$. Then:

$$R_{\text{Th}} = \frac{3\ \Omega \times 6\ \Omega}{3\ \Omega + 6\ \Omega} + 3\ \Omega = 5\ \Omega$$

The Thévenin equivalent voltage is found by removing the capacitor and measuring the voltage across R_b (the voltage across the resistor combination of 4 Ω and 2 Ω). This Thévenin voltage may be calculated by using the voltage division law:

$$V_{\text{Th}} = \frac{R_b}{R_a + R_b} \times 18\ \text{V} = \frac{6\ \Omega}{3\ \Omega + 6\ \Omega} \times 18\ \text{V} = 12\ \text{V}$$

The time constant is $\tau = 5\ \Omega \times 2\ \text{F} = 10\ \text{s}$.

Apply eq. 4.33:

$$v_c(t) = 12\ \text{V} \times (1 - e^{-2/10}) = 2.18\ \text{V}$$

32. B For the average value of this waveform, apply eq. 4.36(b):

$$I_{avg} = \frac{(3\ \text{V} \times 2\ \text{ms}) + (-1\ \text{V} \times 2\ \text{ms})}{4\ \text{ms}} = 1.0\ \text{V}$$

For the rms value of this waveform, apply eq. 4.37 or use

$$I_{rms} = \sqrt{\frac{(9\ \text{V}^2 \times 2\ \text{ms}) + (1\ \text{V}^2 \times 2\ \text{ms})}{4\ \text{ms}}} = 2.24\ \text{V}$$

33. A Solve for the impedance of each branch:

$$Z_1 = 3\ \Omega - j\frac{1}{(2\ \text{rad/s} \times \frac{1}{8}\ \text{H})} = (3 - j4)\ \Omega$$

$$Z_2 = 5\ \Omega + j(2\ \text{rad/s} \times 2\ \text{H}) = (5 + j4)\ \Omega$$

$$Z_3 = 2\ \Omega + j(2\ \text{rad/s} \times \frac{1}{2}\ \text{H}) = (2 + j1)\ \Omega$$

Find the total impedance:

$$Z_T = Z_1 + \frac{Z_2 \times Z_3}{Z_2 + Z_3} = (3 - j4)\ \Omega + \frac{(5 + j4)\ \Omega \times (2 + j1)\ \Omega}{(5 + j4)\ \Omega + (2 + j1)\ \Omega} = (4.45 - j3.18)\ \Omega$$

Apply Ohm's law:

$$I_s = \frac{10\angle 30°\ \text{V}}{(4.45 - j3.18)\ \Omega} = 1.84\angle 66°\ \text{A}$$

34. C Rearrange eq. 4.48 to solve for C:

$$C = \frac{1}{(2\pi \times 31.8 \times 10^3\ \text{Hz})^2 \times (10 \times 10^{-3}\ \text{H})} = 2.5 \times 10^{-9}\ \text{F} = 2.5\ \mu\text{F}$$

Rearrange eq. 4.52(b) to solve for R:

$$R = 2\pi \times BW \times L = 2\pi \times (2 \times 10^3\ \text{Hz}) \times (10 \times 10^{-3}\ \text{H}) = 125.7\ \Omega$$

35. B Determine the impedance of each branch.

Let $Z_a = R + j\omega L = (3 + j(1{,}000)(4\ \text{mH})) = (3 + j4)\ \Omega$

and $Z_b = 0 - j\frac{1}{(1{,}000\ \text{rad/s})(200\ \mu\text{F})} = -j5\ \Omega$

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Then:

$$Z_T = \frac{Z_a Z_b}{Z_a + Z_b} = \frac{(3 + j4)\ \Omega \times (-j5)\ \Omega}{(3 + j4)\ \Omega + (-j5)\ \Omega}$$
$$= (7.5 - j2.5)\ \Omega$$

The input voltage phasor is $10.0\angle 0°$ V. Apply Ohm's law; then:

$$I = \frac{10.0\angle 0°\ \text{V}}{(7.5 - j2.5)\ \Omega} = 1.265\angle 18.4°\ \text{A}$$

Complex power = $10.0\angle 0°$ V × $1.265\angle 18.4°$ A = $12.65\angle 18.4°$ VA

36. D Apply eq. 4.64:

$$V_o = -\left(\left(\frac{50\ \text{k}\Omega}{10\ \text{k}\Omega} \times 3\ \text{V}\right) + \left(\frac{50\ \text{k}\Omega}{20\ \text{k}\Omega} \times (-4\ \text{V})\right)\right)$$
$$= -5.0\ \text{V}$$

37. C
$$R_x = \Sigma F_x = 130\left(\frac{12}{13}\right) - 200(\cos 30)$$
$$= -53.2\ \text{N}$$
$$R_y = \Sigma F_y = 130\left(\frac{5}{13}\right) + 200(\sin 30) - 100$$
$$= 50\ \text{N}$$
$$R = \sqrt{R^2_x + R^2_y} = 73\ \text{N}$$

38. A $\mu_s = \tan\theta = \tan(30) = 0.577 = 0.58$

39. B $\Sigma F_y = 0 \qquad B\cos(30) = 100$

$B = 115.5\ \text{N}$

40. D
$$M_A = 260\left(\frac{12}{13}\right)(0.15) + 260\left(\frac{5}{13}\right)(0.05)$$
$$= 41\ \text{N·m}$$

41. B $\Sigma F_x = 0 \quad \frac{4}{5} T_{AC} = T_{BC}\cos 45$

$\Sigma F_y = 0 \quad \frac{3}{5} T_{AC} + T_{BC}\sin 45 - 1{,}000 = 0$

Solve for T_{AC}: $T_{AC} = 714\ \text{N}$

42. A $\Sigma M_B = 0 \quad A(9) - 1{,}500(1.5) + 375(1) = 0$

$A = 208.33\ \text{N}$

43. B Maximum moment occurs at the support.

$$M_{\max} = 400(2) + 200(1) = 1{,}000\ \text{N·m}$$

44. C Cut the truss through members *BD*, *DE*, and *EG*, and use the FBD of the upper section of the truss:

$$\Sigma F_x = 0 \qquad F_{DE} = 6\ \text{kN}$$

45. B $\Sigma F_y = 0 \qquad 3F = 600$

$F = 200\ \text{N}$

46. D $\overline{Y} = \frac{\Sigma \bar{y} A}{A} = \frac{1(20) + 5(12)}{20 + 12} = 2.5\ \text{cm}$

47. A $\Sigma M_A = 0 \qquad (F_{BC}\cos 45)(2) - 100(4) = 0$

$F_{BC} = 283\ \text{N}$

48. C $I_{x'} = I_x + A d^2$

$$A = \frac{I_{x'} - I_x}{d^2} = \frac{7{,}000 - 2{,}000}{10^2} = 50\ \text{cm}^2$$

49. B See Example 1.4. Sulfur may have more than one valence or oxidation state. Set up a simple algebraic equation, letting x equal the unknown valence of sulfur. Sodium has a valence of +1, and oxygen has a valence of −2. The net charge on $Na_2S_2O_3$ is 0, since it's uncharged. Sum up the individual atomic charges and set them equal to 0. Thus:

$$2(+1) + 2(x) + 3(-2) = 0$$
$$x = +2$$

50. A Look for a metal, since metals generally form positively charged ions, called cations. The only metal listed is lithium, Li.

51. C The molar mass of $Ca(NO_3)_2$ is 164 g/mol. Thus, 10 moles is 1,640 gs.

52. C Iron(III) hydroxide is an ionic compound. Hence, use the criss-cross method of formula writing. Iron(III) is Fe^{3+}. Hydroxide is OH^-. Criss-crossing gives $Fe(OH)_3$.

53. A This is a freezing-point-depression problem, so use eq. 1.11. The solute is $CaCl_2$, and the solvent is water. $CaCl_2$ is a strong electrolyte, so $i = 3$.

The K_f of water is given as 1.86. The molality m of the solution is:

$$m = \frac{111\ \text{g}/111\ \text{g/mol}}{1.0\ \text{kg}} = 1.0\ \text{molal}$$

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Substitute these numbers into eq. 1.11, and solve for ΔT. Thus, $\Delta T = 5.5°C$. Hence, the freezing point T_f of the solution is $-5.5°C$.

54. C The correct expression for the equilibrium constant K_c is given by eq. 1.28. Of the choices given, only $K_c = \dfrac{[NH_3]^2}{[N_2][H_2]^3}$ is in this form.

55. C First convert 100 g to moles of $CaCO_3$; the answer is 1 mol of $CaCO_3$. There are 3 moles of oxygen in 1 mole of $CaCO_3$; this gives 3 mol of O. Then multiply by Avogadro's number to convert to the number of atoms.

$$\text{molar mass} = 40 + 12 + 3(16)$$
$$= 100\text{g/mol } CaCO_3$$
$$1 \text{ mol } CaCO_3 \Rightarrow 3 \text{ mol O}$$
$$3 \text{ mol O}\left(\frac{6.022 \times 10^{23} \text{ atoms O}}{1 \text{ mol O}}\right)$$
$$= 18.066 \times 10^{23} \text{ atoms of oxygen}$$

56. C The oxidizing agent is the reactant species that removes electrons from another reactant. Since copper is in the +2 oxidation state in $CuSO_4$ and goes to the zero or uncharged state on the product side, it must gain 2 electrons by removing them from zinc. Thus, $CuSO_4$ is the oxidizing agent.

57. D The reaction is already balanced; assume O_2 is in excess. Use the factor-label method:

$$300 \text{ g } C_2H_6 \frac{(1.0 \text{ mol } C_2H_6)}{(30.0 \text{ g } C_2H_6)} \frac{(4.0 \text{ mol } CO_2)}{(2.0 \text{ mol } C_2H_6)} \frac{(44.0 \text{ g } CO_2)}{(1.0 \text{ mol } CO_2)} = 880 \text{ g}$$

58. B Since CO_2 can be regarded as an ideal gas, convert 880 g to liters using the ideal gas law:

$$PV = nRT$$

where $n = \text{m/MM} = \dfrac{880 \text{ g}}{44.0 \text{ g/mol}} = 20.0 \text{ mol}$.

STP means that $T = 273$ K and $P = 1.0$ atm. Also $R = 0.0821$ L·atm/K·mol. Substitute the numbers, and solve for V: $V = 448$ L.

59. B Use $(M_A)(V_A) = (M_B)(V_B)$, since moles of acid must equal moles of base.

$M_A = 0.25$ M, $V_A =$ unknown

$$M_B = \frac{20 \text{ g NaOH}/40 \text{ g/mol}}{1.0 \text{ L}} = 0.500 \text{ M}$$

$V_B = 1.0$ L

Substitute the numbers and solve for V_A: $V_A = 2.0$ L.

60. A The values of entropy (S), internal energy (U), and enthalpy (H) are each proportional to the mass of the system. For purposes of analysis, the mass specific forms of these properties are often used. The mass-specific form is defined as the value of the property divided by the mass of the system; thus:

$$s = \frac{S}{m}, \qquad u = \frac{U}{m}, \qquad h = \frac{H}{m}$$

The value of pressure, however, does not depend on the mass of the system, and therefore pressure is an intensive property.

61. C A steady-state system is defined as a system in which the state remains constant with time; therefore the state of a system remains unchanged for a steady-state process, and statement (A) is true.

It can be shown that the state of a system that is comprised of a macroscopically homogeneous substance is determined by the values of any two independent properties, so statement (B) is true.

Because during a work interaction, energy is exchanged between the system and its surroundings, the state of the system must change; therefore statement (C) is false.

One statement of the second law of thermodynamics is that there exists one stable equilibrium state for which the value of entropy is a maximum for a given value of energy. If the value of entropy is lower than this maximum value, it is possible that it will increase spontaneously, so statement (D) is true.

62. A If an energy balance and an entropy balance is applied to the system described in statement (A), it is possible to configure the system in such a way that the values of energy and entropy do not change with time. Device (A) is possible.

By applying an entropy balance to the system described in statement (B), it can be shown

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that the value of entropy must be continuously decreasing; therefore, device (B) is not possible.

By applying an energy balance to statement (D), it can be shown that the value of energy must be continuously increasing; therefore, device (D) is not possible.

An equilibrium state is determined by the amount of energy contained in the material, the amount of material, and the external forces placed on the material; therefore, there cannot be two equilibrium states under these conditions. Device (C) is not possible.

63. B When the energy balance equation, eq. 12.14, is applied to this system, it can be simplified as follows:

$$\Delta E = Q - W$$

where Q = the amount of energy transferred to a system during a heat interaction,
W = the amount of energy transferred from a system during a work interaction.

Substitute the given values:

$$\begin{aligned}\Delta E &= 50\text{ kJ} - 30\text{ kJ}\\ &= 20\text{ kJ}\end{aligned}$$

64. B The relationship between a change in specific enthalpy and the temperature of a system is given by eq. 12.38:

$$\Delta h = c_p(\Delta T)$$

Then:

$$\begin{aligned}c_p &= \frac{\Delta h}{\Delta T}\\ &= \frac{120\ \frac{\text{kJ}}{\text{kg}}}{46°\text{C} - 23°\text{C}}\\ &= 5.22\frac{\text{kJ}}{\text{kg}\cdot°\text{C}} \text{ or } 5.22\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\end{aligned}$$

Therefore, based on the property data tables at the end of Chapter 12, the substance is most likely helium, which has a constant-pressure specific heat of 5.19 kJ/kg·K.

65. C The change in internal energy for a closed system is equal to:

$$\Delta U = m\,\Delta u$$

The change in specific internal energy is related to the change in temperature by eq. 12.38:

$$\Delta u = c_v\,\Delta T$$

so that

$$\begin{aligned}\Delta U &= m\,c_v\,\Delta T\\ &= (4\text{ kg})\left(1.74\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(35°\text{C} - 15°\text{C})\\ &= 139.2\text{ kJ}\end{aligned}$$

66. D For an isentropic process in a system composed of an ideal gas, from eq. 12.41:

$$p_1 v_1^{\,k} = p_2 v_2^{\,k}$$

$$\begin{aligned}v_2 &= v_1\left(\frac{p_1}{p_2}\right)^{1/k}\\ &= \left(8.2\ \frac{\text{ft}^3}{\text{lbm}}\right)\left(\frac{50\text{ psia}}{120\text{ psia}}\right)^{\frac{1}{1.40}}\\ &= 4.39\text{ ft}^3/\text{lbm}\end{aligned}$$

67. D From eq. 12.47, for an isometric process of a system composed of an ideal gas:

$$\frac{T_1}{p_1} = \frac{T_2}{p_2}$$

$$\begin{aligned}p_2 &= p_1\frac{T_2}{T_1}\\ &= (600\text{ kPa})\frac{(110 + 273)\text{K}}{(200 + 273)\text{K}}\\ &= 486\text{ kPa}\end{aligned}$$

68. A One relationship that describes an isothermal process of an ideal gas is Boyle's law, eq. 12.49:

$$p_1 v_1 = p_2 v_2$$

$$\begin{aligned}v_2 &= v_1\frac{p_1}{p_2}\\ &= \left(30\frac{\text{m}^3}{\text{kg}}\right)\frac{(200\text{ kPa})}{(800\text{ kPa})}\\ &= 7.5\text{ m}^3/\text{kg}\end{aligned}$$

69. D For a two-phase mixture of water, the average value of the specific internal energy is given by eq. 12.66b:

$$u_{\text{mix}} = u_f + x(u_g - u_f)$$

From the saturated water table at the end of Chapter 12, at 40°C:

$u_f = 167.56$ kJ/kg, $u_g = 2{,}430.1$ kJ/kg

so that:

$$\begin{aligned}u_{\text{mix}} &= 167.56\frac{\text{kJ}}{\text{kg}} + 0.58\\ &\quad\times\left(2{,}430.1\frac{\text{kJ}}{\text{kg}} - 167.56\frac{\text{kJ}}{\text{kg}}\right)\\ &= 1{,}479.8\text{ kJ/kg}\end{aligned}$$

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70. B For a turbine, from eq. 12.77:

$$\dot{w} = \dot{m}(h_i - h_e)$$

where, from eq. 12.33:

$$h_i - h_e = c_p(T_i - T_e)$$

Find the value of the constant-pressure specific heat of air in the property data table at the end of Chapter 12:

$$c_p = 1.00 \text{ kJ/kg·K}$$

Substitute known values:

$$\dot{w} = \dot{m}\, c_p(T_i - T_e)$$
$$= \left(15\frac{\text{kg}}{\text{s}}\right)\left(1.00\frac{\text{kJ}}{\text{kg·K}}\right)(280°\text{C} - 37°\text{C})$$
$$= 3{,}645\frac{\text{kJ}}{\text{s}} = 3{,}645 \text{ kW}$$

71. D Distance traveled is determined by calculating the absolute value of the area of the velocity graph from $0 \le t \le 25.0$ s. It is necessary to determine the time(s) when $V = 0$.

$$V(t) = t^2 - 6t + 9$$

Factoring, $V = 0$ at $t = 3.0$ s.

Distance traveled =

$$\left|\int_0^3 (t^2 - 6t + 9)\, dt\right| + \left|\int_3^{25} (t^2 - 6t + 9)\right|$$
$$= \left(\frac{t^3}{3} - 3t^2 + 9t\right)$$
$$= |9 - 0| + |(3{,}558.33 - 9)|$$
$$= 3{,}558.3 \text{ ft.}$$

72. A Given: $r(t) = 2.0t^3 - 0.5t^2$ and $\theta(t) = t^2 + 8t$.

Then:

$$\omega = \dot{\theta} = \frac{d\theta}{dt} = 2t + 8 = 20.0 \text{ rad/s}$$
$$t = \frac{20 - 8}{2} = 6.0 \text{ s.}$$

Angular momentum is

$$H = mr^2\dot{\theta} = (2.0 \text{ kg})[2(6)^3 - 0.5(6)^2]^2(20.0 \text{ rad/s})$$
$$= 6.85 \times 10^6 \text{ kg·m}^2\text{/s.}$$

73. A For cord tension when $\theta = 30°$, use the tangential and normal coordinates system.

FBD of ball involves only tension and weight forces (neglecting air friction).

$$\Sigma F_n = \frac{mV^2}{R} = \frac{(15 \text{ kg})(3.0 \text{ m/s})^2}{4.0 \text{ m}} = 33.75 \text{ N}$$

From the FBD of the ball:

$$\Sigma F_n = T - \text{W} \cos 30°$$
$$= T - (15 \text{ kg})(9.81 \text{ m/s}^2)(0.866) = 33.75$$

Then:

$$T = 33.75 + 127.4 = 161.2 \text{ N}$$

74. D Draw the FBD of the block, using inclined and normal coordinates. External forces are normal reaction N_R, weight, and kinetic friction F_{FK}.

Block's acceleration is along inclined direction. Solve:

$$a_1 = \frac{\Sigma F_1}{m} = \frac{\text{W} \sin 40° - F_{FK}}{6.0 \text{ lbm}}$$
$$= \frac{6.0 \sin 40° - 0.25\, N_R}{6.0 \text{ lbm}}$$

From equilibrium in normal direction,

$$N_R = \text{W} \cos 40° = (6.0)(0.766) = 4.6 \text{ lbf}$$

$$a_1 = \frac{3.86 - 0.25(4.6 \text{ lb})}{6.0 \text{ lbm}}$$
$$= \frac{(2.71 \text{ lb})(32.2)}{6.0 \text{ lbm}} = 14.5 \text{ ft/sec}^2$$

75. C Apparent weight is reaction N_R. Draw the FBD of the car, and identify weight and normal and friction forces.

Normal direction is toward center of curvature.

$$\Sigma F_n = \frac{mV^2}{R} = \frac{(1{,}455 \text{ kg})(18.3 \text{ m/s})^2}{61.0 \text{ m}}$$
$$= 7{,}988.0 \text{ N}$$
$$= W_c - N_R = 7{,}988.0 \text{ N}$$
$$N_R = (1{,}455 \text{ kg})(9.81 \text{ m/s}^2) - 7{,}988$$
$$= 6{,}285 \text{ N}$$

76. C Employ the conservation of linear momentum principle for impact.

Let $V_1 = 20.0$ ft/sec and $V_{II} = -5.0$ ft/sec.

Since $m_1 = 3.0$ lbm, postcollision velocity is:

$$V_1' = \frac{mV_1'}{m} = \frac{-34.5 \text{ lb-ft/sec}}{3.0 \text{ lb}} = -11.5 \text{ ft/sec}$$

For coefficient of restitution e:

$$e(V_1 - V_{II}) = V_{II}' - V_1'$$

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Also:

$$m_I V_I + m_{II} V_{II} = m_I V_I' + m_{II} V_{II}' \rightarrow V_{II}'$$

$$= \frac{(3.0)(20.0) + (7.0)(-5.0) - (-34.5)}{7.0}$$

$$= \frac{59.5}{7.0} = 8.5 \text{ ft/sec}$$

Substitute known values and solve for e:

$$e = \frac{8.5 - (-11.5)}{20.0 - (-5.0)} = 0.80$$

77. A The equation for work of a force is $W_{1\rightarrow 2} = \int F \cdot dr$. For a constant-friction force, $W_{1\rightarrow 2} = (F_F)(20.0 \text{ m})$ since the friction force and displacement are in the same direction. $F_F = 0.8N_R$, where N_R is the normal reaction between the tires and pavement, given by $N_R = W \cos 3°$ (from equilibrium in the direction perpendicular to incline). Then

$$N_R = (1{,}300)(9.81)\cos 3° = 12{,}735.5 \text{ N}$$

Energy dissipated is friction work, so:

$$W_{1\rightarrow 2} = (0.8 \times 12{,}735.5\text{N})(20.0 \text{ m})$$
$$= 203{,}768.4 \text{ N·m}$$
$$W_{1\rightarrow 2} \approx 203{,}760 \text{ J}$$

78. D Period of vibration τ is independent of the initial displacement.

$$\tau = \frac{1}{f_n} \quad \text{and} \quad f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Then:

$$\tau = 2\pi\sqrt{\frac{m}{k}}$$

where $k = 0.260$ kN/m $= 260.0$ N/m. Substitute known values:

$$\tau = 2\pi\sqrt{\frac{1.8 \text{ kg}}{260.0 \text{ N/m}}} = 2\pi\sqrt{0.00688\text{s}^2}$$
$$= 6.28(0.083\text{s}) = 0.523 \text{ s}$$

79. C The speed of point P relative to the center C of the wheel is simply the tangential speed.

$$V_{p/c} = r\omega = (0.5\text{m})(120 \text{ rpm})$$
$$= (0.5 \text{ m})(120)\left(\frac{2\pi}{60} \text{ rad/s}\right)$$
$$= 6.28 \text{ m/s} \approx 6.3 \text{ m/s}$$

80. C First determine the speed of the block at point B, utilizing the conservation of energy principle for a frictionless process from A to B:

$$KE_A + PE_A = KE_B + PE_B$$

Let $PE_B = 0$(datum), and $PE_A = mgh_A = (5.0 \text{ lbm})(32.2 \text{ ft/sec}^2)(3.0 \text{ ft}) = 15.0$ ft-lbf. Since $V_A = 0$, then $KE_A = 0$, and the conservation of energy equation becomes

$$0 + 15.0 \text{ ft-lbf} = \frac{1}{2}mV_B^2 + 0$$

Solve:

$$V_B = \sqrt{\frac{2(15.0 \text{ ft-lbf})}{m}}$$
$$= \sqrt{\frac{(30.0 \text{ ft-lb})(32.2)}{5.0 \text{ lbm}}} = 13.9 \text{ ft/sec}$$

Since there is no work on the block between point B and spring C, $V_C = V_B = 13.9$ ft/sec. Use the work and kinetic energy principle from C to the 3.0-in compressed position (point D): $KE_C + W_{\text{net } C\rightarrow D} = KE_D$. The work from C to D is spring work only, so

$$W_{\text{net } C\rightarrow D} = \frac{1}{2}k(X_C^2 - X_D^2)$$

Substitute known values:

$$15.0 \text{ ft-lbf} + \frac{1}{2}(40.0 \text{ lbf/ft})[0 - (0.25 \text{ ft})^2]$$
$$= \frac{1}{2}(5.0 \text{ lbm})V_D^2$$

Solve:

$$V_D = \sqrt{\frac{2(15.0 - 1.25)\text{ft-lbf}(32.2)}{5.0 \text{ lbm}}} = 13.3 \text{ ft/sec}$$

81. A The flow rate of fluid through an orifice flow meter is given by the equation

$$Q = CA\sqrt{2g\left[\left(\frac{p_1}{\gamma} + z_1\right) - \left(\frac{p_2}{\gamma} + z_2\right)\right]}$$

Because z_1 is equal to z_2, this equation can be simplified to:

$$Q = CA\sqrt{2g\left(\frac{p_1 - p_2}{\gamma}\right)}$$

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Substitute known values:

$$Q = (0.61)(0.50\ \text{m}^2) \times \sqrt{2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\left(\frac{1{,}000\ \text{Pa} - 450\ \text{Pa}}{9{,}810\ \frac{\text{N}}{\text{m}^3}}\right)\left(\frac{\frac{\text{N}}{\text{m}^2}}{\text{Pa}}\right)}$$

$$= 0.32\ \text{m}^3/\text{s}$$

82. A Because the density of air has a much smaller value than that of the manometer fluid, pressure differences within the air sections are negligible compared to pressure differences within the manometer fluid. Therefore, p_A is very nearly the pressure at the top of the manometer fluid exposed to vessel A, and p_B is very nearly the pressure at the surface of the manometer fluid exposed to vessel B. The two pressures can be related as follows:

$$p_B - p_A = \rho_m g h_m$$

This equation is rearranged as:

$$p_B = p_A + \rho_m g h_m$$

$$= 600\ \text{Pa} + \left(850\frac{\text{kg}}{\text{m}^3}\right)\left(9.81\frac{\text{m}}{\text{s}^2}\right)(0.20\ \text{m}) \times \left(\frac{\text{N}}{\frac{\text{kg}\cdot\text{m}}{\text{s}^2}}\right)\left(\frac{\text{Pa}}{\frac{\text{N}}{\text{m}^2}}\right)$$

$$= 2{,}268\ \text{Pa}$$

83. This problem is solved using the Bernoulli equation:

$$\frac{p_1}{\gamma} + \frac{\bar{V}_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{\bar{V}_2^2}{2g} + z_2$$

In this case, because the diameter of the tank is assumed to be much larger than the diameter of the siphon tube, the average velocity at section 1 is nearly 0. Also, the pressures of the fluid at section 1 and section 2 are equal to the atmospheric pressure. Using these facts, simplify the Bernoulli equation and solve for $\bar{V}_2$:

$$\frac{\bar{V}_2^2 - \overbrace{\bar{V}_1^2}^{=0}}{2g} = \overbrace{\frac{p_1 - p_2}{\gamma}}^{=0} + (z_1 - z_2)$$

$$\bar{V}_2 = \sqrt{2g(z_1 - z_2)}$$

$$= \sqrt{2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(3\text{m})}$$

$$= 7.67\ \text{m/s}$$

The volumetric flow rate is determined from the average velocity using $Q = \bar{V}A$, where the area A is given by the equation

$$A = \frac{\pi D^2}{4}$$

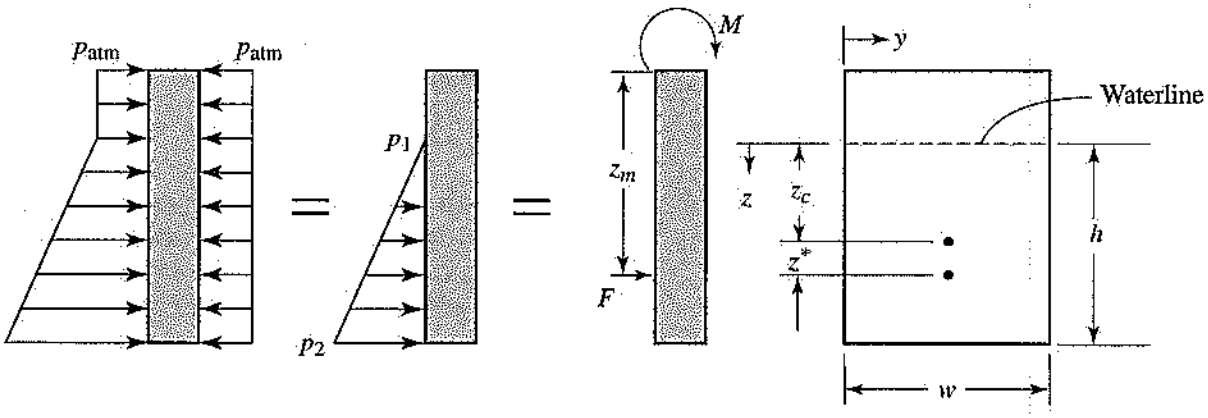
and where D is the diameter of the siphon tube.

$$Q = \bar{V}\left(\frac{\pi D^2}{4}\right)$$

$$= \left(7.67\frac{\text{m}}{\text{s}}\right)\left[\frac{\pi(0.03\ \text{m})^2}{4}\right]\left(\frac{3{,}600\ \text{s}}{\text{h}}\right)$$

$$= .00542\ \frac{\text{m}^3}{\text{s}}$$

84. C The total force acting on the gate, applied by the water pressure, can be modeled as a point force located at the center of pressure. Using the magnitude of this point force and its distance from the axis of the hinge, the required moment is determined. (Refer to the figure below.)



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To determine the magnitude of the point force, use these equations:

$$F = \frac{p_1 + p_2}{2} A_v$$

$$p_1 = \overbrace{\rho g z_1}^{=0}$$

$$= 0 \frac{\text{lbf}}{\text{ft}^2}$$

$$p_2 = \rho g z_2$$

$$= \left(62.4 \frac{\text{lbm}}{\text{ft}^3}\right)\left(32.2 \frac{\text{ft}}{\text{sec}^2}\right)(4 \text{ ft})\left(\frac{\text{lbf}}{32.2 \frac{\text{lbm-ft}}{\text{sec}^2}}\right)$$

$$= 249.6 \frac{\text{lbf}}{\text{ft}^2}$$

$$A_v = (3 \text{ ft})(4 \text{ ft})$$

$$= 12 \text{ ft}^2$$

$$F = \frac{0 \frac{\text{lbf}}{\text{ft}^2} + 249.6 \frac{\text{lbf}}{\text{ft}^2}}{2}(12 \text{ ft}^2)$$

$$= 1{,}497.6 \text{ lbf}$$

To determine the center of pressure, use these equations:

$$z^* = \frac{\rho g I_{y_c} \overbrace{\sin(90°)}^{=1}}{F}$$

$$I_{y_c} = \frac{wh^3}{12}$$

$$= \frac{(3 \text{ ft})(4 \text{ ft})^3}{12}$$

$$= 16 \text{ ft}^4$$

$$z^* = \frac{\left(62.4 \frac{\text{lbm}}{\text{ft}^3}\right)\left(32.2 \frac{\text{ft}}{\text{sec}^2}\right)(16 \text{ ft}^4)}{1497.6 \text{ lbf}}\left(\frac{\text{lbf}}{32.2 \frac{\text{lbm-ft}}{\text{sec}^2}}\right)$$

$$= 0.667 \text{ ft}$$

To determine the length of the moment arm, use this equation:

$$z_m = 1 \text{ ft} + z_c + z^*$$

$$= 1 \text{ ft} + 2 \text{ ft} + 0.667 \text{ ft}$$

$$= 3.667 \text{ ft}$$

And the total moment is:

$$M = F z_m$$

$$= (1{,}497.6 \text{ lbf})(3.667 \text{ ft})$$

$$= 5{,}496 \text{ lbf-ft}$$

85. C Solve this problem using the Bernoulli equation, modified to include the frictional head loss:

$$\frac{p_1}{\gamma} + \frac{\bar{V}_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{\bar{V}_2^2}{2g} + z_2 + h_f$$

In this case, the average velocity at the outlet is equal to the average velocity at the inlet, so the pressure loss is given by the equation

$$p_1 - p_2 = \gamma(z_2 - z_1 + h_f)$$

The frictional head loss is expressed as

$$h_f = f\left(\frac{L}{D}\right)\left(\frac{\bar{V}^2}{2g}\right)$$

Also:

$$V = \frac{\dot{m}}{\rho A}$$

where

$$A = \frac{\pi(0.20 \text{ m})^2}{4}$$

$$= 0.157 \text{ m}^2$$

Then:

$$\bar{V} = \frac{\left(1000 \frac{\text{kg}}{\text{s}}\right)}{\left(1{,}000 \frac{\text{kg}}{\text{m}^3}\right)(0.157 \text{ m}^2)}$$

$$= 6.37 \text{ m/s}$$

and

$$h_f = (0.027)\left(\frac{9 \text{ m}}{0.20 \text{ m}}\right)\left(\frac{\left(6.37 \frac{\text{m}}{\text{s}}\right)^2}{2\left(9.81 \frac{\text{m}}{\text{s}^2}\right)}\right)$$

$$= 2.51 \text{ m}$$

Finally:

$$p_1 - p_2 = \left(9{,}810 \frac{\text{N}}{\text{m}^3}\right)(-2.3 \text{ m} + 2.51 \text{ m})\left(\frac{\text{Pa}}{\frac{\text{N}}{\text{m}^2}}\right)$$

$$= 2{,}060 \text{ Pa}$$

86. B The Reynolds number for flow in a pipe is given by the equation:

$$\text{Re} = \frac{\rho \bar{V} D}{\mu}$$

At first glance, the numerator of this expression looks like mass flow rate per unit area, so answer (A) seems reasonable. However, if the Reynolds number were purely a measure of the

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rate of fluid flow, there would be no need of viscosity in the relationship, so it can be inferred that the meaning may be somewhat broader. Examine the other choices for a better fit.

The pressure of the fluid does not appear in the relationship for the Reynolds number, nor is it easy to derive a pressure-force term from the parameters used. Answer (C) seems unlikely.

An inertial force term would be proportional to

$$F_i \approx \rho \bar{V}^2$$

And a viscous force would be proportional to

$$F_v \approx \mu \frac{V}{L}$$

where L is a characteristic length for the flow. The ratio of these two forces is as follows:

$$\frac{F_i}{F_v} \approx \frac{\rho \bar{V}^2}{\mu \frac{\bar{V}}{L}} = \frac{\rho \bar{V} L}{\mu} = \mathrm{Re}_L$$

Therefore, the answer is (B).

Choice (D) is incorrect because the Reynolds number is always an important parameter when analyzing a fluid-flow situation. It gives an indication of whether the flow is likely to be laminar or turbulent.

87. A Since the head loss through each pipe must be equal:

$$f_a \frac{L_a}{D_a} \frac{\bar{V}_a^2}{2g} = f_b \frac{L_b}{D_b} \frac{\bar{V}_b^2}{2g}$$

With $f_a = f_b$:

$$\frac{V_a}{V_b} = \sqrt{\frac{D_a}{D_b} \frac{L_b}{L_a}} = \sqrt{\frac{4\text{ in}}{2\text{ in}} \frac{4\text{ ft}}{8\text{ ft}}} = 1$$

$$(\bar{V}_a = \bar{V}_b)$$

Use this result in the continuity equation:

$$\begin{aligned} \bar{V}A &= \bar{V}_a A_a + \bar{V}_b A_b \\ &= \bar{V}_a (A_a + A_b) \\ &= \bar{V}_a \left(\frac{\pi D_a^2}{4} + \frac{\pi D_b^2}{4} \right) \\ &= \frac{\pi \bar{V}_a}{4} (D_a^2 + D_b^2) \end{aligned}$$

Then:

$$\begin{aligned} \bar{V}_a &= (\bar{V}A) \frac{4}{\pi (D_a^2 + D_a^2)} \\ &= \left(260 \frac{\text{ft}^3}{\text{min}} \right) \left[\frac{4}{\pi [(4\text{ in})^2 + (2\text{ in})^2]} \right] \\ &\quad \left(\frac{144\text{ in}^2}{\text{ft}^2} \right) \left(\frac{\text{min}}{60\text{ sec}} \right) \\ &= 39.7\ \text{ft}^3/\text{sec} \end{aligned}$$

88. D Use the impulse-momentum principle to solve this problem:

$$\vec{F} = \dot{m}\vec{V}_2 - \dot{m}\vec{V}_1$$

In this case, only the horizontal, or x-direction, component is important. The x-direction equation for the impulse momentum principle is as follows:

$$|\vec{F}_x| = \dot{m}|\vec{V}_2| \cos\alpha - \dot{m}|\vec{V}_1|$$

Also, since $A_1 = A_2$, $|\vec{V}_1| = |\vec{V}_2| = 20$ m/s, so

$$\begin{aligned} |\vec{F}_x| &= \dot{m}|\vec{V}_1|(\cos\alpha - 1) \\ &= \left(50 \frac{\text{kg}}{\text{s}} \right) \left(20 \frac{\text{m}}{\text{s}} \right) (\cos 45 - 1) \left(\frac{\text{N}}{\frac{\text{kg·m}}{\text{s}^2}} \right) \\ &= -293\text{ N} \end{aligned}$$

89. A Use the standard Emf Series in Table 8.4 as an approximation of the behavior of dissimilar metals in salt water. Iron is anodic with respect to copper. Therefore, the anodic oxidation half-cell reaction for iron will occur.

90. A Although the solute and solvent may be in different phases (as, e.g., when the solute is table salt, a solid, and the solvent is water, a liquid), the solution has only a single phase (in this example, liquid).

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91. A In choices (B) and (C), the 1p level does not exist. In choice (D), the 3d level is a higher energy level than the 4s level.

92. D Engineering stress, σ, is defined as load per area. Here, the load is 500 N and the area is

$$\pi r^2 = 7.85\ \text{mm}^2 = 7.85 \times 10^{-6}\ \text{m}^2$$

Then the stress is

$$\sigma = \frac{500\ \text{N}}{7.85 \times 10^{-6}} = 6.36 \times 10^6 \frac{\text{N}}{\text{m}^2} = 6.36\ \text{MN/m}^2$$

93. B For a plain carbon steel containing 0.5 wt. %C, the lever law for amount of pearlite should be drawn in the austenite plus ferrite region, just above the eutectoid temperature. The amount of austenite at this temperature is the amount of pearlite when the austenite is cooled to room temperature:

$$f_\gamma = \frac{0.5 - 0.02}{0.8 - 0.02} \cong \frac{5}{8} \cong 63\%$$

94. B Refer to the phase diagram for steel-carbon.

95. A Reduction in area can be espressed as a fraction:

$$RA = \frac{A_0 - A}{A_0}$$

True strain ϵ, is $\ln\frac{1}{l_0} = \ln\frac{A_0}{A}$. Therefore,

$$\epsilon = \ln\frac{1}{1 - RA} = \ln\frac{1}{0.7} = 0.357$$

96. C Metal is a conductor; free electrons can easily move through it. A semiconductor allows some electrons to move through it. A superconductor has no electrical resistance.

97. B The net moment is equal to the area under the shear diagram. Therefore, area = 48 kN·m, the net moment is 48 kN·m. The area of the trapezoid is equal to the area of the rectangle = 48 kN·m.

98. C For a solid shaft, $\tau = \frac{16\,T}{\pi d^3}$. Then

$$T = \frac{\tau \pi d^3}{16} = 596.4\ \text{N·m}$$

99. A Here, $M_{max} = 100$ N·m. Then:

$$\sigma_{max} = \frac{M_{max}C}{I} = \frac{100(0.03)}{\frac{1}{12}(0.04)(0.06)^3}$$
$$= 4.166\ \text{MPa}$$

100. D Here, $V_{max} = 300$ N. For a circular cross section,

$$\tau = \frac{4\,V}{3A} = \frac{4(300)}{3\frac{\pi}{4}(0.1)^2} = 50.9\ \text{kPa}$$

101. C The critical load is given by the equation

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 EI}{4^2} = 0.617\ \text{EI}$$

Note that $L_e = 2L = 2(2) = 4$ m.

102. B The normal stress at point A is given by the equation

$$\sigma_A = \frac{Mc}{I} - \frac{P}{A}$$
$$= \frac{(400)(0.02)}{\frac{\pi}{64}(0.04)^4} - \frac{10{,}000}{\frac{\pi}{4}(0.04)^2} = 55.7\ \text{MPa}$$

103. A For each 3-cm-diameter bolt:

$$\tau = \frac{V}{A_s} = \frac{300}{\frac{\pi}{4}(0.03)^2} = 0.424\ \text{MPa}$$

104. D The normal stress in section AB is given by the equation

$$\sigma_{AB} = \frac{P_{AB}}{A_{AB}} = \frac{30{,}000}{\frac{\pi}{4}(0.08)^2} = 5.968\ \text{MPa}$$

105. C

$2^0 = 1$
$2^1 = 2$
$2^2 = 4$
$2^3 = 8$
$2^4 = 16$
$2^5 = 32$

$1 + 2 + 4 + 32 = 39$
or
00100111_2

106. B Use values given for Problem 105. Then:

$32 + 8 + 4 + 2 + 1 = 47$
or
101111_2

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107. A First time: J = 2 + 4
I = 6
I = 6^2 or 36
Second time: J = 6 + 36 = 42
I = 42
I = 42^2 or 1764
Third time: J > 100

108. D

	A	B	C	D
1	23	24	25	26
2	7	**49**	**49*23=1127**	
3	8	**64**	**64*24=1536**	
4	9	**81**	**81*25=2025**	
5	10	**100**	**100*26=2600**	

109. A Refer to the solution for problem 108 and look at Column C.

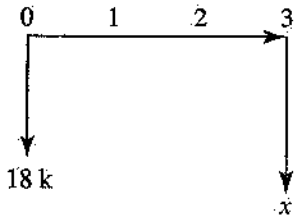
110. C A flowchart consists of graphic symbols to show how information would flow through a program running on a computer. Choices (A), (B), and (D) indicate that written instructions are involved. Instructions are expressed as symbols, making choice (C) the correct answer.

111. C Effective annual rate of interest
= (1 + rate per month)^n − 1
= $(1 + 0.25)^{12} - 1$
= 14.55 − 1 = 13.55
= 1,355%

112. A Use the following equation:

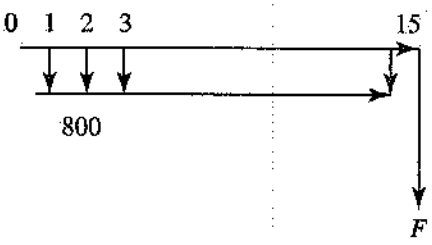
$$4.00x + 4{,}000 = 1.00x + 40{,}000$$
$$3x = 36{,}000$$
$$x = 12{,}000$$

113. C



x = 18,000(F/P, 12, 3)
= 18,000(1.4049) = $25,288

114. B



F = 800(F/A, 10, 15)
= 800(31.7725)
= $25,418

115. C

Year	Opening Balance	Depreciation Write-off %	Depreciation	Balance at End of Year
1	25,000	0.20	5,000	20,000
2	20,000	0.32	6,400	13,600
3	13,600	0.192	2,611	10,989
4	10,989	0.115	**1,264**	9,725

116. B Choices (A) and (C) are professional societies representing PEs and specific engineering disciplines including PEs. They do not have power to settle disputes of any nature. Ethics violations are civil suits and could be settled in court; however, the Board of Registration in each state is empowered to handle ethics violations. Choice (B) is the best response.

117. C Insolvency of the bond holder would not release a party. There is no mention of a time constraint; therefore, (A) and (B) are incorrect. Ice recurs based on temperature, not an Act of God, and should be a design consideration. Therefore, (C) is the correct answer.

118. D Choices (B) and (C) are unethical at this point in time. Formal resignation is required but again not now. A discussion with her supervisor is the first step in the process. Choice (D) is the correct response.

119. D The hierarchy of ethical responsibility is stated in the Code of Ethics. Choice (D) is the correct response.

120. A Knowledge transfer does not include replicating a previous design. A contact paid for the original design and ownership resides with the purchaser. The absence of a nondisclosure statement does not make the assignment legitimate. Choices (B), (C), and (D) are not acceptable. Choice (A) is the correct response.

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ANSWER SHEET: Afternoon Session

1. Ⓐ Ⓑ Ⓒ Ⓓ	13. Ⓐ Ⓑ Ⓒ Ⓓ	25. Ⓐ Ⓑ Ⓒ Ⓓ	37. Ⓐ Ⓑ Ⓒ Ⓓ	49. Ⓐ Ⓑ Ⓒ Ⓓ
2. Ⓐ Ⓑ Ⓒ Ⓓ	14. Ⓐ Ⓑ Ⓒ Ⓓ	26. Ⓐ Ⓑ Ⓒ Ⓓ	38. Ⓐ Ⓑ Ⓒ Ⓓ	50. Ⓐ Ⓑ Ⓒ Ⓓ
3. Ⓐ Ⓑ Ⓒ Ⓓ	15. Ⓐ Ⓑ Ⓒ Ⓓ	27. Ⓐ Ⓑ Ⓒ Ⓓ	39. Ⓐ Ⓑ Ⓒ Ⓓ	51. Ⓐ Ⓑ Ⓒ Ⓓ
4. Ⓐ Ⓑ Ⓒ Ⓓ	16. Ⓐ Ⓑ Ⓒ Ⓓ	28. Ⓐ Ⓑ Ⓒ Ⓓ	40. Ⓐ Ⓑ Ⓒ Ⓓ	52. Ⓐ Ⓑ Ⓒ Ⓓ
5. Ⓐ Ⓑ Ⓒ Ⓓ	17. Ⓐ Ⓑ Ⓒ Ⓓ	29. Ⓐ Ⓑ Ⓒ Ⓓ	41. Ⓐ Ⓑ Ⓒ Ⓓ	53. Ⓐ Ⓑ Ⓒ Ⓓ
6. Ⓐ Ⓑ Ⓒ Ⓓ	18. Ⓐ Ⓑ Ⓒ Ⓓ	30. Ⓐ Ⓑ Ⓒ Ⓓ	42. Ⓐ Ⓑ Ⓒ Ⓓ	54. Ⓐ Ⓑ Ⓒ Ⓓ
7. Ⓐ Ⓑ Ⓒ Ⓓ	19. Ⓐ Ⓑ Ⓒ Ⓓ	31. Ⓐ Ⓑ Ⓒ Ⓓ	43. Ⓐ Ⓑ Ⓒ Ⓓ	55. Ⓐ Ⓑ Ⓒ Ⓓ
8. Ⓐ Ⓑ Ⓒ Ⓓ	20. Ⓐ Ⓑ Ⓒ Ⓓ	32. Ⓐ Ⓑ Ⓒ Ⓓ	44. Ⓐ Ⓑ Ⓒ Ⓓ	56. Ⓐ Ⓑ Ⓒ Ⓓ
9. Ⓐ Ⓑ Ⓒ Ⓓ	21. Ⓐ Ⓑ Ⓒ Ⓓ	33. Ⓐ Ⓑ Ⓒ Ⓓ	45. Ⓐ Ⓑ Ⓒ Ⓓ	57. Ⓐ Ⓑ Ⓒ Ⓓ
10. Ⓐ Ⓑ Ⓒ Ⓓ	22. Ⓐ Ⓑ Ⓒ Ⓓ	34. Ⓐ Ⓑ Ⓒ Ⓓ	46. Ⓐ Ⓑ Ⓒ Ⓓ	58. Ⓐ Ⓑ Ⓒ Ⓓ
11. Ⓐ Ⓑ Ⓒ Ⓓ	23. Ⓐ Ⓑ Ⓒ Ⓓ	35. Ⓐ Ⓑ Ⓒ Ⓓ	47. Ⓐ Ⓑ Ⓒ Ⓓ	59. Ⓐ Ⓑ Ⓒ Ⓓ
12. Ⓐ Ⓑ Ⓒ Ⓓ	24. Ⓐ Ⓑ Ⓒ Ⓓ	36. Ⓐ Ⓑ Ⓒ Ⓓ	48. Ⓐ Ⓑ Ⓒ Ⓓ	60. Ⓐ Ⓑ Ⓒ Ⓓ

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Practice Examination

Afternoon Session

60 Questions

Directions: For each incomplete statement or question, select the answer that best completes the sentence or answers the question. Then fill in the corresponding space on the answer sheet.

Questions 1–3

A weight attached to a spring moves up and down, so that the equation of motion is $\frac{d^2x}{dt^2} + 64x = 0$, where x is the displacement in feet of the weight from its equilibrium position at any time t. Suppose that at $t = 0$ second the displacement is 3 inches $[x(0) = \frac{1}{4}]$ and the velocity is 1 foot per second $[x'(0) = 1]$.

1. The displacement x of the weight at any time t is given by:
   (A) $x = -\sin 8t$
   (B) $x = \frac{1}{4}\cos 8t + \frac{1}{8}\sin 8t$
   (C) $x = 2\cos 8t$
   (D) $x = \cos 8t - \sin 8t$

2. What is the amplitude of motion?
   (A) $\frac{1}{4}$ ft
   (B) $\frac{1}{3}$ ft
   (C) $\frac{\sqrt{3}}{2}$ ft
   (D) $\frac{\sqrt{5}}{8}$ ft

3. The period of motion is:
   (A) $\frac{\pi}{8}$ sec
   (B) $\frac{\pi}{3}$ sec
   (C) $\frac{\pi}{2}$ sec
   (D) $\frac{\pi}{4}$ sec

Questions 4–6

A function f is defined by a determinant as follows:

$$f(x) = \begin{vmatrix} 2x^2 & x \\ 8 & 2 \end{vmatrix}$$

4. The x-intercept(s) of the graph of this function is (are):
   (A) 1
   (B) -3
   (C) 0 and 2
   (D) 2 and -2

5. What is the minimum value of this function?
   (A) -4
   (B) -2
   (C) 1
   (D) 0

6. The area bounded by this curve and the x-axis is:
   (A) 1 square unit
   (B) 3 square units
   (C) $\frac{16}{3}$ square units
   (D) $\frac{32}{3}$ square units

Questions 7–9

Vectors **A** and **B** are defined as follows:

$$\mathbf{A} = 2\mathbf{i} - 2\mathbf{j} - \mathbf{k}, \mathbf{B} = \mathbf{i} - \mathbf{k}$$

7. The dot product of vectors **A** and **B** is:
   (A) 12
   (B) 3
   (C) -6
   (D) 0

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8. What is the magnitude of the cross product of vectors **A** and **B**?
   (A) 7
   (B) 12
   (C) 1
   (D) 3

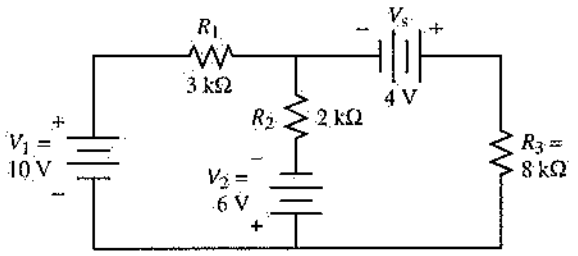
9. The angle between vectors **A** and **B** is:
   (A) 45°
   (B) 38°
   (C) 105°
   (D) 97°

Questions 10–12
An area in Quadrant I is bounded by $y = x^3$, $x = 2$, and the x-axis.

10. What is the area of this region?
    (A) 6 square units
    (B) 8 square units
    (C) 4 square units
    (D) 2 square units

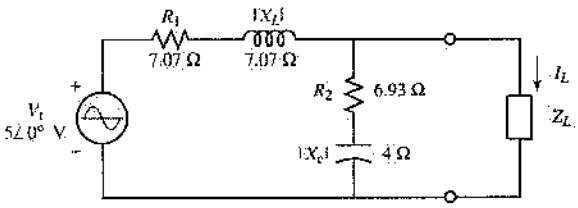
11. The volume generated by revolving this area about the y-axis is:
    (A) 8π cubic units
    (B) 14π cubic units
    (C) $\frac{64\pi}{5}$ cubic units
    (D) $\frac{8\pi}{3}$ cubic units

12. The first moment of this area with respect to the x-axis is:
    (A) 6
    (B) $\frac{64}{7}$
    (C) $\frac{46}{5}$
    (D) $\frac{13}{2}$



13. What is the power dissipated by R_3 in the circuit shown in the diagram?
    (A) 3.42 mW
    (B) 2.18 mW
    (C) 1.83 mW
    (D) 5.76 mW

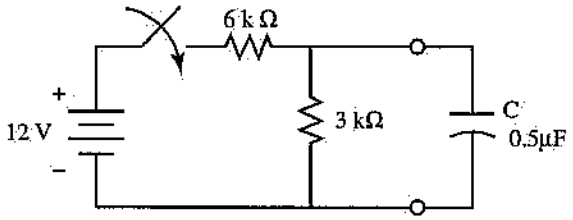
Questions 14 and 15 relate to the figure shown below.



14. Determine the Thévenin equivalent circuit, V_{Th} and Z_{Th}, for the load impedance, Z_L, in the circuit.
    (A) 5.51 ∠32.2° V; 3.67 ∠−17.4° Ω
    (B) 2.79 ∠−42.4° V; 5.58 ∠2.63° Ω
    (C) 1.9 ∠ 63.4° V; 4.12 ∠−31.5° Ω
    (D) 3.47 ∠−17.8° V; 7.83 ∠8.9° Ω

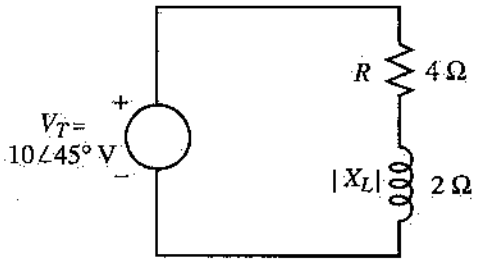
15. What is the load current, I_L, if the load impedance is $(5 - j7)$ Ω?
    (A) 0.223 ∠−9.87° A
    (B) $(1.24 + j0.84)$ A
    (C) 0.517 ∠−21.3° A
    (D) $(3.13 + j1.87)$ A

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16. For the circuit shown in the diagram, how long will the capacitor take to charge to 2 volts after the switch is closed? The initial voltage across the capacitor is 0 volt.
(A) 2.33 ms
(B) 1.86 ms
(C) 0.318 ms
(D) 0.693 ms

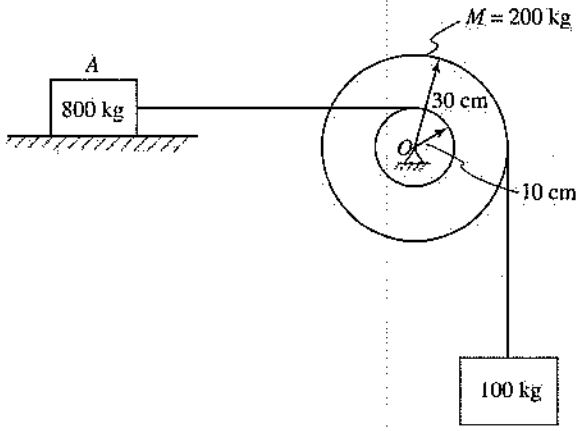
Questions 17 and 18 relate to the figure shown below.



17. The total volt amperes (VA) for the circuit are:
(A) $(15.3 + j12)$ VA
(B) $18.6 \angle -16.4°$ VA
(C) $22.4 \angle -26.6°$ VA
(D) $(19 - j4.3)$ VA

18. If the frequency of the source is 400 hertz, what capacitor value must be connected in series with R and L so that the circuit has unity power factor?
(A) 87.3 μF
(B) 26.6 μF
(C) 142 μF
(D) 199 μF

Questions 19–21 relate to the figure shown below.

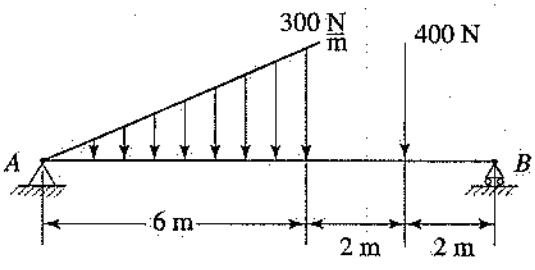


19. The minimum coefficient of static friction between block A and the surface is most nearly:
(A) 0.3
(B) 0.375
(C) 1
(D) 0.5

20. The resultant bearing reaction at O is most nearly:
(A) 2,943 N
(B) 424 N
(C) 141 N
(D) 4,162 N

21. The tension in the cable connected to block A is most nearly:
(A) 2,943 N
(B) 300 N
(C) 100 N
(D) 981 N

Questions 22–24 relate to the figure shown below.



22. The reaction at point B is most nearly:
(A) 350 N
(B) 680 N
(C) 1,040 N
(D) 450 N

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23. The maximum moment in the beam is most nearly:
(A) 900 N·m
(B) 1,029 N·m
(C) 1,507 N·m
(D) 2,058 N·m

24. The maximum shear load in the beam is most nearly:
(A) 620 N
(B) 330 N
(C) 680 N
(D) 1,040 N

Questions 25–29

Steam enters an adiabatic turbine at 1 megapascal and 300°C and expands to atmospheric pressure. The turbine efficiency is 80%.

25. What would the temperature of the water exiting the turbine be if the turbine were ideal?
(A) 60°C
(B) 80°C
(C) 100°C
(D) 120°C

26. What would be the approximate work output of an ideal turbine with the same inlet and exit conditions of water?
(A) 375 kJ/kg
(B) 461 kJ/kg
(C) 789 kJ/kg
(D) 2,632 kJ/kg

27. What is the approximate actual work of this turbine?
(A) 369 kJ/kg
(B) 461 kJ/kg
(C) 576 kJ/kg
(D) 2,100 kJ/kg

28. What is the value of enthalpy at the exit of this turbine?
(A) 419 kJ/kg
(B) 1,859 kJ/kg
(C) 2,257 kJ/kg
(D) 2,682 kJ/kg

29. What is the exit temperature of the water?
(A) 85°C
(B) 103°C
(C) 135°C
(D) 183°C

Questions 30–32

A reagent bottle contains 250 milliliters of a 26.3%-by-mass NaCl solution. Its density is known to be 1.20 grams per milliliter.

$$K_b(H_2O) = 0.512°C \cdot kg/mol$$

$$K_f(H_2O) = 1.86°C \cdot kg/mol$$

30. What is the molal concentration of the solution?
(A) 0.539 m
(B) 3.75 m
(C) 6.10 m
(D) 26.3 m

31. What is the boiling point of this solution?
(A) 106.2°C
(B) 102.6°C
(C) 101.9°C
(D) 98.5°C

32. What is the freezing point of this solution?
(A) −6.25°C
(B) −10.0°C
(C) −20.0°C
(D) −25.5°C

Questions 33–35

The following combustion reaction of propane, C_3H_8, is carried out in a closed vessel:

$$C_3H_8(g) + 5O_2(g) \rightarrow 4H_2O(l) + 3CO_2(g)$$

33. How many grams of CO_2 will be produced from 88.0 grams of propane and 360 grams of oxygen?
(A) 88.0 g
(B) 132 g
(C) 176 g
(D) 264 g

34. The limiting reagent is:
(A) propane
(B) oxygen
(C) water
(D) carbon dioixde

35. How many grams of oxygen will remain at the end of the reaction?
(A) 20.0 g
(B) 40.0 g
(C) 60.0 g
(D) 80.0 g

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Questions 36–38

A mother is pulling her child, seated on a sled, at a constant speed of 2.5 feet per second up a snow-covered incline that is 10° from the horizontal. The mass of the child plus the sled is 50.0 pounds mass, and the kinetic friction coefficient between the sled and snow surfaces is 0.1. The mother pulls with a force that is constantly directed at 20° from the inclined surface.

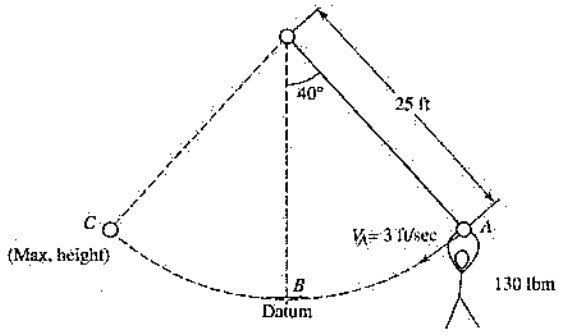
36. The magnitude of the mother's pulling force is necessarily:
(A) 8.9 lbf **(C)** 17.8 lbf
(B) 6.6 lbf **(D)** 14.0 lbf

37. The friction force acting on the sled is nearly:
(A) 13.1 lbf **(C)** 5.3 lbf
(B) 4.5 lbf **(D)** 6.6 lbf

38. The work done by the pulling force for a 120.0-foot displacement of the sled along the incline is approximately
(A) 1,580 ft-lbf **(C)** 660 ft-lbf
(B) 0 **(D)** 1,680 ft-lbf

Questions 39–41

A flying-trapeze acrobat jumps off a platform while holding onto an inextensible rope having a length of 25.0 feet. As indicated in the diagram below, the mass of the acrobat is 130.0 pounds mass, and he swings from the rope at point A with an initial tangential velocity of 3.0 feet per second. The effects of rope friction at the hinge point and of wind resistance are considered negligible.

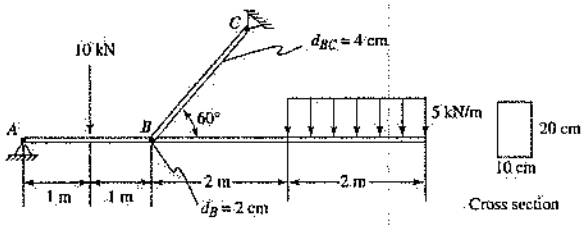


39. The speed of the acrobat at elevation datum point B should be:
(A) 25.8 ft/sec
(B) 61.3 ft/sec
(C) 8.7 ft/sec
(D) 19.6 ft/sec

40. What is the angular velocity of the rope when the acrobat's normal acceleration is 1.0 foot per second squared?
(A) 0.2 rad/sec
(B) 6.7 rad/sec
(C) 2.5 rad/sec
(D) 0.88 rad/sec

41. When the acrobat has swung to the maximum height at point C, the elevation of the free end of the rope above the datum is:
(A) 0.8 ft
(B) 0.15 ft
(C) 6.0 ft
(D) 1.6 ft

Questions 42–45 relate to the figure shown below.



42. The average normal stress in bar BC is most nearly:
(A) 10 MPa
(B) 28.8 MPa
(C) 27.56 MPa
(D) 120 MPa

43. The maximum bending stress in the beam is most nearly:
(A) 180 MPa
(B) 30 MPa
(C) 90 MPa
(D) 15 MPa

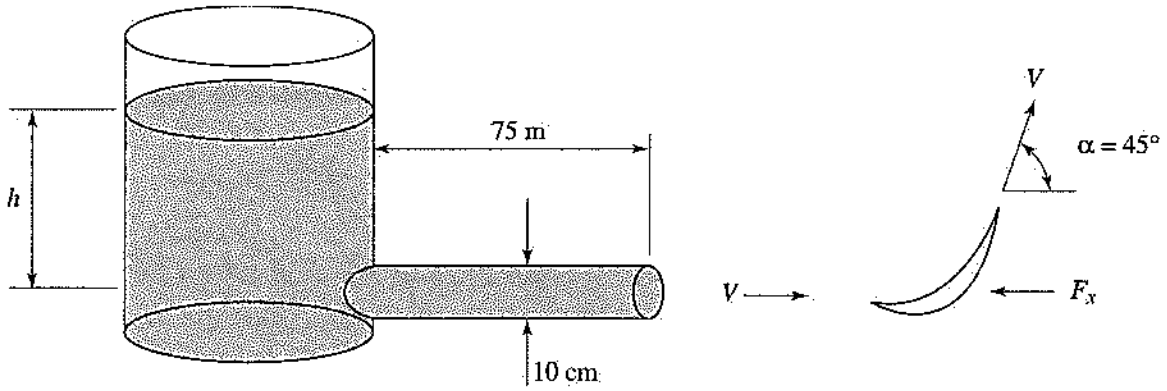
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44. The average shear stress in the pin at B (double shear) is most nearly:
(A) 55.6 MPa
(B) 55.13 MPa
(C) 87.8 MPa
(D) 25 MPa

45. The maximum transverse shear stress in the beam is most nearly:
(A) 3.75 MPa
(B) 4.5 MPa
(C) 1.5 MPa
(D) 3 MPa

Questions 46–49

Water flows from the tank shown in the diagram through a cast-iron pipe that is 75 meters long and has a 10-centimeter inside diameter. The jet of water leaving the pipe is expected to turn a turbine. The turbine operates most efficiently when the horizontal force (F_x) on the blade is at least 100 newtons. (The density of water is 1,000 kg/m^3, and its viscosity is 8×10^{-4} N·s/m^2. The roughness for cast-iron pipe is 0.25 mm.)



46. What should the velocity of the water leaving the pipe be in order to produce a horizontal force of 100 newtons?
(A) 6.59 m/s
(B) 8.00 m/s
(C) 8.50 m/s
(D) 5.44 m/s

47. What is the Reynolds number of the water at this velocity?
(A) 1.00×10^{-6}
(B) 6.80×10^{-5}
(C) 5.30×10^{-5}
(D) 1.06×10^{-5}

48. What is the friction factor for this water velocity and this pipe?
(A) 0.25
(B) 0.026
(C) 0.05
(D) 0.018

49. What is the head loss in the pipe due to friction?
(A) 43.16 m
(B) 4.05 m
(C) 34.36 m
(D) 12.37 m

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50. The crystal structure of iron at room temperature is:
(A) FCC
(B) BCC
(C) HCP
(D) BCT

51. When carbon is dissolved in iron to a concentration of 0.5 weight percent carbon and the alloy is heated to 900°C, what phase is present?
(A) ferrite
(B) austenite
(C) pearlite
(D) martensite

52. What is (are) present in the alloy described in Problem 51 after it cools slowly to 720°C from 900°C?
(A) ferrite
(B) ferrite and pearlite
(C) austenite
(D) martensite

53. The diffusion coefficient of carbon in iron at 720°C is:
(A) 4.4×10^{-13} cm^2/s
(B) 2.4×10^{-7} cm^2/s
(C) 2.7×10^{-9} cm^2/s
(D) 8.5×10^{-11} cm^2/s

54. What is the most common number of bits per byte?
(A) 2
(B) 4
(C) 8
(D) 16

55. What is 17 in straight binary code?
(A) 10000
(B) 10001
(C) 1111
(D) 1001

56. How many times will the second line of the following program be executed?

```
            D = 38
LOOPSTART   D = D-1
            F = INTEGER PART OF (D/2)
            IF F > 12, THEN GO TO
              LOOPSTART, OTHERWISE
              GO TO END
END         PRINT "DONE"
```

(A) 14
(B) 13
(C) 9
(D) 17

57. An unlicensed engineer interviews for a position as Director of Plant Operations at a university. On his resume, he states that he is a member of the National Society of Professional Engineers (NSPE) and includes his membership number. He is, in fact, an associate member of NSPE. Does his claim that he is a member violate the Engineers' code of ethics?
(A) Yes. It implies that he is a licensed engineer.
(B) No. He was implying membership, not licensure.
(C) Yes. A member number could be construed as a PE registration number.
(D) No. He was indicating professional society involvement.

58. A forensic engineer accepts a retainer from an attorney for a plaintiff and receives selected documentation from the attorney. Subsequently the engineer bills for work that includes a review of the case documentation. Still later, because of a disagreement with the attorney, she ceases to perform the work without delivering a report to the attorney or receiving any additional payment for her services. Thereafter, the engineer returns the full retainer fee to the attorney and all of the file documentation provided earlier. Several months later, the engineer is approached by the defense attorneys in the same case and accepts an assignment to function as an expert for the defense in the same legal dispute. Was it ethical for the engineer to accept the assignment?
(A) No. She drew no conclusions during the first review of the material.
(B) Yes. She returned all moneys paid to her.
(C) No. She did not have the consent of all concerned parties.
(D) Yes. Several months passed before she was approached by the defense attorneys.

59. Superior Air Flights is considering the purchase of a helicopter to connect service between its base airport and a new field being built about 25 miles away. The choppers are assumed to be needed for only 6 years until a rapid transit service is completed. Estimates for the two craft under consideration are as follows:

	Whirl 2B	ROT8
First cost	$95,000	$120,000
Annual maintenance	$3,000	$9,000
Salvage value	$12,000	$25,000
Life in years	3	6

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What is the annual cost advantage of selecting ROT8?
(A) $0
(B) $700
(C) $2,100
(D) $4,216

60. It costs Superior Air Flights $1,200 to run a particular flight, empty or full. Additionally, each passenger generates costs of $40. Regular tickets sell for $90. A plane holds 65 people, but each flight carries only about 35 passengers. The Marketing Manager proposes to sell special tickets for $50 to people not normally flying this route. He thinks he can sell 15 additional seats for each flight without cutting into the regular business. If he is correct, how much will the total profit on this flight be?
(A) $200
(B) $500
(C) $700
(D) $1,000

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Answer Key

Afternoon Session

1. B	16. D	31. A	46. A
2. D	17. C	32. C	47. C
3. D	18. D	33. D	48. B
4. C	19. B	34. A	49. A
5. A	20. D	35. B	50. B
6. C	21. A	36. D	51. B
7. B	22. B	37. B	52. B
8. D	23. D	38. A	53. B
9. A	24. C	39. D	54. C
10. C	25. C	40. A	55. B
11. C	26. B	41. C	56. B
12. B	27. A	42. C	57. A
13. C	28. D	43. A	58. C
14. B	29. B	44. B	59. D
15. A	30. C	45. A	60. C

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Diagnostic Chart

Afternoon Session Practice Exam

			Reason for Incorrect Answer				
Subject Area	*Total Number of Questions*	*Number Correct*	*Number Incorrect*	*Lack of Knowledge*	*Misread Problem*	*Careless or Mathematical Error*	*Wrong Guess*
Mathematics (1–12)	12						
Electric Circuits (13–18)	6						
Statics (19–24)	6						
Thermodynamics (25–29)	5						
Chemistry (30–35)	6						
Dynamics (36–41)	6						
Mechanics of Materials (42–45)	4						
Fluid Mechanics (46–49)	4						
Materials Science (50–53)	4						
Computers (54–56)	3						
Ethics (57–58)	2						
Economics (59–60)	2						
Total	60						

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Answers Explained

1. B The solutions of the auxiliary equation $m^2 + 64 = 0$ are $\pm 8i$. Hence:

$$x = c_1 \cos 8t + c_2 \sin 8t \text{ and}$$
$$x' = -8c_1 \sin 8t + 8c_2 \cos 8t$$

The initial condition $x(0) = \frac{1}{4}$ implies $c_1 = \frac{1}{4}$, and the initial condition $x'(0) = 1$ implies $c_2 = \frac{1}{8}$. Thus, the displacement of the weight at any time t is given by

$$x = \frac{1}{4}\cos 8t + \frac{1}{8}\sin 8t$$

2. D The amplitude of motion is $\sqrt{c_1^2 + c_2^2}$

$$= \sqrt{\left(\frac{1}{4}\right)^2 + \left(\frac{1}{8}\right)^2} = \frac{\sqrt{5}}{8} \text{ ft.}$$

3. D The period of motion is $\frac{2\pi}{\omega} = \frac{2\pi}{8} = \frac{\pi}{4}$ secs.

4. C The function f may be rewritten as $f(x) = 4x^2 - 8x$. The x-intercepts of the graph of f are the real solutions of the quadratic equation $4x^2 - 8x = 0$. Hence, the x-intercepts are 0 and 2.

5. A For $f(x) = 4x^2 - 8x$:

$$f'(x) = 8x - 8 \quad \text{and} \quad f''(x) = 8$$

Solving yields $f'(x) = 0$, which implies $x = 1$. Since $f''(1) = 8 > 0$, there is a minimum at $x = 1$, and this minimum value is $f(1) = -4$.

6. C Area $= \int_0^2 [0 - (4x^2 - 8x)]\,dx$

$$= -\frac{4x^3}{3} + 4x^2\Big|_0^2 = \frac{16}{3} \text{ square units.}$$

7. B $\mathbf{A}\cdot\mathbf{B} = (2)(1) + (-2)(0) + (-1)(-1) = 3.$

8. D $\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -2 & -1 \\ 1 & 0 & -1 \end{vmatrix} = \mathbf{i}\begin{vmatrix} -2 & -1 \\ 0 & -1 \end{vmatrix}$

$$+\mathbf{j}\begin{vmatrix} 2 & -1 \\ 1 & -1 \end{vmatrix} + \mathbf{k}\begin{vmatrix} 2 & -2 \\ 1 & 0 \end{vmatrix} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}.$$

Hence, $|\mathbf{A} \times \mathbf{B}| = \sqrt{2^2 + (-1)^2 + 2^2} = 3.$

9. A Angle $\theta = \cos^{-1}\left(\frac{\mathbf{A}\cdot\mathbf{B}}{|\mathbf{A}||\mathbf{B}|}\right) = \cos^{-1}\frac{1}{\sqrt{2}} = 45°.$

10. C Area $= \int_0^2 x^3\,dx = \frac{x^4}{4}\Big|_0^2 = 4$ square units.

11. C Take the volume of the outer cylinder, and subtract from it the volume of the hollowed-out region:

$$\text{Volume} = \pi(2)^2(8) - \pi\int_0^8 (\sqrt[3]{y})^2\,dy$$
$$= 32\pi - \frac{3\pi}{5}y^{5/3}\Big|_0^8 = \frac{64\pi}{5} \text{ cubic units.}$$

12. B $M_x = \int_0^8 y(2 - \sqrt[3]{y})\,dy$

$$= \int_0^8 (2y - y^{4/3})dy = y^2 - \frac{3}{7}y^{7/3}\Big|_0^8 = \frac{64}{7}.$$

13. C Apply Kirchhoff's voltage law to both loops:
Loop A includes V_1, V_2, R_1, and R_2.
Loop B includes V_2, V_3, R_2, and R_3.
Choose a clockwise direction for loop currents. Then:

Loop A: $5I_A - 2I_B = 10\text{ V} + 6\text{ V}$
Loop B: $-2I_A + 10I_B = -6\text{ V} + 4\text{ V}$

Solve for I_B:

$$I_B = \frac{\begin{vmatrix} 5 & 16 \\ -2 & -2 \end{vmatrix}}{\begin{vmatrix} 5 & -2 \\ -2 & 10 \end{vmatrix}} = \frac{-10 + 32}{50 - 4} = 0.478 \text{ mA}$$

Apply eq. 4.9:

$$P = I_B^2 \times R_3 = (0.478\text{ mA})^2 \times 8\text{ k}\Omega = 1.83\text{ mW}$$

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14. B The Thévenin equivalent voltage is determined by using the voltage division law:

$$V_{Th} = \frac{Z_2}{Z_1 + Z_2} \times V_T$$
$$= \frac{(6.93 - j4)\ \Omega}{(7.07 + j7.07)\ \Omega + (6.93 - j4)\ \Omega} \times 5\angle 0°\ \text{V} = 2.79\angle -42.4°\ \text{V}$$

The Thévenin equivalent impedance is determined as follows:

$$Z_{Th} = \frac{Z_1 \times Z_2}{Z_1 + Z_2}$$
$$= \frac{(7.07 + j7.07)\ \Omega \times (6.93 - j4)\ \Omega}{(7.07 + j7.07)\ \Omega + (6.93 - j4)\ \Omega}$$
$$= (5.57 + j0.256)\ \Omega = 5.58\angle 2.63°\ \Omega$$

15. A Use the Thévenin equivalent values to obtain the load current:

$$I_L = \frac{V_{Th}}{Z_{Th} + Z_L}$$
$$= \frac{2.79\angle 42.4°\ \text{V}}{(5.57 + j0.256)\ \Omega + (5 - j7)\ \Omega}$$
$$= 0.223\angle -9.87°\ \text{A}$$

16. D The maximum or final voltage the capacitor could charge to is

$$V_f = \frac{3\ \text{k}\Omega}{6\ \text{k}\Omega + 3\ \text{k}\Omega} \times 12\ \text{V} = 4\ \text{V}$$

The resistance "seen" by the capacitor is

$$R = \frac{3\ \text{k}\Omega \times 6\ \text{k}\Omega}{3\ \text{k}\Omega + 6\ \text{k}\Omega} = 2\ \text{k}\Omega$$

The time constant for this circuit is $\tau = 2\ \text{k}\Omega \times 0.5\ \mu\text{F} = 1.0$ ms.

Apply eq. 4.33:

$$2\ \text{V} = 4\ \text{V}(1 - e^{-t/\tau})$$

Substitute for τ and solve for t:

$$t = 0.693\ \text{ms}.$$

17. C Solve for the load current:

$$I_L = \frac{10\angle 45°\ \text{V}}{(4 + j2)\ \Omega} = 2.24\angle 18.4°\ \text{A}$$

The phase angle from voltage to current is 26.6° (45° − 18.4°).

Apply eqs. 4.45 and 4.46:

$$P = 10\ \text{V} \times 2.24\ \text{V} \times \cos(26.6°) = 20\ \text{W}$$
$$Q = 10\ \text{V} \times 2.24\ \text{A} \times \sin(26.6°) = 10\ \text{VAR}$$

For an inductive circuit the power factor is lagging, so applying eq. 4.47:

$$S = (20 - j10)\ \text{VA} = 22.4\angle -26.6°\ \text{VA}$$

18. D $X_L = X_C = \frac{1}{\omega C} = 2\ \Omega$

Solve for C:

$$C = \frac{1}{(2\ \Omega)(2\pi)(400\ \text{Hz})} = 199\ \mu\text{F}$$

19. B Use the FBD of the pulley, and take the moment about its center:

$$T(10) = 981(30) \qquad T = 2{,}943\ \text{N}$$

Use the FBD of the block.

Then:

$$\Sigma F_x = 0 \qquad F = T = 2{,}943\ \text{N}$$
$$\Sigma F_y = 0 \qquad N = 800(9.81) = 7{,}848\ \text{N}$$
$$\mu_s = \frac{F}{N} = \frac{2{,}943}{7{,}848} = 0.375$$

20. D $O = \sqrt{(2{,}943)^2 + (2{,}943)^2} = 4{,}162\ \text{N}$

21. A Refer to the solution to Problem 19: $T = 2{,}943$ N.

22. B $\Sigma M_A = 0 \qquad B(10) - 400(8) - 900(4) = 0$
$B = 680\ \text{N}$

23. D Draw the shear and bending-moment diagrams. The maximum moment will occur about 4.98 m to the left of point A, where shear is equal to 0.

The equation for the shear in the section where the distributed load is acting is

$$V = -25x^2 + 620x$$

and the equation for the bending moment in the same section is

$$M = \frac{-25}{3}x^3 + 620$$

Then:

$$M_{max} = 2{,}058\ \text{N·m}$$

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24. C The maximum shear load can be determined from the shear diagram drawn for Problem 23. But by inspection the maximum shear load is equal to the reaction at B, which is 680 N. Then:

$$V_{max} = 680 \text{ N}$$

25. C From the superheated steam tables at the end of Chapter 12, the value of entropy at the entrance state of the turbine is 7.1229 kJ/kg·K. Since normal atmospheric pressure is 101.35 kPa, the table for superheated steam at 100 kPa will provide values very close to those expected at atmospheric pressure. Since the value of entropy for saturated vapor at this pressure is greater than the value at the turbine inlet, some condensation will occur. Therefore, the temperature of the exiting stream of water will be equal to the saturation temperature of water at atmospheric pressure, or 100°C.

26. B Turbine work is determined using eq. 12.77:

$$\dot{W} = \dot{m}(h_i - h_e) \qquad \text{or} \qquad w = (h_i - h_e)$$

The value of enthalpy in the inlet is taken from the superheated steam table:

$$h_i = 3051.2 \text{ kJ/kg}$$

To find the value of enthalpy in the outlet stream of an ideal turbine, the quality of the exit stream for the ideal turbine must be determined. Use eq. 12.66d:

$$s_{mix} = s_f + x(s_g - s_f) \qquad \text{or} \qquad x = \frac{(s_{mix} - s_f)}{(s_g - s_f)}$$

Find the values of s_f and s_g using the saturated water table:

$$s_f = 1.3069 \text{ kJ/kg·K}, \qquad s_g = 7.3549 \text{ kJ/kg·K}$$

In this case, the value of s_{mix} is equal to the value of entropy in the inlet stream of the turbine:

$$s_{mix} = 7.1229 \text{ kJ/kg·K}$$

Substitute known values:

$$x = \frac{s_{mix} - s_f}{s_g - s_f} = \frac{\left(7.1229 \frac{\text{kJ}}{\text{kg·K}} - 1.3069 \frac{\text{kJ}}{\text{kg·K}}\right)}{\left(7.3549 \frac{\text{kJ}}{\text{kg·K}} - 1.3069 \frac{\text{kJ}}{\text{kg·K}}\right)} = 0.962$$

Using this value for quality, determine the value of enthalpy for the exit stream of the ideal turbine by applying eq. 12.66c, where $h_e = h_{mix}$:

$$h_e = h_f + x(h_g - h_f)$$

Find the values of h_f and h_g using the saturated water table:

$$h_f = 419.04 \text{ kJ/kg}, \qquad h_g = 2{,}676.1 \text{ kJ/kg}$$

Then:

$$h_e = 419.04 \frac{\text{kJ}}{\text{kg}} + 0.962\left(2{,}676.1 \frac{\text{kJ}}{\text{kg}} - 419.04 \frac{\text{kJ}}{\text{kg}}\right) = 2590.3 \text{ kJ/kg}$$

The work produced in an ideal turbine under these conditions is:

$$\dot{w} = (h_I - h_e) = \left(3{,}051.2 \frac{\text{kJ}}{\text{kg}} - 2{,}590.3 \frac{\text{kJ}}{\text{kg}}\right) = 460.9 \text{ kJ/kg}$$

27. A Turbine efficiency as given is defined as the amount of work a turbine produces, divided by the amount of work produced in an ideal turbine operating at the same conditions:

$$\eta_{turb} = \frac{w_{real}}{w_{ideal}}$$

Then:

$$w_{real} = (\eta_{turb})(w_{ideal}) = (0.80)\left(461 \frac{\text{kJ}}{\text{kg}}\right) = 368.8 \text{ kJ/kg}$$

28. D Determine the value of enthalpy for the exit state of the real turbine as follows:

$$w = (h_i - h_e)$$
$$h_e = h_i - w = 3{,}051.2 \frac{\text{kJ}}{\text{kg}} - 368.8 \frac{\text{kJ}}{\text{kg}} = 2{,}682.4 \text{ kJ/kg}$$

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29. B Use the 100 kPa (0.10 MPa) section of the saturated steam table and the value of enthalpy determined for Problem 28 to find the outlet temperature of the turbine.

	Temperature (°C)	*Enthalpy*
Saturated steam table at 100 kPa	100	2,676.2
Exit state of real turbine	t_e	2,682.4
Saturated steam table at 100 kPa	150	2,776.4

Use linear interpolation to generate a more accurate estimate of t_e:

$$\frac{t_e - 100}{150 - 100} = \frac{2{,}682.4 - 2{,}676.2}{2{,}776.4 - 2{,}676.2}$$

$$t_e = (150 - 100)\left(\frac{2{,}682.4 - 2{,}676.2}{2{,}776.4 - 2{,}676.2}\right) + 100$$
$$= 103.1°C$$

30. C Use the factor-label method. Remember to distinguish among solute, solvent, and solution.

$$\frac{(26.3\ \text{g NaCl})}{(100\ \text{g sol'n})}\frac{(100\ \text{g sol'n})}{(73.7\ \text{g } H_2O)}\frac{(1.0\ \text{mol NaCl})}{(58.5\ \text{g NaCl})}\frac{(1{,}000\ \text{g } H_2O)}{(1.0\ \text{kg } H_2O)} = 6.10\ \text{m}$$

31. A Use eq. 1.10, and substitute the molality calculated for Problem 30. The van't Hoff factor $i = 2$ for NaCl (a strong electrolyte), and $K_b(H_2O)$ is given in the introductory data for Problems 30–32.

32. C Use eq. 1.11, and substitute the known molality and i values. $K_f(H_2O)$ is given in the introductory data for Problems 30–32.

33. D Since amounts of *both* reactants are given, the limiting reagent must be determined. First:

$$88.0\ \text{g } C_3H_8 = 2.0\ \text{mol } C_3H_8 \quad \text{and}$$
$$360\ \text{g } O_2 = 11.25\ \text{mol } O_2$$

The ratio of propane to oxygen is 1 mol to 5 mol in the balanced reaction. Since 2 mol C_3H_8 are actually present, 10 mol O_2 are required to completely react with the propane. However, 11.25 mol O_2 are actually present and available for reaction. Hence, O_2 is present in excess while C_3H_8 is in short supply and hence is the limiting reagent.

Use the factor-label method, and start with the limiting reagent, since it determines the maximum product yield:

$$2.0\ \text{mol } C_3H_8\frac{(3\ \text{mol } CO_2)}{(1.0\ \text{mol } C_3H_8)}\frac{(44.0\ \text{g } CO_2)}{(1.0\ \text{mol } CO_2)}$$
$$= 264\ \text{g } CO_2$$

34. A The limiting reagent was evaluated in Problem 33 as C_3H_8 (propane).

35. B Find the amount of O_2 that will remain after the reaction as follows:

$$11.25\ \text{mol (initially present)}$$
$$-\ 10.0\ \text{mol (used up)}$$
$$=\ 1.25\ \text{mol (left over)}$$

Convert moles into grams: 1.25 mol O_2 is 40.0 g O_2.

36. D Use the inclined and normal coordinate system. Consider child plus sled as a combined free body. Draw the FBD of the child/sled system, and identify external forces: normal reaction N_R, weight W, pulling force F_p, and kinetic friction F_{FK}.

Solve F_p using Newton's second law (let the positive inclined direction be along the 10° upward slope).

$\Sigma F_1 = ma_1$ Since the speed along the incline is constant, $a_1 = 0$ and $\Sigma F_1 = 0$.

Also, $F_p \cos 20° - F_{FK} - W_1 = 0$
$0.94F_p - 0.1N_R - W \sin 10° = 0$

Then:

$0.94F_p - 0.1N_R - (50.0\ \text{lbf})(0.174) = 0$ (eq. I)

$\Sigma F_n = 0$

$F_p \sin 20° + N_R - (50.0\ \text{lbf}) \cos 10° = 0$

Then:

$0.342F_p + N_R - 49.24\ \text{lbf} = 0$ (eq. II)

Solve eqs. I and II simulataneously for N_R:

$$N_R = \frac{0.94F_p - 8.68}{0.1} = 9.4F_p - 86.8$$

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Substitute into II:

$$0.342F_p + (9.4F_p - 86.8) - 49.24 = 0$$

$$F_p = \frac{136.04}{9.74} = 13.96 \text{ lbf} \approx 14.0 \text{ lbf}$$

37. B The kinetic friction force acting on the sled is $0.1N_R$. Use the value of F_p obtained for Problem 36: $N_R = 9.4(14.0 \text{ lb}) - 86.8 = 44.8$ lbf.

$$F_{FK} = 0.1(44.8) = 4.48 \text{ lbf}$$

38. A Work done by pulling force F_p for a 120.0-ft displacement:

$$W_{1\to 2} = (F_{pl})(\Delta l) = (14.0 \text{ lb})\cos 20°(120.0 \text{ ft})$$
$$= 1{,}578.7 \text{ ft-lbf} \approx 1{,}580 \text{ ft-lbf}$$

39. D Employ the conservation of energy equation between points A and B. Since the elevation datum is at B, potential energy at B is 0. The equation is:

$$KE_A + PE_A = KE_B + PE_B$$

Solve KE_B, and then obtain velocity V_B.

$$\frac{1}{2}(130.0 \text{ lbm})(3.0 \text{ ft/sec})^2 + mgh_A = \frac{1}{2}mV_B^2 + 0$$
$$h_A = 25.0 \text{ ft} - (25.0 \text{ ft})\cos 40° = 5.85 \text{ ft}$$

Substitute and solve:

$$585.0 \text{ lbm ft}^2/\text{sec}^2 + (130.0 \text{ lbm})(32.2 \text{ ft/sec}^2) \times (5.85 \text{ ft}) = 65.0 \text{ lbm } V_B^2$$
$$V_B^2 = 9.0 \text{ ft}^2/\text{sec}^2 + 376.7 \text{ ft}^2/\text{sec}^2$$
$$V_B = \sqrt{385.7} = 19.6 \text{ ft/sec}$$

40. A Angular velocity is ω. Use the tangential and normal coordinate system. Tangential speed $V = R\omega$.

$$\text{Normal acceleration} = a_N = \frac{V^2}{R}$$
$$= 1.0 \text{ ft/sec}^2 \text{ (given)}.$$

Then:

$$V = \sqrt{(R)(a_N)} = \sqrt{(25.0 \text{ ft})(1.0 \text{ ft/sec}^2)}$$
$$= 5.0 \text{ ft/sec}$$

and

$$\omega = \frac{V}{R} = \frac{5.0 \text{ ft/sec}}{25.0 \text{ ft}} = 0.20 \text{ rad/sec}$$

41. C At maximum height, point C, let $V_C = 0$ and $KE_C = 0$. Use the conservation of energy equation from point B to C:

$$KE_B + PE_B = KE_C + PE_C$$

Since elevation h_C is desired, solve PE_C and use $PE_C = mgh_C$.

$$\frac{1}{2}(130.0 \text{ lbm})(19.6 \text{ ft/sec})^2 + 0 = 0 + (130.0 \text{ lbm})(32.2 \text{ ft/sec}^2)h_C$$

$$\frac{24{,}970.4 \text{ lbm-ft}^2/\text{sec}^2}{4{,}186.0 \text{ lbm-ft/sec}^2} = h_C$$
$$h_C = 5.96 \text{ ft} \approx 6.0 \text{ ft}$$

42. C $\Sigma M_A = 0$

$$F_{BC} \sin 60(2) - 10(1) - 10(5) = 0$$
$$F_{BC} = 34.64 \text{ kN}$$

Then:

$$\sigma_{AB} = \frac{F_{BC}}{A_{BC}} = \frac{34{,}640}{\frac{\pi}{4}(0.04)^2} = 27.56 \text{ MPa}$$

43. A The maximum moment occurs at the right end of the beam and is equal to

$$M_{\max} = 120 \text{ kN·m}$$

Therefore:

$$\sigma_{\max} = \frac{M_{\max}C}{I} = \frac{120{,}000(0.1)}{\frac{1}{12}(0.1)(0.2)^3} = 180 \text{ MPa}$$

44. B $\tau = \dfrac{V}{A_s} = \dfrac{\frac{34{,}640}{2}}{\frac{\pi}{4}(0.02)^2} = 55.13 \text{ MPa}$

45. A $V_{\max} = 50{,}000$ kN

Then:

$$\tau = \frac{3V}{2A} = \frac{3(50{,}000)}{2(0.1)(0.2)} = 3.75 \text{ MPa}$$

46. A The force on a fixed blade is given by the equation

$$|\mathbf{F}_x| = \dot{m}(|\mathbf{V}_1| - |\mathbf{V}_2| \cdot \cos\alpha)$$

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Because friction along the face of the blade can be neglected, $|\mathbf{V}_1| = |\mathbf{V}_2|$, and

$$|\mathbf{F}_x| = \dot{m}|\mathbf{V}_1|(1 - \cos\alpha)$$
$$\dot{m} = \rho Q$$
$$= \rho|\mathbf{V}_1|A$$
$$= \rho|\mathbf{V}_1|\left(\frac{\pi D^2}{4}\right)$$

and

$$|\mathbf{F}_x| = \rho\left(\frac{\pi D^2}{4}\right)|\mathbf{V}_1|^2(1 - \cos\alpha)$$

Then:

$$|\mathbf{V}_1| = \sqrt{\left[\frac{1}{\rho}\left(\frac{4}{\pi D^2}\right)\left(\frac{1}{1 - \cos\alpha}\right)\right]|\mathbf{F}_x|}$$

$$= \sqrt{\left[\frac{1}{\left(1{,}000\frac{\text{kg}}{\text{m}^3}\right)}\left(\frac{4}{\pi(0.10\text{ m})^2}\right)\left(\frac{1}{1 - \cos 45^\circ}\right)\right](100\text{ N})\left(\frac{\frac{\text{kg}\cdot\text{m}}{\text{s}^2}}{\text{N}}\right)}$$

$$= 6.59\text{ m/s}$$

47. C Find the Reynolds number as follows:

$$\text{Re} = \frac{\rho\bar{V}D}{\mu}$$
$$= \frac{\left(1{,}000\frac{\text{kg}}{\text{m}^3}\right)\left(6.59\frac{\text{m}}{\text{s}}\right)(0.10\text{ m})}{8 \times 10^{-4}\frac{\text{N}\cdot\text{s}}{\text{m}^2}}\left(\frac{\text{N}}{\frac{\text{kg}\cdot\text{m}}{\text{s}^2}}\right)$$
$$= 0.000053 \text{ or } 5.30 \times 10^{-5}$$

48. B The friction factor is determined using the Moody diagram at the end of Chapter 7. The parameters required are the Reynolds number and the relative roughness. The Reynolds number was determined for Problem 47. The relative roughness is found as follows:

$$\frac{e}{D} = \frac{0.25\text{ mm}}{10\text{ cm}}$$
$$= \frac{0.00025\text{ m}}{0.10\text{ m}}$$
$$= 0.0025$$

From the Moody diagram, the friction factor is found to be 0.026.

49. A The head loss in the pipe due to friction is found as follows:

$$h_f = f\left(\frac{L}{D}\right)\left(\frac{\bar{V}^2}{2g}\right)$$
$$= 0.026\left(\frac{75\text{ m}}{0.10\text{ m}}\right)\left(\frac{\left(6.59\frac{\text{m}}{\text{s}}\right)^2}{2\left(9.81\frac{\text{m}}{\text{s}^2}\right)}\right)$$
$$= 43.16\text{ m}$$

50. B Refer to the iron-carbon phase diagram.

51. B Refer to the iron-carbon phase diagram.

52. B Refer to the iron-carbon phase diagram.

53. B At 720°C, iron is body centered cubic. According to Table 8.3, D_0 is 3.9×10^{-7} m²/s and Q is 80 kJ/mol. Therefore:

$$D = D_0 e^{-\frac{Q}{RT}} = 3.9 \times 10^{-7}\, e^{-\frac{80{,}000}{(8.31)(993)}}\text{ m}^2/\text{s}$$
$$= 2.4 \times 10^{-11}\text{ m}^2/\text{s} = 2.4 \times 10^{-7}\text{ cm}^2/\text{s}$$

54. C The most common number of bits per byte is 8.

55. B

2^0	1	
2^1		
2^2		or 10001
2^3		
2^4	16	
	17	

56. B

1st, 2nd	F=18
3rd, 4th	F=17
5th, 6th	F=16
7th, 8th	F=15
9th, 10th	F=14
11th, 12th	F=13
13th	F=12

57. A Choices (B) and (D) are the least reasonable responses. A registered engineer would state membership and a number, making (C) a possible selection. However, (A) is the best choice because the implication is that the applicant is registered, which is unethical.

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58. C Not writing written conclusions, returning moneys paid, and passing time do not make it ethical to accept the other assignment. If all parties had agreed and thus they are informed, the engineer could have accepted the assignment. Choices (A), (B), and (D) are not acceptable.

59. D $AC(2B) = 95K + 3K - 12K(A/F, 10, 3)$
$= 95K + 3K - 12K(0.3021)$
$= \$37,566$

and

$AC(ROT) = 120K + 9K - 25K(A/F,10,6)$
$= 120K + 9K - 25K(0.1296)$
$= \$33,350$

Then:

Difference in annual cost $= \$37,566 - \$33,350$
$= \$4,216$

60. C Use this equation:

Profit = revenues − costs
$= [(35 \cdot \$90) + (15 \cdot \$50)] - [(35+15) \cdot \$40] - \$1,200$
$= [(\$3,150) + (\$750)] - [(50)(\$40)] - \$1,200$
$= \$700$

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